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On Reliability and Ratio in the Beta Case

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Abstract

We consider distribution, reliability and moment of ratio in two independent beta random variables X and Y, and reliability and k^{th} moment of ratio are represented by a mathematical generalized hypergeometric function. We introduce an approximate maximum likelihood estimate(AML) of reliability and right-tail probability in the beta distribution.

Keywords: Beta distribution, generalized hypergeometric function, psi-function, reliability, right tail probability.

1. Introduction

For two independent random variables X and Y, a real number c, the probability $P(X < cY)$ induces the following facts, (i) it is reliability when a real number $c = 1$, (ii) it is a distribution of $X/(X + Y)$ when $c = t/(1-t)$ for $0 < t < 1$ and (iii) we obtain the density of a skewed-symmetric random variable if X and Y are symmetric random variable about origin (Woo, 2006).

The reliability will increase the need for industry to perform systematic study for the identifications and reduction of causes of failures. These reliability studies must be performed by persons who (a) can identify and quantify the modes of failures, (b) know how to obtain and analyze the statistics of failure occurrences, and (c) can construct mathematical models of failure that depend on, for example, the parameters of material strength or design quality, fatigue or wear resistance, and the stochastic nature of the anticipated duty cycle (Saunders, 2007).

Many authors had considered properties of beta distribution in Johnson *et al.* (1994). Ali and Woo (2005) studied inference on reliability in a power function distribution, and Ali *et al.* (2005) studied ratio for a power function distribution. Saunders (2007) introduced reliability, life testing, and prediction of service lives for engineers and scientists. Woo (2007) studied reliability in two independent half-triangle distributions. Son and Woo (2007a, 2007b) studied AML of parameter by means of deriving AML in Balakrishnan and Cohen (1991).

In this paper, we consider a quotient distribution, reliability, and moment of ratio in two independent beta random variables, whose reliability and k^{th} moment of ratio are represented by a mathematical generalized hypergeometric function. We introduce the approximate maximum likelihood estimate(AML) of reliability and right-tail probability in a beta distribution.

2. Quotient Distribution and Reliability

Let X be a beta random variable with the density:

$$f_X(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1,\ \alpha > 0,\ \beta > 0, \tag{2.1}$$

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where $B(a, b)$ is the beta function.

Let $x_1, x_2, \ldots, x_m$ be observed values from the density (2.1) with parameter α and β. Then moment estimates of α and β are well-known in Johnson *et al.* (1994):

$$\tilde{\alpha} = \bar{x}\left(\bar{x} - \frac{1}{m}\sum_{i=1}^{m} x_i^2\right) \Big/ \frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2 \text{ and } \tilde{\beta} = (1-\bar{x})\left(\bar{x} - \frac{1}{m}\sum_{i=1}^{m} x_i^2\right) \Big/ \frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2, \tag{2.2}$$

where $\bar{x} = \sum_{i=1}^{m} x_i/m$.

Let $W = Y/X$. From the quotient density in Rohatgi (1976, p.141) and formula 2.33 in Oberhettinger (1974), we obtain the density of a quotient W:

Fact 1. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1, β_1) and (α_2, β_2), respectively, then the following density of $W = Y/X$ is given by:

If $0 < w \leq 1$,

$$f_W(w) = \begin{cases} \dfrac{B(\alpha_1 + \alpha_2, \beta_1)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot w^{\alpha_2 - 1} \cdot {}_2F_1(1-\beta_2, \alpha_1+\alpha_2; \alpha_1+\alpha_2+\beta_1; w), & \text{if } \beta_2 \neq 1, \\ \dfrac{B(\alpha_1 + \alpha_2, \beta_1)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot w^{\alpha_2 - 1}, & \text{if } \beta_2 = 1, \end{cases}$$

If $w > 1$,

$$f_W(w) = \begin{cases} \dfrac{B(\alpha_1 + \alpha_2, \beta_2)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot w^{-\alpha_1 - 1} \cdot {}_2F_1\left(1-\beta_1, \alpha_1+\alpha_2; \alpha_1+\alpha_2+\beta_2; \dfrac{1}{w}\right), & \text{if } \beta_2 \neq 1, \\ \dfrac{B(\alpha_1 + \alpha_2, \beta_2)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot w^{-\alpha_1 - 1}, & \text{if } \beta_2 = 1, \end{cases} \tag{2.3}$$

where ${}_2F_1(a, b; c; x)$ is the hypergeometric function.

Remark 1. If $\alpha_i = 1$ or $\beta_i = 1$ then the beta distribution is a power function distribution, whose reliability and ratio were considered in Ali and Woo (2005).

From Remark 1, it's sufficient for us to consider it only when $\alpha_i \neq 1 \neq \beta_i$, $i = 1$ and 2. From the density (2.3) of $W = Y/X$ and formula 7.512(5) in Gradshteyn and Ryzhik (1965, p.849), we can obtain k^{th} moment of W:

Fact 2. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1, β_1) and (α_2, β_2), respectively, k^{th} moment of $W = Y/X$ is:

$$\begin{aligned} E\left(W^k\right) = &\frac{B(\alpha_1 + \alpha_2, \beta_1)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot \frac{1}{\alpha_2 + k} \cdot {}_3F_2(1-\beta_2, \alpha_1+\alpha_2 . \alpha_2 + k; \alpha_1+\alpha_2+\beta_1, \alpha_2+1+k; 1) \\ &+ \frac{B(\alpha_1 + \alpha_2, \beta_2)}{B(\alpha_1, \beta_1)B(\alpha_2, \beta_2)} \cdot \frac{1}{\alpha_1 - k} \cdot {}_3F_2(1-\beta_1, \alpha_1+\alpha_2 . \alpha_1 - k; \alpha_1+\alpha_2+\beta_2, \alpha_1+1-k; 1), \end{aligned}$$

if $\alpha_i > k$, $\beta_i > k$, by symmetry of parameters, where ${}_3F_2(a, b, c; d, e; x)$ is the generalized hypergeometric function.

From Fact 2 and formula 9.14(1) in Abramowitz and Stegun (1970, p.1045), Table 1 in the Appendix gives approximate numerical values of mean, variance and skewness of a quotient $W = Y/X$.

Table 1: Mean, variance, and skewness of the quotient $W = Y/X$.

(α_1, β_1) (α_2, β_2)	(9.0, 10.0) (3.5, 5.5)	(9.0, 10.0) (4.5, 7.5)	(9.5, 10.5) (5.5, 7.5)	(9.0, 9.0) (10.0, 10.0)	(3.5, 4.5) (4.0, 5.0)
mean	0.87500	0.84375	0.94570	1.06250	1.24440
variance	0.19063	0.15502	0.15943	0.14320	0.94070
skewness	0.45784	0.55533	0.51530	0.65346	4.12790

From Table 1, we observe that the density (2.3) is skewed to the right when true parameters are as given in Table 1.

Next we consider reliability $P(Y < X)$ when X and Y are independent random variables which they have the density (2.1) with parameters (α_1, β_1) and (α_2, β_2), respectively. From formula 7.512(5) in Gradshteyn and Ryzhik (1965, p.849) and the density (2.3) in Fact 1, reliability is obtained as:

Fact 3. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1, β_1) and (α_2, β_2), respectively, reliability $P(Y < X)$ is given:

$$P(Y < X) = \frac{B(\alpha_1 + \alpha_2, \beta_1)}{B(\alpha_1, \beta_1) \cdot B(\alpha_2, \beta_2) \cdot \alpha_2} \cdot {}_3F_2(1 - \beta_2, \alpha_1 + \alpha_2 . \alpha_2; \alpha_1 + \alpha_2 + \beta_1, \alpha_2 + 1; 1). \quad (2.4)$$

Remark 2. From Fact 3, especially if X and Y are *iid* rv's (that is, if $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$), then $P(Y < X) = 1/2$ is obvious.

Now we consider right tail probability of a beta random variable X with the density (2.1), right tail probability $R(t) = P(X > t)$ of X is:

$$R(t) = I_{1-t}(\alpha, \beta), \quad 0 < t < 1, \quad (2.5)$$

where $I_x(a, b)$ is the formula 6.6.2 in Abramowitz and Stegun (1970).

Because AML of a parameter is more efficient in a sense of mean squared error than moment estimator (Son and Woo, 2007a, 2007b)), and as usual AML is more efficient than other estimates (MLE, *etc.* in Balakrishnan and Cohen, 1991), we've got a satisfaction with introducing estimates of reliability (2.4) and right-tail probability (2.5) by means of AML of parameters:

For a log-likelihood function $l(\alpha, \beta)$ based on observed values $x_1, x_2, \ldots, x_m$ from the density (2.1), we obtain the following approximate forms by Taylor series for $h(\alpha, \beta) \equiv \partial/\partial\alpha l(\alpha, \beta) = 0$ and $g(\alpha, \beta) \equiv \partial/\partial\beta l(\alpha, \beta) = 0$:

$$0 = \frac{\partial l(\alpha, \beta)}{\partial \alpha} \equiv h(\alpha, \beta) \approx h(a, b) + \left(\frac{\partial h(a, b)}{\partial \alpha}(\alpha - a) + \frac{\partial h(a, b)}{\partial \beta}(\beta - b)\right), \quad \forall\, (a, b) \in R^2 \text{ and}$$
$$0 = \frac{\partial l(\alpha, \beta)}{\partial \beta} \equiv g(\alpha, \beta) \approx g(a, b) + \left(\frac{\partial g(a, b)}{\partial \alpha}(\alpha - a) + \frac{\partial g(a, b)}{\partial \beta}(\beta - b)\right), \quad \forall\, (a, b) \in R^2, \quad (2.6)$$

where a and b are moment estimates (2.2) of α and β, respectively, and $h(\alpha, \beta) = m[\psi(\alpha + \beta) - \psi(\alpha)] + \sum_{i=1}^{m} \ln x_i$, and $g(\alpha, \beta) = m[\psi(\alpha + \beta) - \psi(\alpha)] + \sum_{i=1}^{m} \ln(1 - x_i)$, where $\psi(x)$ is the psi-function.
From two Equations (2.6), AML $\hat{\alpha}$ and $\hat{\beta}$ of α and β are obtained as:

$$\hat{\alpha} \approx a - \left[h(a, b) \cdot \frac{\partial g(a, b)}{\partial \beta} - g(a, b) \cdot \frac{\partial h(a, b)}{\partial \beta}\right] \Big/ \left[\frac{\partial h(a, b)}{\partial \alpha} \cdot \frac{\partial g(a, b)}{\partial \beta} - \frac{\partial h(a, b)}{\partial \beta} \cdot \frac{\partial g(a, b)}{\partial \alpha}\right] \text{ and}$$
$$\hat{\beta} \approx b - \left[h(a, b) \cdot \frac{\partial g(a, b)}{\partial \alpha} - g(a, b) \cdot \frac{\partial h(a, b)}{\partial \alpha}\right] \Big/ \left[\frac{\partial h(a, b)}{\partial \beta} \cdot \frac{\partial g(a, b)}{\partial \alpha} - \frac{\partial h(a, b)}{\partial \alpha} \cdot \frac{\partial g(a, b)}{\partial \beta}\right]. \quad (2.7)$$

Let $x_1, x_2, \ldots, x_m$ and $y_1, y_2, \ldots, y_n$ be independent observed values from the density (2.1) each with parameters (α_1, β_1) and (α_2, β_2), respectively. As we replace AML $\hat{\alpha}_i$ and $\hat{\beta}_i$ instead of α_i and β_i (i = 1 and 2) in reliability (2.4) and right tail probability (2.5), we obtain the following estimates of reliability and right-tail probability:

(A) An estimate of the reliability (2.4) in Fact 3 is given as:

$$\widehat{P(Y < X)} = \frac{B\left(\hat{\alpha}_1 + \hat{\alpha}_2, \hat{\beta}_1\right)}{B\left(\hat{\alpha}_1, \hat{\beta}_1\right) \cdot B\left(\hat{\alpha}_2, \hat{\beta}_2\right) \cdot \hat{\alpha}_2} \cdot {}_3F_2\left(1 - \hat{\beta}_2,\ \hat{\alpha}_1 + \hat{\alpha}_2.\hat{\alpha}_2;\ \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\beta}_1,\ \hat{\alpha}_2 + 1;\ 1\right).$$

(B) An estimate of the right tail probability (2.5) is given as:

$$\widehat{R(t)} = \widehat{P(X < t)} = I_{1-t}\left(\hat{\alpha}, \hat{\beta}\right), \quad 0 < t < 1, \qquad \text{where } \hat{\alpha} \equiv \hat{\alpha}_1 \text{ and } \hat{\beta} \equiv \hat{\beta}_1 \text{are given,}$$

which AML $\hat{\alpha}_i$ and $\hat{\beta}_i$ (i = 1 and 2) are given by (2.6) and (2.7) as followings:

$$\hat{\alpha}_1 = a - \frac{1}{m} \cdot \frac{[h(a,b) - g(a,b)]\,\psi'(a+b) - h(a,b)\psi'(b)}{\psi'(a)\psi'(b) - \psi'(a+b)[\psi'(a) + \psi'(b)]} \quad \text{and}$$

$$\hat{\beta}_1 = b - \frac{1}{m} \cdot \frac{[h(a,b) - g(a,b)]\psi'(a+b) + g(a,b)\psi'(a)}{\psi'(a+b)[\psi'(a) + \psi'(b)] - \psi'(a)\psi'(b)}$$

and

$$\hat{\alpha}_2 = c - \frac{1}{n} \cdot \frac{[p(c,d) - q(c,d)]\,\psi'(c+d) - p(c,d)\psi'(d)}{\psi'(c)\psi'(d) - \psi'(c+d)[\psi'(c) + \psi'(d)]} \quad \text{and}$$

$$\hat{\beta}_2 = d - \frac{1}{n} \cdot \frac{[p(c,d) - q(c,d)]\psi'(c+d) + q(c,d)\psi'(c)}{\psi'(c+d)[\psi'(c) + \psi'(d)] - \psi'(c)\psi'(d)},$$

where $\psi'(x) = d\psi(x)/dx$.

$$a = \bar{x}\left(\bar{x} - \sum_{i=1}^{m} \frac{x_i^2}{m}\right) \Big/ \left[\sum_{i=1}^{m} \frac{(x_i - \bar{x})^2}{m}\right], \qquad b = (1 - \bar{x})\left(\bar{x} - \sum_{i=1}^{m} \frac{x_i^2}{m}\right) \Big/ \left[\sum_{i=1}^{m} \frac{(x_i - \bar{x})^2}{m}\right],$$

$$p(c,d) = n[\psi(c+d) - \psi(c)] + \sum_{i=1}^{n} \ln y_i, \qquad q(c,d) = n[\psi(c+d) - \psi(c)] + \sum_{i=1}^{n} \ln(1 - y_i),$$

$$c = \bar{y}\left(\bar{y} - \sum_{i=1}^{n} \frac{y_i^2}{n}\right) \Big/ \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n}, \qquad d = (1 - \bar{y})\left(\bar{y} - \sum_{i=1}^{n} \frac{y_i^2}{n}\right) \Big/ \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n}.$$

3. The Ratio Distribution

If the density (2.1) has $\alpha = 1$ or $\beta = 1$then the beta distribution is a power function distribution, whose ratio was considered by Ali and Woo (2005). Hence, it's sufficient for us to consider distribution of the ratio only when $\alpha \neq 1 \neq \beta$.

From the density (2.3) in Fact 1, the density of $V = X/(X + Y)$ can be obtained as:

Fact 4. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1, β_1) and (α_2, β_2), respectively, then the following density of $V = X/(X + Y)$ is given by:

$$f_V(v)=\begin{cases}\dfrac{B(\alpha_1+\alpha_2,\beta_1)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)}\left(\dfrac{1-v}{v}\right)^{\alpha_2-1}{}_2F_1\left(1-\beta_2,\alpha_1+\alpha_2;\alpha_1+\alpha_2+\beta_1;\dfrac{1-v}{v}\right)\dfrac{1}{v^2}, & \text{if } \dfrac{1}{2}\le v<1\\[2ex] \dfrac{B(\alpha_1+\alpha_2,\beta_2)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)}\left(\dfrac{1-v}{v}\right)^{-(\alpha_1-1)}{}_2F_1\left(1-\beta_1,\alpha_1+\alpha_2;\alpha_1+\alpha_2+\beta_2;\left(\dfrac{1-v}{v}\right)^{-1}\right)\dfrac{1}{(1-v)^2}, & \text{if } 0<v<\dfrac{1}{2}\end{cases} \tag{3.1}$$

Especially if $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then distribution of the ratio V is symmetric about 1/2.

From the density (3.1) of $V = X/(X + Y)$ in Fact 4, formulas 1.110 and 7.512(5) in Gradshteyn and Ryzhik (1965, p.21 & p.849, respectively), we obtain k^{th} moment of ratio $V = X/(X + Y)$:

Fact 5. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1,β_1) and (α_2,β_2), respectively, then k^{th} moment of $V = X/(X + Y)$ is: For $\alpha_i > k$ and $\beta_i > k$, $i = 1$ and 2,

$$\begin{aligned}E\left(V^k\right)=\sum_{j=0}^{\infty}\binom{-k}{j}\cdot\Bigg[&\frac{B(\alpha_1+\alpha_2,\beta_1)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_2+j)}{}_3F_2(1-\beta_2,\alpha_1+\alpha_2,\alpha_2+j;\alpha_1+\alpha_2+\beta_1,\\ &\alpha_2+1+j;1)+\frac{B(\alpha_1+\alpha_2,\beta_2)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_1+k+j)}{}_3F_2(1-\beta_1,\alpha_1+\alpha_2,\alpha_1+k+j;\\ &\alpha_1+\alpha_2+\beta_2,\alpha_1+k+j+1;1)\Bigg].\end{aligned}$$

where $\binom{-k}{j}\equiv(-k)(-k-1)\cdots(-k-j+1)/j!$.

Especially if $k = 1$ and 2, from formulas 1.112 (1) and (2) in Gradshteyn and Ryzhik (1965, p.21), then Ist and 2^{nd} moments of ratio V is specified as the following:

Fact 6. If X and Y are independent random variables which they have the density (2.1) with parameters (α_1,β_1) and (α_2,β_2), respectively, then expectation and 2^{nd} moment of $V = X/(X + Y)$ is:

$$\begin{aligned}E(V)=\sum_{j=0}^{\infty}(-1)^j\cdot\Bigg[&\frac{B(\alpha_1+\alpha_2,\beta_1)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_2+j)}{}_3F_2(1-\beta_2,\alpha_1+\alpha_2,\alpha_2+j;\alpha_1+\alpha_2+\beta_1,\alpha_2+1+j;1)\\ &+\frac{B(\alpha_1+\alpha_2,\beta_2)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_1+1+j)}{}_3F_2(1-\beta_1,\alpha_1+\alpha_2,\alpha_1+1+j;\alpha_1+\alpha_2+\beta_2,\alpha_1+j+2;1)\Bigg],\end{aligned}$$

$$\begin{aligned}E\left(V^2\right)=\sum_{j=0}^{\infty}(j+1)(-1)^j\Bigg[&\frac{B(\alpha_1+\alpha_2,\beta_1)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_2+j)}{}_3F_2(1-\beta_2,\alpha_1+\alpha_2,\alpha_2+j;\alpha_1+\alpha_2+\beta_1,\alpha_2+1+j;1)\\ &+\frac{B(\alpha_1+\alpha_2,\beta_2)}{B(\alpha_1,\beta_1)B(\alpha_2,\beta_2)\cdot(\alpha_1+2+j)}{}_3F_2(1-\beta_1,\alpha_1+\alpha_2,\alpha_1+2+j;\alpha_1+\alpha_2+\beta_2,\alpha_1+j+3;1)\Bigg].\end{aligned}$$

Table 2: Mean, variance and skewness of the ratio $V = X/(X+Y)$.

(α_1, β_1) / (α_2, β_2)	mean	variance	skewness
(6.0, 6.0) / (7.0, 7.0)	0.49857	0.00954	0.02130
(9.5, 10.5) / (5.5, 7.5)	0.53363	0.01003	0.15695
(9.0, 10.0) / (4.5, 6.5)	0.54338	0.01178	0.18819
(9.0, 10,0) / (3.5, 5.5)	0.55985	0.01416	0.22903
(9.0, 10.0) / (4.5, 7.5)	0.56465	0.01192	0.19319
(4.0, 5.0) / (6.0, 7.0)	0.48541	0.01446	−0.16762
(3.5, 4.5) / (4.0, 5.0)	0.49388	0.01884	−0.09174
(5.5, 6.5) / (6.5, 7.5)	0.49502	0.01144	−0.18836
(3.5, 3.5) / (4.5, 4.5)	0.49638	0.01629	−0.16416
(5.5, 5.5) / (6.5, 6.5)	0.49835	0.01075	−0.20464
(9.0, 9.0) / (10.0, 10.0)	0.49933	0.00672	−0.24356

Remark 3. The generalized hypergeometric function is numerically evaluated by formula 9.14(1) in Abramowitz and Stegun (1970, p.1045), when α_i and β_i $(i = 1$ and $2)$ are greater than k.

From Fact 5 and formula 9.14(1) in Abramowitz and Stegun (1970, p.1045), Table 2 in the Appendix provides approximate numerical values of mean, variance, and coefficient of skewness of ratio $V = X/(X+Y)$ when true parameters vary. From Table 2, we observe the following:

Fact 7. (a) The density (3.1) of ratio is symmetric about 0.5 when $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, (b) the density (3.1) is skewed to the right when (α_1, β_1) and (α_2, β_2) are (6.0, 6.0) and (7.0, 7.0), (9.5, 10.5) and (5.5, 7.5), (9.0, 10.0) and (4.5, 6.5), (9.0, 10,0) and (3.5, 5.5), (9.0, 10.0) and (4.5, 7.5), respectively, and the density (3.1) is skewed to the left when (α_1,β_1) and (α_2,β_2) are (4.0, 5.0) and (6.0, 7.0), (3.5, 4.5) and (4.0, 5.0), (5.5, 6.5) and (6.5, 7.5), (3.5, 3.5) and (4.5, 4.5), (5.5, 5.5) and (6.5, 6.5), (9.0, 9.0) and (10.0, 10.0), respectively.

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