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FUZZY RISK MEASURES AND ITS APPLICATION TO PORTFOLIO OPTIMIZATION

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ABSTRACT. In possibility framework, we propose two risk measures named Fuzzy Value-at-Risk and Fuzzy Conditional Value-at-Risk, based on Credibility measure. Two portfolio optimization models for fuzzy portfolio selection problems are formulated. Then a chaos genetic algorithm based on fuzzy simulation is designed, and finally computational results show that the two risk measures can play a role in possibility space similar to Value-at-Risk and Conditional Value-at-Risk in probability space.

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1. Introduction

Risk measurement is essentially important factor in financial decision making under uncertainty which is generally understood to have two aspects: probability uncertainty and fuzzy uncertainty. Most of the existing risk measures are based on the probability theory. Variance was first proposed by Markowitz to measure the risk associated with the return of assets in probability framework. Since the middle of 1990s, Value-at-Risk (VaR), a measure of downside risk, has become popular in financial risk management. It has even been recommended as a standard on banking supervision by Basel Committee[17]. One can find plenty of materials on the theory, modeling, algorithms, and applications related to VaR at http://www.gloriamundi.org which is updated on-line. As an alternative measure of risk, Conditional Value-at-Risk (CVaR), a coherent risk measure[15], defined as the mean of the tail distribution exceeding VaR, has been proposed as a natural remedy for the deficiencies of VaR which is not a coherent risk measure in general in the sense of Artzner et al.[3]. Moreover, minimizing

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CVaR can be achieved by minimizing a more tractable auxiliary function without predetermining the corresponding VaR first, and at the same time, VaR can be calculated as a by-product[18, 19]. Up to now, VaR and CVaR are investigated and applied extensively in financial management[1, 2, 8, 14].

Though probability theory is one of the main tools used for analyzing uncertainty in finance, it cannot describe uncertainty completely since there are some other uncertain factors that differ from the random ones found in financial markets. In reality, many events with fuzziness are characterized by probabilistic approaches although they are not random events [22]. Some other techniques have also been applied to handle the uncertainty of the financial markets, for instance, fuzzy set theory [25]. Fuzzy set theory provides a framework to deal with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables and provides an excellent framework for analysis. Fuzzy set theory has been widely used to solve many practical problems including financial risk management. By using fuzzy approaches, quantitative analysis, qualitative analysis, the experts' knowledge and the investors' subjective opinions can be better integrated in a financial optimization model. Recently, a few authors, such as Ramaswamy [16], Tanaka and Guo [20] and Carlsson and Fuller[5] studied fuzzy financial optimization problem. Inuiguchi and Ramik [9] surveyed the advantages and disadvantages of such mathematical programming approaches compared with stochastic programming and reviewed the newly developed ideas and techniques in fuzzy mathematical programming. Wang and Zhu [22] summarized on fuzzy portfolio selection. Yu er al.[23, 24] investigated a nonlinear ensemble forecasting model and optimal portfolio problems with artificial neural networks. One can refer to Bellman and Zadeh [4] and Zimmermann [27] for a detailed discussion on the fuzzy decision theory.

Possibility theory was proposed by Zadeh [26] and advanced by Dubois and Prade [7] where fuzzy variables are associated with possibility distributions in a similar way that random variables are associated with probability distributions in the probability theory. The possibility distribution function of a fuzzy variable is usually defined by the membership function of the corresponding fuzzy set. Possibility and necessity measures play a key role in possibility theory and are used to model financial optimization problems. However, it is clear, a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. The credibility measure, defined by the average of the possibility measure and necessity measure, might deserve to be used in financial optimization modelling. A fuzzy event must hold if its credibility achieves 1, and fail if its credibility is 0. One can refer to [10] for details.

Cherubini and Lunga [6] presented a VaR measure which accounts for market liquidity and showed that taking into account market liquidity implies a decoupling of valuation of long and short positions. Zmeskal [28] described an approach to model uncertainty of the international index portfolio by a VaR methodology under soft conditions by fuzzy-stochastic methodology. Vercher et al. [21] presented two fuzzy portfolio selection models where the objective is to minimize the downside risk constrained by a given expected return. However, few papers are reported on VaR and CVaR defined in fuzzy environments and solved portfolio optimization with them.

Possibility, necessity or credibility distributions can use to characterize experts' knowledge, historical data, and prediction results. In this paper, we propose two novel risk measures named Fuzzy Value-at-Risk(FVaR) and Fuzzy Conditional Value-at-Risk(FCVaR), based on credibility measure introduced by Liu and coauthors [11, 13], and apply the two fuzzy risk measures to portfolio optimization problems in a fuzzy financial environment. The rest of the paper is organized as follows. In Section 2, we give some related symbols and concepts of FVaR and FCVaR. In Sections 3, two portfolio optimization models for fuzzy portfolio selection problems are formulated, a Chaos Genetic Algorithm based on Fuzzy Simulation(CGAFS) is designed, and a practical case is given to demonstrate the effectiveness of CGAFS. Conclusions are discussed in section 4.

2. Some symbols and concepts

Let Θ be a nonempty set, and $P(\Theta)$ the power set of Θ. Then Pos is called a possibility measure if it satisfies the following three axioms.

Axiom 1. $P(\Theta) = 1$;

Axiom 2. $P(\emptyset) = 0$;

Axiom 3. $Pos\{\cup_i A_i\} = Sup_i Pos\{A_i\}$, for any collection A_i in $P(\Theta)$.

Then the triplet $(\Theta, P(\Theta), Pos)$ is called a possibility space.

A fuzzy variable ξ is defined as a function from a space $(\Theta, P(\Theta), Pos)$ to the set of real numbers. For a fuzzy variable ξ, its membership function can be derived from the possibility measure by the expression

$$\mu(x) = Pos\{\theta \in \Theta \,|\xi(\theta) = x\}, x \in R.$$

Dubois and Prade[7] developed the possibility measure and necessity measure as follows. Let r be a real number and ξ be a fuzzy variable. The possibility and necessity measure of $\{\xi \le r\}$ are respectively defined as:

$$Pos\{\xi \le r\} = \sup_{x \le r} \mu(x),$$

$$Nec\{\xi \le r\} = 1 - \sup_{x > r} \mu(x).$$

Remark 1. The possibility measure is a conjugate or dual of the necessity measure.

A $n-$ dimensional fuzzy vector ξ is defined as a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the set of $n-$ dimensional real vectors. It can be proved that the vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T$ is a fuzzy vector if and only if $\xi_i(i = 1, 2, \cdots, n)$ are fuzzy variables. Throughout, vectors will be denoted in bold, to be distinguished from variables.

Let $f : R^n \to R$ be a function and $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T$ be a fuzzy vector on the possibility space $(\Theta, P(\Theta), Pos)$. Then $\eta = f(\xi_1, \xi_2, \cdots, \xi_n)$ is a fuzzy variable defined as $\eta(\theta) = f(\xi_1(\theta), \xi_2(\theta), \cdots, \xi_n(\theta))$ for any $\theta \in \Theta$.

A fuzzy variable is said to be normal if there exists a real number r such that $\mu(r) = 1$. We always assume that the fuzzy variables are normal in this paper.

The definitions of credibility measure and its expected value of a fuzzy variable were introduced in [13]. The credibility measure has self-duality property which is not possessed of possibility measure or necessary measure.

Definition 1.[13] The credibility measure of $\{\xi \leq r\}$ is defined as:

$$Cr\{\xi \leq r\} = \frac{1}{2}\left(Pos\{\xi \leq r\} + Nec\{\xi \leq r\}\right).$$

Remark 2. The credibility measure is a monotone, self dual and sub-additive measure.

Definition 2. [13] Let ξ be a fuzzy variable. Then the expected value of a fuzzy variable ξ is defined by

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr,$$

provided that at least one of the two integrals is finite.

The expected value of a fuzzy variable is a Choqute Integeral since the credibility measure is self dual.

There are n securities $i = 1, \cdots, n$ to be invested in a financial market that we consider. The return rate of each security is assumed to be a fuzzy variable. The fuzzy return rate of security i is ξ_i, $i = 1, \cdots, n$.

Let $f(\mathbf{x}, \xi) : R^n \times R^n \to R$ be the loss associated with the decision vector $\mathbf{x}$, to be chosen from a certain subset $\mathbf{X} \subseteq R^n$. The vector $\mathbf{x}$ can be interpreted as representing a portfolio, with $\mathbf{X}$ as the set of available portfolios subject to various constraints. The vector ξ stands for the uncertainties that can affect the loss. The portfolio $\mathbf{x}$ is optimal to portfolio $\mathbf{y}$, i.e., $\mathbf{x} \geq \mathbf{y}$ means $f(\mathbf{x}, \xi) \leq f(\mathbf{y}, \xi)$.

For each $\mathbf{x}$, the loss function $f(\mathbf{x}, \xi)$ is a fuzzy variable. The expected value of $f(\mathbf{x}, \xi)$ for any $\mathbf{x} \in \mathbf{X}$ is

$$E[f(\mathbf{x}, \xi)] = \int_0^{\infty} Cr\{f(\mathbf{x}, \xi) \geq r\}dr - \int_{-\infty}^{0} Cr\{f(\mathbf{x}, \xi) \leq r\}dr.$$

The credibility of $f(\mathbf{x}, \xi)$ not exceeding a threshold r is given by

$$\psi(\mathbf{x}, r) = Cr\{f(\mathbf{x}, \xi) \leq r\}.$$

Definition 3. Let ξ be a fuzzy vector. FVaR is defined by

$$FVaR_\beta(\mathbf{x}) = \inf\{r \in R \,|\, \psi(\mathbf{x}, r) > \beta\}.$$

It values for the loss associated with $\mathbf{x}$ and any prescribed confidence level $\beta \in (0, 1)$, commonly, β is close to one. $FVaR_\beta(\mathbf{x})$ is an increasing and left-continuous function of β.

Definition 4. Let ξ be a fuzzy vector. FCVaR is defined by

$$FCVaR_\beta(\mathbf{x}) = FVaR_\beta(\mathbf{x}) + (1-\beta)^{-1}\int_0^{+\infty} Cr\{f(\mathbf{x},\xi) - FVaR_\beta(\mathbf{x}) \geq r\}dr,$$

provided the integral is finite.

It values for the conditional expectation of losses above that amount $FVaR_\beta(\mathbf{x})$ associated with $\mathbf{x}$ and any prescribed confidence level $\beta \in (0,1)$.

Remark 3. If the possibility and necessity of $f(\mathbf{x},\xi)$ not exceeding a threshold r are given by

$$\psi(\mathbf{x},r) = Pos\{f(\mathbf{x},\xi) \leq r\},$$
$$\text{and } \psi(\mathbf{x},r) = Nec\{f(\mathbf{x},\xi) \leq r\},$$

we can obtain the different definitions of $FVaR_\beta(\mathbf{x})$ and $FCVaR_\beta(\mathbf{x})$.

3. Portfolio optimization with fuzzy risk measures

We consider the situation that problems space is a possibility space, and formulate a portfolio management problem utilizing or as the measure of risk. Then, we give a hybrid intelligent algorithm for these models and a numerical example.

3.1. Portfolio selection models

Suppose there exits n risk securities that can be chosen by the investor in the financial market. Let $\mathbf{x} = (x_1, \cdots, x_n)^T \in R^n$ denote the amount of the investments in the n risk securities decided by the investor, and fuzzy vector $\xi = (\xi_1, \cdots, \xi_n)^T \in R^n$ denote the uncertain returns of the n risk securities, $\xi_1, \cdots, \xi_n$ are fuzzy variables. The loss function in Section 2 is concretely defined as

$$f(\mathbf{x},\xi) = -\mathbf{x}^T\xi.$$

Portfolio optimization tries to find an optimal trade-off between the risk and the return according to the investor's preference, while the portfolio selection is performed through the analysis of risk and return. Thus the fuzzy portfolio selection problem using $FVaR$ as a risk measure can be represented as

$$\min_{\mathbf{x}\in\mathbf{X}} FVaR_\beta(\mathbf{x}).$$

The fuzzy portfolio selection problem using $FCVaR$ as a risk measure can be represented as

$$\min_{\mathbf{x}\in\mathbf{X}} FCVaR_\beta(\mathbf{x}).$$

Where $\mathbf{X}$ denotes the constraints on the portfolio position, which usually includes the requirement such as initial wealth, bound and short-selling constraints, etc. We specify the constraints set $\mathbf{X}$ below.

Suppose the investor has an initial wealth unit 1. Thus the portfolio satisfies

$$\sum_{i=1}^{n} x_i = 1. \tag{1}$$

To ensure diversification and satisfy the regulations, we impose the bound constraints on the portfolio

$$l_i \le x_i \le u_i, \quad i = 1, \cdots, n, \tag{2}$$

where $\mathbf{l} = (l_1, \cdots, l_n)^T$ and $\mathbf{u} = (u_1, \cdots, u_n)^T$ are the given lower and upper bounds on the portfolios. Suppose short position of each security is not allowed

$$x_i \ge 0, \quad i = 1, \cdots, n. \tag{3}$$

Let σ be the minimum expected by the investor, $m_i = E[\xi_i]$, and $\mathbf{m} = (m_1, \cdots, m_n)^T$. Then, we have

$$\mathbf{x}^T\mathbf{m} \ge \sigma. \tag{4}$$

Generally, the model for minimizing $FVaR$ is the problem

$$\begin{cases} \min & FVaR_\beta(\mathbf{x}) \\ s.t. & \mathbf{x}^T\mathbf{m} \ge \sigma, \\ & \mathbf{l} \le \mathbf{x} \le \mathbf{u} \\ & \sum_{i=1}^n x_i = 1 \\ & \mathbf{x} \ge 0 \end{cases} \tag{5}$$

The model for minimizing $FCVaR$ is the problem

$$\begin{cases} \min & FCVaR_\beta(\mathbf{x}) \\ s.t. & \mathbf{x}^T\mathbf{m} \ge \sigma, \\ & \mathbf{l} \le \mathbf{x} \le \mathbf{u} \\ & \sum_{i=1}^n x_i = 1 \\ & \mathbf{x} \ge 0 \end{cases} \tag{6}$$

3.2. Chaos genetic algorithm based on fuzzy simulation(CGAFS)

In this section, a novel hybrid intelligent algorithm, the chaos genetic algorithm based on fuzzy simulation developed by Liu and Iwamura[12], is designed to solve the afore mentioned models. The chaos genetic algorithm is employed to search the optimal portfolio. Firstly, it selects randomly a set of initial feasible portfolio strategies as the initial population and codes them into chromosomes. Secondly, it calculates the objective function of each chromosome by using fuzzy simulation and converts it to the value of fitness function. Thirdly, it selects and crosses the chromosomes according to certain probabilities. Finally, it mutates chaotically the chromosomes according to a well-known logistic map. This process is iterated population after population until an optimal portfolio strategy is obtained. This CGAFS procedure can be described in detail as follows:

Step 1: *Input the parameters of CGAFS, such as population size, cross probability, chaos mutate probability, chaos factor, and confident level.*

Step 2: *Initialize population size chromosomes randomly* $\mathbf{x}^j$*, and satisfy the formulas (1),(2), and (3).*

Step 3: *Compute the* $\mathbf{x}^T\mathbf{m}$ *value in formulas (4) for all chromosomes by the fuzzy simulation:*

Substep 1: $e = 0$. *N is a sufficiently large number.*
Substep 2: *Randomly generate θ_k from Θ such that $v_k = Pos\{f(\mathbf{x}, \xi(\theta_k))\} \geq \varepsilon$ for $k = 1, \cdots, N$, where ε is a sufficiently small number.*
Substep 3: *Set $a = f(\mathbf{x}, \xi(\theta_1)) \wedge \cdots \wedge f(\mathbf{x}, \xi(\theta_N))$, $b = f(\mathbf{x}, \xi(\theta_1)) \vee \cdots \vee f(\mathbf{x}, \xi(\theta_N))$.*
Substep 4: *Randomly generate r from $[a, b]$.*
Substep 5: *If $r \geq 0$, then $e \leftarrow e + Cr\{f(\mathbf{x}, \xi) \geq r\}$, where the credibility can be estimated by*

$$\frac{1}{2}\left(\max_{1 \leq k \leq N} \langle v_k \,| f(\mathbf{x}, \xi(\theta_k)) \geq r \rangle + \min_{1 \leq k \leq N} \{1 - v_k \,| f(\mathbf{x}, \xi(\theta_k)) < r\}\right)$$

Substep 6: *If $r \leq 0$, then $e \leftarrow e - Cr\{f(\mathbf{x}, \xi) \leq r\}$, where the credibility can be estimated by*

$$\frac{1}{2}\left(\max_{1 \leq k \leq N} \langle v_k \,| f(\mathbf{x}, \xi(\theta_k)) \leq r \rangle + \min_{1 \leq k \leq N} \{1 - v_k \,| f(\mathbf{x}, \xi(\theta_k)) > r\}\right)$$

Substep 7: *Repeat Substep 4 to Substep 6 for N times.*
Substep 8: $\mathbf{x}^T\mathbf{m} = -(a \vee 0 + b \wedge 0 + e \times (b - a)/N)$.

Step 4: *Check $\{\mathbf{x}^j\}$ satisfy the formulas (4) and go to Step 2 until $j =$ pop$-$size.*
Step 5: *Compute the objective values for all chromosomes by the fuzzy simulation for model (5):*
Substep 1: *Randomly generate θ_k from Θ such that $v_k = Pos\{f(\mathbf{x}, \xi(\theta_k))\} \geq \varepsilon$, $k = 1, \cdots N$.*
Substep 2: $L(r) =$

$$\frac{1}{2}\left(\max_{1 \leq k \leq N} \langle v_k \,| f(\mathbf{x}, \xi(\theta_k)) \leq r \rangle + \min_{1 \leq k \leq N} \{1 - v_k \,| f(\mathbf{x}, \xi(\theta_k)) > r\}\right) \quad (7)$$

Substep 3: *Find the minimal value r such that $L(r) \geq \beta$ holds.*
Substep 4: *Let the objective value be equal to r.*

Step 6: *Calculate the fitness of each chromosome according to the objective value by index.*
Step 7: *Select the chromosomes by steady-state selection operation. Update the chromosomes by crossover and chaos mutation operations, and check the feasibility.*
Step 8: *Repeat the third to sixth steps for a given number of cycles.*
Step 9: *Return the best chromosome as the approximate optimal portfolio strategy.*

The procedure above is for solving the problem (5). To solve problem (6), the Step 5 needs to be modified as follows, with other steps remain unchanged.

Step 5: *Compute the objective values for all chromosomes by the fuzzy simulation for model (6):*
Substep 1: *Set $e = 0$.*
Substep 2: *Randomly generate θ_k from Θ such that $v_k = Pos\{f(\mathbf{x}, \xi(\theta_k))\} \geq \varepsilon$, $k = 1, \cdots N$.*
Substep 3: *$L(r)$ is definite in formulas (7).*

TABLE 1. Stocks

Stocks	Code	Company	Stocks	Code	Company
1	600000	Pudong Dev Bank	2	600001	Handan Steel
3	600004	Baiyun Airport	4	600009	ShanghaiAirport
5	600016	Minsheng Banking	6	600019	Baoshan Steel
7	600026	China Ship Dev	8	600028	Sinopec Corp
9	600029	Southern Airline	10	600050	China unicom
11	600058	Minmetals Dev	12	600085	Tongrentang
13	600098	Guangzhou Devel	14	600205	Shandong Alumini
15	600583	Offshore Oil	16	600649	Raw Water Sup
17	600688	Shanghai Pechem	18	600832	Oriental Pearl
19	600887	Yili Company	20	600895	Zhangjiang Hi-Te

Substep 4: *Find the minimal value r^* such that $L(r^*) \geq \beta$ holds.*
Substep 5: *Set $a = r^*$, $b = f(\mathbf{x}, \xi(\theta_1)) \vee \cdots \vee f(\mathbf{x}, \xi(\theta_N))$.*
Substep 6: *Randomly generate r from $[0, b-a]$.*
Substep 7: *$e \leftarrow e + Cr\{f(\mathbf{x}, \xi) - r^* \geq r\}$, where the credibility can be estimated by*

$$\frac{1}{2}\left(\max_{1\leq k\leq N} \langle v_k \,|f(\mathbf{x}, \xi(\theta_k)) - r^* \geq r\rangle + \min_{1\leq k\leq N} \{1 - v_k \,|f(\mathbf{x}, \xi(\theta_k)) - r^* < r\}\right)$$

Substep 8: *Repeat the Substep 6 and Substep 7 for N times.*
Substep 9: *Let objective value be equal to $r^* \vee 0 + b \wedge 0 + e \times (b-a)/(N(1-\beta))$.*

3.3. Computational results

In this section, we give a numerical example applying CGAFS to models (5) and (6). All the computations were performed using the program for CGAFS designed by ourselves within Matlab7.0R14 on a Dell Dimension 5150 running Microsoft Windows XP. The rate of transaction costs and tax for stocks is 0.0075 in the two securities markets on the Chinese mainland. Assume that an investor chooses 20 different stocks from the Shanghai Stock Exchange for his investment. The exchange codes and names of companies of the 20 stocks are given in Table 1.

Now we use the model (5) or (6) to reallocate the investor's assets. Because the Shanghai Stock Exchange is very young, the arithmetic means are not good estimates of the actual returns that the investor will receive in the future. In the situation, the expected return of stocks is regarded as fuzzy number may be better than as crisp number. The expected return of stocks denote by triangular fuzzy variables that their centre value, left spread value and right spread value can be estimated as follows.

First, we collect historical data on the 20 stocks from January, 2002 to August, 2006. The data are downloaded from the web-site www.gw.com.cn. Then we use one month as a period to obtain the historical rates of returns for 56 periods.

Second, compute the average of historical rates of returns for 56 periods of each stock and subtract transaction cost and tax from the average. And the difference is denoted as the history arithmetic mean R_a of each stock.

Third, the number N_h of latest periods that represents the trend of stock in future is provided by experts' knowledge. Compute the average of historical trend rates of returns for N_h periods of each stock and subtract transaction cost and tax from the average. And the difference is denoted as the history trend mean R_h of each stock.

Fourth, based on the corporations' financial reports, the predicted rate of return R_f of a period in future is estimated by experts of investment corporations.

Finally, the left spread, the center and the right spread of triangular fuzzy number of each stock are minimum, middle and maximum of R_a, R_h and R_f respectively.

In the following, we give the estimation example for the triangular fuzzy numbers of rates of returns for stock of Sinopec Corp in detail. First, we use historical data (month price at the open and at the close from January, 2002 to August, 2006) to calculate the historical rates of returns. These data are listed in Table 2. The average of historical rates of returns from January, 2002 to August, 2006 of Sinopec Corp is 0.0123, the transaction cost and tax is 0.0075, and then the history arithmetic mean R_a is 0.0048.

In this example, we choose $N_h = 6$. The average of historical rates of returns from March, 2006 to August, 2006 of Sinopec Corp is 0.0235, the transaction cost and tax is 0.0075, and then the history trend mean R_h is 0.0160. The predicted rate of return R_f of Sinopec Corp in a period in future estimated by experts of investment corporations is 0.0096. Thus, the left spread is 0.0048, the center is 0.0096 and the right spread is 0.0160. Using a similar method, we obtain the triangular fuzzy rates of all 20 stocks. These are listed in Table 3.

The parameters of CGAFS are set as follows: the population size is 60, cross probability is 0.3, chaos mutate probability is 0.04, $\varepsilon = 0.0001$, $N = 10000$. The values of l_i, u_i, σ are given by investors. They are as follows:

$$l_i = 0.0, u_i = 1.0, i = 1, \cdots, n, \sigma = 0.0020.$$

Giving different value of confident level β, we obtain expected return, FVaR and an optimal portfolio strategy by solving (5). The corresponding computational results are listed in Table 4 and Table 5.

Giving different value of confident level β, we obtain expected return, FCVaR and an optimal portfolio strategy by solving model (6). The corresponding computational results are listed in Table 6 and Table 7.

From the above results, we find that we can obtain the different portfolio strategies by solving (5) or (6) in which the different confident level are given. Through choosing the values of the confident level according to the investor's frame of mind, the investor may achieve a favorite portfolio strategy. Through choosing different expected return constraint, the investor may also achieve alternative portfolio strategy.

TABLE 2. The Rates of Returns of Sinopec Corp from January, 2002 to August, 2006

Month	Open	Close	Rate	Month	Open	Close	Rate
200608	5.78	6.14	0.0623	200404	5.53	5.24	-0.0524
200607	6.26	5.78	-0.0767	200403	5.08	5.50	0.0827
200606	6.45	6.25	-0.0310	200402	5.30	5.08	-0.0415
200605	6.08	6.46	0.0625	200401	4.96	5.05	0.0181
200604	5.20	6.07	0.1673	200312	3.92	4.95	0.2628
200603	5.28	5.05	-0.0436	200311	3.72	3.92	0.0538
200602	5.01	5.29	0.0559	200310	3.43	3.72	0.0845
200601	4.66	4.98	0.0687	200309	3.61	3.44	-0.0471
200512	4.10	4.66	0.1366	200308	3.70	3.60	-0.0270
200511	3.91	4.11	0.0512	200307	3.74	3.71	-0.0080
200510	4.08	3.92	-0.0392	200306	3.83	3.74	-0.0235
200509	4.41	4.13	-0.0635	200305	3.75	3.80	0.0133
200508	4.08	4.41	0.0809	200304	3.61	3.73	0.0332
200507	3.50	4.08	0.1657	200303	3.52	3.60	0.0227
200506	3.55	3.53	-0.0056	200302	3.52	3.53	0.0028
200505	4.15	3.55	-0.1446	200301	3.01	3.50	0.1628
200504	4.18	4.15	-0.0072	200212	3.23	3.01	-0.0681
200503	4.49	4.18	-0.0690	200211	3.27	3.25	-0.0061
200502	4.00	4.49	0.1225	200210	3.43	3.27	-0.0467
200501	4.35	4.00	-0.0805	200209	3.67	3.44	-0.0627
200412	4.44	4.36	-0.0180	200208	3.47	3.67	0.0576
200411	4.46	4.44	-0.0045	200207	3.90	3.47	-0.1103
200410	4.70	4.47	-0.0489	200206	3.15	3.90	0.2381
200409	4.67	4.71	0.0086	200205	3.37	3.15	-0.0653
200408	4.58	4.67	0.0196	200204	3.2	3.36	0.0500
200407	4.80	4.60	-0.0417	200203	3.19	3.21	0.0063
200406	5.03	4.79	-0.0477	200202	3.16	3.19	0.0095
200405	5.28	5.03	-0.0474	200201	3.45	3.16	-0.0841

TABLE 3. The triangular fuzzy numbers of the expected rates of returns

Stock	L.Spread	Center	R.Spread	Stock	L.Spread	Center	R.Spread
1	0.0024	0.0158	0.0337	2	-0.0076	0.0098	0.0484
3	-0.0039	0.0081	0.0109	4	-0.0039	0.0081	0.0109
5	0.0090	0.0127	0.0384	6	-0.0114	0.0004	0.0096
7	0.0128	0.0284	0.0498	8	0.0048	0.0096	0.0160
9	-0.0102	0.0227	0.0352	10	-0.0008	0.0063	0.0571
11	0.0096	0.0519	0.1292	12	0.0031	0.0203	0.0217
13	-0.0081	0.0134	0.0182	14	0.0118	0.0183	0.0215
15	0.0193	0.0351	0.0747	16	-0.0046	0.0031	0.0377
17	0.0021	0.0072	0.0121	18	0.0081	0.0257	0.0337
19	0.0236	0.0286	0.0327	20	0.0081	0.0296	0.0451

TABLE 4. Expected Return and FVaR when $\sigma = 0.0020$

Confident level	Expected Return	FVaR
$\beta = 0.90$	0.0302	-0.0215
$\beta = 0.95$	0.0299	-0.0208
$\beta = 0.99$	0.0296	-0.0187

TABLE 5. The optimal portfolio strategy

Stock	Ratio($\beta = 0.90$)	Ratio($\beta = 0.95$)	Ratio($\beta = 0.99$)
1	0.0201	0.0160	0.0313
2	0.0238	0.0173	0.0205
3	0.0158	0.0038	0.0134
4	0.0168	0.0102	0.0216
5	0.0167	0.0319	0.0233
6	0.0171	0.0123	0.0269
7	0.0279	0.0233	0.0309
8	0.0207	0.0116	0.0193
9	0.0239	0.0107	0.0196
10	0.0249	0.0241	0.0131
11	0.0187	0.0199	0.0329
12	0.0230	0.0193	0.0213
13	0.0134	0.0051	0.0096
14	0.0237	0.0160	0.0148
15	0.1592	0.1980	0.2562
16	0.0137	0.0063	0.0093
17	0.0205	0.0194	0.0148
18	0.0218	0.0251	0.0319
19	0.4740	0.5091	0.3524
20	0.0244	0.0207	0.0369

TABLE 6. Expected Return and FCVaR when $\sigma = 0.0020$

Confident level	Expected Return	FCVaR
$\beta = 0.90$	0.0281	-0.0193
$\beta = 0.95$	0.0279	-0.0187
$\beta = 0.99$	0.0262	-0.0186

In fact, the process in this example describes a method which is combination of quantitative analysis with history data and qualitative analysis with experts' knowledge can obtain robust solution and decrease turnover ratio when statistical parameters have estimating error or history data lack.

4. Conclusions

TABLE 7. The optimal portfolio strategy

Stock	Ratio($\beta = 0.90$)	Ratio($\beta = 0.95$)	Ratio($\beta = 0.99$)
1	0.0256	0.0242	0.0133
2	0.0227	0.0219	0.0113
3	0.0228	0.0115	0.0148
4	0.0247	0.0194	0.0130
5	0.0227	0.0441	0.0349
6	0.0091	0.0282	0.0145
7	0.0295	0.0275	0.0579
8	0.0158	0.0124	0.0323
9	0.0138	0.0103	0.0218
10	0.0223	0.0170	0.0369
11	0.0267	0.0324	0.0320
12	0.0082	0.0140	0.0380
13	0.0170	0.0284	0.0062
14	0.0367	0.0433	0.0460
15	0.1560	0.1955	0.0409
16	0.0196	0.0324	0.0074
17	0.0145	0.0202	0.0117
18	0.0262	0.0405	0.0347
19	0.4463	0.3277	0.5040
20	0.0395	0.0490	0.0285

In probability framework, both VaR and CVaR are important instruments in risk management and portfolio selection. In possibility framework, we propose two risk measures FVaR and FCVaR, formulat two portfolio optimization programming models for fuzzy portfolio selection problems, and design a chaos genetic algorithm based on fuzzy simulation. Computational results show that FVaR and FCVaR can play a role in possibility space similar to VaR and CVaR in probability space and that robust solutions can be obtained when they are applied to portfolio selection problems.

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