On the Complex-Valued Recursive Least Squares Escalator Algorithm with Reduced Computational Complexity

Namyong Kim* *Lifelong Member*

ABSTRACT

In this paper, a complex-valued recursive least squares escalator filter algorithm with reduced computational complexity for complex-valued signal processing applications is presented. The local tap weight of RLS-ESC algorithm is updated by incrementing its old value by an amount equal to the local estimation error times the local gain scalar, and for the gain scalar, the local input autocorrelation is calculated at the previous time. By deriving a new gain scalar that can be calculated by using the current local input autocorrelation, reduced computational complexity is accomplished. Compared with the computational complexity of the complex-valued version of RLS-ESC algorithm, the computational complexity of the proposed method can be reduced by 50% without performance degradation. The reduced computational complexity of the proposed algorithm is even less than that of the LMS-ESC. Simulation results for complex channel equalization in 64QAM modulation schemes demonstrate that the proposed algorithm has superior convergence and constellation performance.

Key Words : Least-square; Escalator; Complexity; QAM; Complex channel; Equalization.

I. Introduction

The escalator (ESC) filter structure orthogonalizes the input signal using Gram-Schmidt orthogonalization procedure[1]. The ESC using least mean square (LMS-ESC) in which mean squared local estimation errors are minimized was used in the multi-channel filtering problems[2]. In [3], the LMS-ESC has been used in order to improve the performance of the transform-domain LMS (TRLMS) adaptive filtering. The method improves the performance of the TRLMS algorithm by eliminating nontrivial nondiagonal entrees of the correlation matrix of the transformed input process by using the escalator structure. computational complexity of the escalator structure is inherently $O(N^2)$ for the filter length N but depends on the sparseness of the correlation matrix of the transformed vector. In the case of DWT, it is $O(N\log N)$ operations per iteration[3].

Recently, by introducing least squares (LS) approach to the local errors of the ESC structure, a recursive least squares-ESC (RLS-ESC) for real-valued signal processing has been proposed[4]. The RLS-ESC algorithm has faster convergence than the LMS-ESC algorithm but it requires four third times the computational complexity of the LMS-ESC. The escalator structure has superior performance, but its adaptation algorithms require more studies for reducing computational complexity and for some complex-valued signal processing applications. The present study proposes a complex-valued recursive least squares ESC (CRLS-ESC) algorithm that has reduced computational complexity significantly even less than the LMS-ESC.

This paper is organized as follows. In Section II, we briefly describe the escalator filter structure. In Section III, algorithms for escalator filter structure are described. The complex-valued recursive least squares ESC (CRLS-ESC) algorithm that has reduced computational complexity is proposed in Section IV. Simulation results are presented in Section V, and conclusions are drawn in Section VI.

* 강원대학교 공학대학 정보통신공학과(namyong@kangwon.ac.kr)
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Ⅱ. Escalator Filter Structure

Given a symmetric matrix R, there exists a unit lower triangular (ULT) matrix W, such that WRW^T is a diagonal matrix. The ULT matrix W can be computed in the form of $W = W_N W_{N-1} \ldots W_1$. The ULT transformation $Y(k) = W \cdot X(k)$ means that system W generates the uncorrelated output vector $Y(k)$ for the N - dimensional vector $X(k)$ where its symmetric autocorrelation matrix $R = E[X(k)X^T(k)]$. If we define $X(k)$ as an input vector augmented with the desired sample $d(k)$, $X(k) = [x(k-N+1), x(k-N+2), \ldots, x(k), d(k)]^T$ and $Y(k) = [y(k-N+1), y(k-N+2), \ldots, y(k), e(k)]^T$ as an output vector augmented with the error sample $e(k)$, $e(k) = d(k) - y(k)$, $Y(k) = W \cdot X(k)$ becomes filtering process of ESC structure.

We can realize the ULT transformation sequentially like $Y_1(k) = W_1 \cdot X(k)$, $Y_2(k) = W_2 \cdot Y_1(k)$ and $Y_3(k) = W_3 \cdot Y_2(k)$ etc. The final stage's output vector $Y_N(k)$ becomes $Y(k)$. The corresponding ESC filter realization for N=3 (N is the total number of stages) is shown in Fig. 1 and the general ESC filter equations for the weight $\alpha_j^i(k)$ are

$$y_{i,j}(k-m) = y_{i-1,j+1}(k-m) - \alpha_j^i(k) y_{i-1,1}(k-n),$$
$$y_{0,N+1}(k+1) = d(k), y_{0,j}(\cdot) = x(\cdot), y_{i,j}(k+1) = e_i(k), \quad (1)$$

for $1 \le i \le n$, $1 \le j \le N-i+1$, $m = N-i-j$, $n = N-i$ and i is the stage number. This ESC filter structure orthogonalizes the input signal vector.

Ⅲ. Adaptive Escalator Algorithms

The adaptive implementation to estimate the weight $\alpha_j^i(k)$ exploits the fact that the prediction errors at each stage, $y_{i,j}(k-m)$, are local and utilizes the method of steepest descent to minimize the square of these errors. With time-varying convergence parameters $\mu_i(k)$, LMS-ESC algorithm[3] is

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \mu_i(k) y_{i,j}(k-m) y_{i-1,1}(k-n) \quad (2)$$

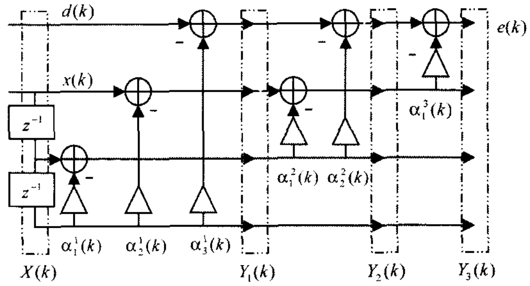

Fig. 1. ESC filter structure for N=3

where, $\mu_i(k) = 2\mu / v_y{}^i(k)$ and $v_y{}^i(k)$ is estimated using a recurrence relation given by

$$v_y{}^i(k) = \theta v_y{}^i(k-1) + (1-\theta) y_{i-1,1}^2(k-n) \quad (3)$$

and $0 < \theta < 1$.

Building an extension to (2) and (3), the complex form of LMS-ESC algorithm (CLMS-ESC) can be acquired (the asterisk designates complex conjugate).

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \frac{2\mu}{v_y{}^i(k)} y_{i,j}(k-m) y_{i-1,1}^*(k-n) \quad (4)$$

where μ is convergence parameter, and

$$v_y{}^i(k) = \theta v_y{}^i(k-1) + (1-\theta) | y_{i-1,1}(k-n) |^2 \quad (5)$$

For faster convergence than the LMS algorithm, the recursive least squares (RLS) algorithm can be applied to ESC filter weight adaptation with some modification. Adopting LS criterion to the local ESC filter structure for updating $\alpha_j^i(k)$, the performance index $J(k)$ to be minimized is

$$J(k) = \sum_{p=0}^{k} w^{k-p} y_{ij}^2(p-m)$$
$$= \sum_{p=0}^{k} w^{k-p} [y_{i-1,j+1}(p-m) - y_{i-1,1}(p-n) \cdot \alpha_j^i(k)]^2 \quad (6)$$

where w represents a weighting factor $0 < w < 1$.

Minimization of $J(k)$ in (6) with respect to $\alpha_j^i(k)$ yields

$$\alpha_j^i(k) = \frac{\sum_{p=0}^{k} w^{k-p} y_{i-1,j+1}(p-m) y_{i-1,1}(p-n)}{\sum_{p=0}^{k} w^{k-p} y_{i-1,1}^2(p-n)} \quad (7)$$

Defining the numerator and denominator in (7) as $A(k)$ and $B(k)$, respectively, $\alpha_j^i(k)$ can be

$$\alpha_j^i(k) = \frac{A(k)}{B(k)} \quad (8)$$

The weight $\alpha_j^i(k)$ can be computed in recursive forms[4] as

$$\alpha_j^i(k) = \alpha_j^i(k-1) + g(k) y_{i,j}(k-m) \quad (9)$$

where,

$$g(k) = \frac{y_{i-1,1}(k-n) B^{-1}(k-1)}{w + y_{i-1,1}^2(k-n) B^{-1}(k-1)} \quad (10)$$

and

$$B^{-1}(k)$$
$$= w^{-1} \cdot [B^{-1}(k-1) - g(k) y_{i-1,1}(k-n) B^{-1}(k-1)] \quad (11)$$

The initial value of $B(k)$ is a small positive constant to avoid $B(k)$ from being ill-conditioned. The RLS-ESC algorithm consisting of (9) and (10) has faster convergence than the LMS-ESC algorithm but it requires four thirds times the computational complexity of the LMS-ESC[4].

Ⅳ. Complex-valued RLS-ESC (CRLS-ESC) Algorithm with Reduced Computational Complexity

The least-squares performance index for complex signals can be expressed as

$$Q(k) = \sum_{p=0}^{k} w^{k-p} \, | y_{ij}(p-m) |^2 \quad (12)$$

Defining complex-valued version of $A(k)$ as $C(k)$ and complex-valued version of $B(k)$ as $D(k)$, complex-valued $\alpha_j^i(k)$, $\beta_j^i(k)$, can be

$$\beta_j^i(k) = \frac{C(k)}{D(k)} \quad (13)$$

where $C(k)$ and $D(k)$ can be expressed recursively in time as

$$C(k) = w \cdot C(k-1) + y_{i-1,j+1}(k-m) y_{i-1,1}^*(k-n) \quad (14)$$

$$D(k) = w \cdot D(k-1) + | y_{i-1,1}(k-n) |^2 \quad (15)$$

The denominator $D(k)$ is a local input autocorrelation which is weighted by the exponential factor w^{k-p}.

The inverse of (15) can be rearranged as

$$D^{-1}(k) = [w \cdot D(k-1) + | y_{i-1,1}(k-n) |^2]^{-1}$$
$$= w^{-1} \cdot [D^{-1}(k-1) - f(k) y_{i-1,1}(k-n) D^{-1}(k-1)] \quad (16)$$

where,

$$f(k) = \frac{y_{i-1,1}^*(k-n) D^{-1}(k-1)}{w + | y_{i-1,1}(k-n) |^2 \, D^{-1}(k-1)} \quad (17)$$

Instead of (10), using (17) and (16), we can acquire a complex form of RLS-ESC algorithm.

$$\beta_j^i(k) = \beta_j^i(k-1) + f(k) y_{i,j}(k-m) \quad (18)$$

The local tap weight $\beta_j^i(k)$ is updated by incrementing its old value by an amount equal to the a local estimation error $y_{i,j}(k-m)$ times the local gain scalar $f(k)$.

Equation (17) can be rearranged into

$$y_{i-1,1}^*(k-n) D^{-1}(k-1)$$
$$= f(k)[w + | y_{i-1,1}(k-n) |^2 \, D^{-1}(k-1)] \quad (19)$$

By multiplying the both sides of (16) with $y_{i-1,1}^*(k-n)$, we acquire

$$D^{-1}(k) y_{i-1,1}^*(k-n) = w^{-1} \cdot [D^{-1}(k-1) y_{i-1,1}^*(k-n)$$

$$- f(k)\,|\,y_{i-1,1}(k-n)\,|^2\, D^{-1}(k-1)] \qquad (20)$$

Replacing $D^{-1}(k-1)y^{*}_{i-1,1}(k-n)$ in (20) with (19), (20) becomes

$$D^{-1}(k)y^{*}_{i-1,1}(k-n) = f(k) \qquad (21)$$

It is noticeable that the local input autocorrelation for the conventional RLS-ESC is calculated at the previous time as described in (17) but the new gain scalar (21) is calculated by using the current local input autocorrelation.

Substituting (21) for $f(k)$ in (18), a complex-valued RLS-ESC algorithm with reduced computational complexity (CRLS-ESC) can be obtained as

$$\beta^{i}_{j}(k) = \beta^{i}_{j}(k-1) + D^{-1}(k)y^{*}_{i-1,1}(k-n)y_{i,j}(k-m)$$
$$= \beta^{i}_{j}(k-1) + \frac{y^{*}_{i-1,1}(k-n)y_{i,j}(k-m)}{D(k)} \qquad (22)$$

where $D(k)$ is computed by (15).

The initial value of $D(k)$ is also a small positive constant to avoid $D(k)$ from being ill-conditioned.

The ESC filter structure with N stages inherently has $0.5(N^2+N)$ weights. The number of complex multiplications and divisions of the proposed algorithm can be computed in (22) and (15). The proposed algorithm, therefore, requires only $2(N^2+N)$. On the other hand, the number of complex computations in RLS-ESC using (17) and (16) is $4(N^2+N)$. It can be noticed that the proposed method can reduce the computational burden of the CRLS-ESC algorithm by half. Compared with the computations in (4) and (5), it has computational complexity even less than the LMS-ESC. The proposed method can be a successful alternative to the escalator coefficient adaptation algorithms in order to improve the performance and reduce the computational complexity of the transform-domain escalator filtering[3] which has $O(N\log N)$ operations in case of DWT and uses the LMS-ESC.

V. Simulation Results and Discussion

In this section the performance of proposed algorithm is investigated in complex channel equalization for 64 QAM modulation schemes. The three complex algorithms are considered: the complex LMS algorithm in tapped delay line (CLMS-TDL), CLMS-ESC in (4) and CRLS-ESC in (20). The complex channel models[5] are

CH 1:

$$H_1(z) = (0.944 - j0.87)[1-(0.0787+j0.0768)z]\cdot$$
$$[1-(0.188+j0.075)z^{-1}][1+(0.158+j0.426)z^{-1}] \qquad (23)$$

CH 2:

$$H_2(z) = (0.54 - j0.63)[1-(0.675-j0.145)z]\cdot$$
$$[1-(0.15+j0.28)z][1+0.2z^{-1}] \qquad (24)$$

The CLMS-TDL algorithm has 22 tap weights. The CLMS-ESC and the CRLS-ESC algorithm consist of 22 stages. A zero mean white Gaussian noise sequence with variance 0.001 is added to yield the equalizer input. The convergence parameters 2μ of the CLMS-TDL are 0.02 and 0.05 for CH 1 and CH 2, respectively. This choice of convergence parameter results in approximately the same minimum MSE as that of the ESC algorithms. The convergence parameter 2μ and smoothing parameter θ of CLMS-ESC are 0.02 and 0.98, respectively. The weighting factor w for CRLS-ESC is 0.98. The convergence results are illustrated in Fig. 2 and 3, and their constellation performance for 64 QAM constellation at 300 samples for CH 2 is depicted in Fig. 4-6. The results show that the proposed algorithm has superior convergence performance in complex channel equalization.

To choose proper weighting factor w for the proposed, MSE performance comparison for the varying parameter value in CH 2 is presented in Fig. 7. Clearly, small weighting factor makes the performance fast and large factor induces slow

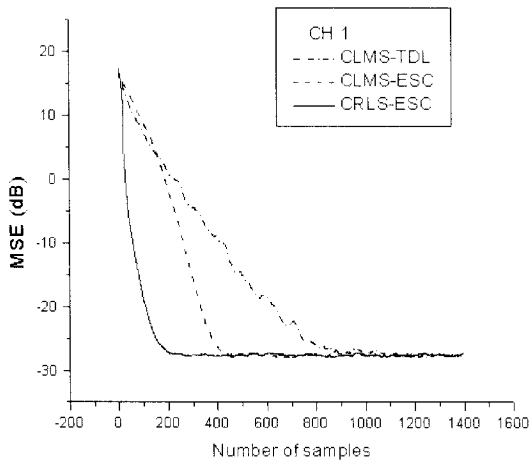

Fig. 2. MSE convergence performance in CH 1.

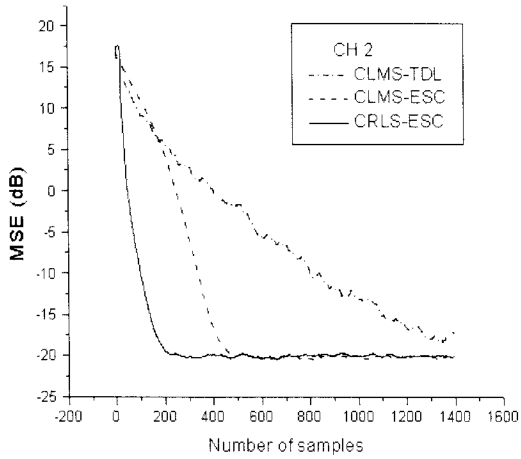

Fig. 3. MSE convergence performance in CH 2.

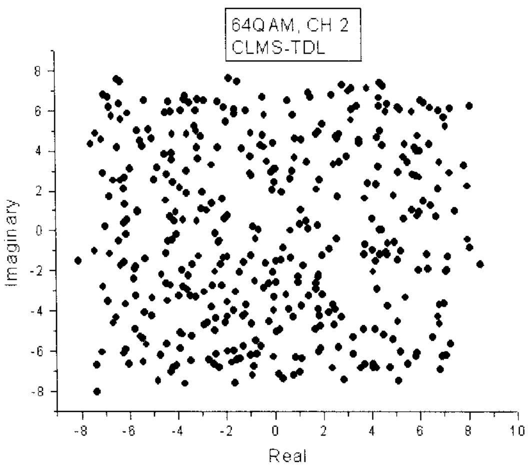

Fig. 4. 64 QAM constellation performance of CLMS-TDL.

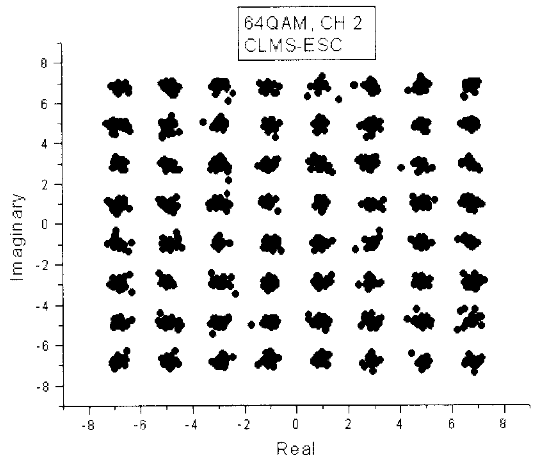

Fig. 5. 64 QAM constellation performance of CLMS-ESC.

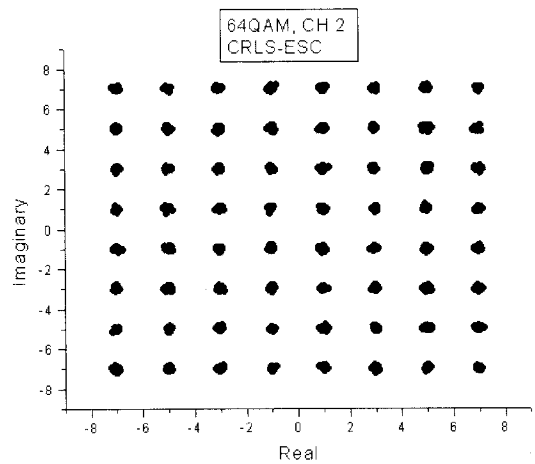

Fig. 6. 64 QAM constellation performance of CRLS-ESC.

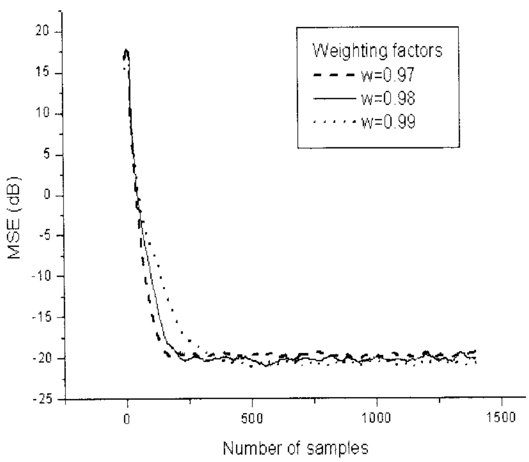

Fig. 7. MSE convergence comparison with varying weighting factor of the proposed algorithm.

learning speed but decreased minimum MSE. In this simulation, 0.98 is chosen as the best factor and it is noticeable that a more in-depth research on the effect of parameters on the proposed algorithm is needed.

Ⅵ. Conclusions

In this paper, a new escalator-weight adaptation algorithm for complex-valued signal processing is presented and the problem of designing RLS-ESC filter algorithms with reduced complexity was investigated. The computational complexity of the proposed algorithm is reduced by 50% without performance degradation comparing to complex-valued version of original RLS-ESC. It even has lower computational complexity than the LMS-ESC. In order to improve the performance and reduce the computational complexity of the transform-domain escalator filtering, the proposed method can be a successful alternative to the escalator coefficient adaptation algorithms. Simulation results for complex channel equalization in 64 QAM modulation schemes demonstrate that the proposed algorithm has superior convergence performance and is readily applicable to complex-valued signal processing applications.

References

〔1〕 Ahmed and D. H. Youn, "On realization and related algorithm for adaptive prediction." IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-28, pp. 493-497, Oct. 1980.

〔2〕 K. M. Kim, I. W. Cha and D. H. Youn, "Adaptive Multichannel Digital Filter with Lattice-Escalator Hybrid Structure," Proceedings of the 1990 ICASSP, New Mexico, USA, Vol. 3, pp. 1413-1416, April. 1990.

〔3〕 V. N. Parikh and A. Z. Baraniecki, "The Use of the Modified Escalator Algorithm to Improve the Performance of Transform-Domain LMS Adaptive Filters."IEEE Trans. on Signal Processing, vol. 46, No. 3, pp. 625-635, March 1998.

〔4〕 N. Kim, "A Least Squares Approach to Escalator Algorithms for Adaptive Filtering", ETRI Journal, vol. 28, No. 2, pp. 155-161, April 2006.

〔5〕 S. Haykin, Adaptive Filter Theory, Prentice Hall, Upper Saddle River, 4th edition, 2001.

〔6〕 D. Hatzinakos and C. L. Nikias, "Blind Equalization Using a Triceptrum-Based Algorithm", IEEE Trans. on Communications, vol. 39, pp. 669-682, May 1991.

김 남 용 (Namyong Kim) 종신회원

1986년 2월 연세대학교 전자공학과 졸업

1988년 2월 연세대학교 전자공학과 석사

1991년 8월 연세대학교 전자공학과 박사

1992년 8월~1998년 2월 관동대학교 전자통신공학과 부교수

1998년 3월~현재 강원대학교 공학대학 정보통신공학과 교수

<관심분야> Adaptive equalization, RBFN algorithms, ITL algorithms, Odor sensing systems.