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RELATION BETWEEN $B_p(3)$ AND $C_p(3)$ WITH THEIR ORDER COMPONENTS WHERE p IS AN ODD PRIME

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ABSTRACT. It is proved that if $M = B_p(3)$ or$C_p(3)$, p is an odd prime, G is a finite group and has the same order components of M, then $G \cong B_p(3)$ or $C_p(3)$.

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1. Introduction

If G is a finite group, we define the prime graph $\Gamma(G)$ as following: its vertices are the primes dividing the order of G, and two vertices p and q are joined by an edge if and only if there is an element in G of order pq. We denote the set of all the connected components of graph $\Gamma(G)$ by $T(G) = \{\pi_i(G)$, for $i = 1, 2, \cdots, t(G)\}$ where $t(G)$ is the number of connected components of $\Gamma(G)$, and if G is of even order we always assume 2 in $\pi_1(G)$. We also denote the set of all the primes dividing n by $\pi(n)$ where n is a natural number. Obviously $|G|$ can be expressed as a product of $m_1, m_2, \cdots, m_{t(G)}$, where m_i is a positive integer with $\pi(m_i) = \pi_i(G)$. All m_i are called the order components of G. Let $OC(G) = \{m_1, m_2, \cdots, m_{t(G)}\}$ be the set of order components of G. The order components of non-abelian simple groups having at least two prime graph components have been attained in [3].

J.G.Thompson has conjectured that 'let M be a non-abelian simple group, if G is a finite group satisfying $Z(G) = 1$ and $N(G) = N(M)$, where $N(G) = \{n \in N | G$has a conjugate class C, such that $|C| = n\}$, then $G \cong M$. W.J.Shi had put forward another conjecture that if$M$ is a finite simple group and G is a finite group satisfying $|G| = |M|$ and $\pi_e(G) = \pi_e(M)$, where $\pi_e(G)$ denotes the set of orders of elements of G, then $G \cong M$. In [5], we had proved that if M is a simple group with non-connected prime graph and G is a finite group

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satisfying the conditions of J.G. Thompson's conjecture then $OC(G) = OC(M)$, and it's obviously that if G is a finite group satisfying the conditions of W.J. Shi's conjecture, $OC(G) = OC(M)$ too , consequently, these two conjecture holds nature for simple group M which has at least two order components and can be characterized by it's order components, hence, it's an important topic to find out those simple groups satisfying above mentioned properties.

We have known that the following simple groups have a non-connected prime graph and can be characterized by their order components: a finite simple group with at least three prime graph components [5], sporadic simple groups [3], Suzuki-Ree groups [6], $G_2(q)$ where $q \equiv 0 \pmod 3$ [4], $E_8(q)$ [1], $PSL_2(q)$ [7], ${}^3D_4(q)$ [8], ${}^2D_n(3), 9 \leq n = 2^m + 1 \neq p$ [9], ${}^2D_{p+1}(2)$, $5 \leq p \neq 2^m - 1$ [24], A_p where p and $p-2$ are primes [12], $PSL(5,q)$ [13], $PSL(3,q)$ where q is an odd prime power [14], $PSL(3,q)$ for $q = 2^n$ [15], $F_4(q)$ where q is even [16], $C_2(q)$ where $q > 5$ [17], $PSU_5(q)$ [18], $PSU(3,q)$ for $q > 5$ [19], ${}^2D_4(q)$ [20], ${}^2E_6(q)$ [22], $E_6(q)$ [21]. Of course, it isn't true that all simple groups with non-connected prime graph can be characterized by their order components, for example, $B_p(3)$ and $C_p(3)$, where p is an odd prime, but we have the following result:

Theorem. *Let $M = B_p(3)$ or $C_p(3)$, where p is an odd prime. If a finite group G has the same order components of M, then $G \cong B_p(3)$ or $C_p(3)$.*

2. Preliminary results

Lemma 1. [[3] Lemma 6] *If $t(G) \geq 2$, H is a π_i subgroup of G, and $H \lhd G$, then $\Pi_{j=1,j\neq i}^{t(G)} m_i \mid (|H| - 1)$.*

Lemma 2. [[2] Theorem 2] *Let G be a 2-Frobenius group of even order. Then $t(G) = 2$, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that $|K/H| = m_2, |H| \cdot |G/K| = m_1, |G/K| \mid (|K/H| - 1), |G/K| \mid \varphi(|K/H|)$, and H is nilpotent.*

Lemma 3. [[25] Lemma 3] *If M is a simple group with $t(M) = 2$, G is a finite group and $OC(G) = OC(M)$, then one of the following holds:*

(1) G is a Frobenius group or 2-Frobenius group.

(2) G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H is a nilpotent π_1-group, K/H is a non-abelian simple group, the odd order component of M is equal to one of those of K/H, G/K is a cyclic π_1-group, and $|G/K| \mid |Out(K/H)|$.

Lemma 4. [[11] Remark] *The only solution of the equation $p^m - q^n = 1$, where p , q are primes and $m, n > 1$, is $3^2 - 2^3 = 1$.*

Lemma 5. [26] *Let p be a prime and n be a natural number, $n \geq 2$. Then there exists a prime divisor r of $p^n - 1$ which does not divide $p^m - 1$ for any natural number $m < n$, except $n = 6$, $p = 2$ or $n = 2$, $p + 1$ is a power of 2. Such r is called a primitive prime divisor of $p^n - 1$.*

Of course a primitive prime divisor of $p^n - 1$ can't divide $p^n + 1$ or $p^m - 1$ for $n \nmid m$.

Lemma 6. [[23] Lemma 1] *If $n \geq 6$ is a natural number, then there exists at least $s(n)$ primes p_i such that $\frac{n+1}{2} < p_i < n$:*
$s(n) = 6$ *for* $n \geq 49$;
$s(n) = 5$ *for* $42 \leq n \leq 47$;
$s(n) = 4$ *for* $38 \leq n \leq 41$;
$s(n) = 3$ *for* $18 \leq n \leq 37$;
$s(n) = 2$ *for* $14 \leq n \leq 17$;
$s(n) = 1$ *for* $6 \leq n \leq 13$.

Lemma 7. *Let p be a prime, $q > 1$ be a natural number $e = min\{d : p \mid (q^d - 1)\}$, $q^e = 1 + p^r k$, $p \nmid k$, t be a natural number satisfying $t = p^s u$ and $p \nmid u$. If $p > 2$ or $r > 2$, then $p^{r+s} \parallel (q^{et} - 1)$.*

Proof. $q^{et} - 1 = (1 + p^r k)^t - 1 = tp^r k + \sum_{i=2}^{t} \binom{t}{i} (p^r k)^i$. If $s = 0$, then $p \nmid t$, $p^r \parallel q^{et} - 1$. If $s > 0$, by calculation we can prove that $p^{r+s+1} \mid \binom{t}{i}(p^r k)^i$ for $2 \leq i \leq t$, hence $p^{r+s} \parallel (q^{et} - 1)$.
So we have that $p^{r+s} \parallel (q^{et} - 1)$. □

Lemma 8. *Assume $q > 1$ is a natural number, $s = \prod_{i=1}^{n}(q^i - 1)$, p is a prime, $p \mid s$. We denote the power of p in the standard factorization of s by s_p. $e = min\{d : p \mid q^d - 1\}$, $q^e = 1 + p^r k$, $p \nmid k$. If $p > 2$ or $r > 2$, then $s_p < q^{\frac{np}{p-1}}$.*

Proof. Let $a = [\frac{n}{e}]$, $w = \prod_{i=1}^{a}(q^{ei} - 1)$, hence, $s_p = w_p = p^{ra + \sum_{j=1}^{\infty}[\frac{a}{p^j}]} \leq p^{ra + \frac{a}{p-1}} < q^{\frac{np}{p-1}}$ by Lemma 7 and $p > 2$ or $r > 2$. □

Lemma 9. *Let q be an odd natural number, $s = \prod_{i=1}^{n}(q^i - 1)$. Then $s_2 < q^{1.5n}$.*

Proof. Set $2^r \parallel q - 1$. We divide the proof into two cases based on r being 1 or not.
Case 1. When $r = 1$, $s = \prod_{i=1}^{n}(q^i - 1) = \prod_{i=1,2|i}^{n}(q^i - 1) \cdot \prod_{j=1,2\nmid j}^{n}(q^j - 1)$.
For $2 \mid i$, set $v = q^2$, $v = 1 + 2^{r'} k$, $2 \nmid k$, clearly $r' \geq 2$, hence, the power of 2 in the standard factorization of $\prod_{i=1,2|i}^{n}(q^i - 1) = \prod_{i=1}^{[\frac{n}{2}]}(v^i - 1)$ is less than $q^{[\frac{n}{2}]\cdot 2}$ by Lemma 8. For $2 \nmid j$, we have that $2 \parallel q^j - 1$, so $2^{[\frac{n+1}{2}]} \parallel \prod_{j=1,2\nmid j}^{n}(q^j - 1)$.
Hence $s_2 < 2^{[\frac{n+1}{2}]} q^{[\frac{n}{2}]\cdot 2} < q^{1.5n}$.
Case 2. When $r \neq 1$, by Lemma 7, we have that $s_2 = 2^{rn + \sum_{j=1}^{\infty}[\frac{n}{2^j}]} < 2^{rn} \cdot 2^n < q^n \cdot 2^n < q^{1.5n}$ since $r \neq 1$. □

Definition 1. Let a and f be expressions of integers with integral coefficients. If $f \mid a$ and $(f,\ a/f) = 1$, then we say that f is a *hall factor* of a.

Lemma 10. [[10] Theorem 1] *If q is a power of a prime number, $c = \prod_{i=1}^{n}(q^{2i} - 1)$ or $(q^n \pm 1) \cdot \prod_{i=1}^{n-1}(q^{2i} - 1)$, then there exists a hall factor f of c satisfying:*
(1) *If* $n \geq 23$ *then* $f > q^{8n}$;
(2) *If* $n = 22$ *then* $f > q^{7n}$;
(3) *If* $18 \leq n \leq 21$ *then* $f > q^{6n}$;
(4) *If* $16 \leq n \leq 17$ *then* $f > q^{5n}$;

(5) *If* $14 \le n \le 15$ *then* $f > q^{4n}$.
And if the standard factorization of $f = \prod_{k=1}^{t} r_k^{\delta_k}$, *then* $r_k^{\delta_k} \le \frac{q^{n-1}-1}{q-1}$.

3. Proof of the theorem

Proof. Because $M = B_p(3)$ or $C_p(3)$, p is an odd prime, and G has the same order components with M, so the even order component of G is $m_1 = 2\cdot 3^{p^2}(3^p+1)\prod_{i=1}^{p-1}(3^{2i}-1)$, the odd order component of G is $m_2 = (3^p-1)/2$.

We divide the proof into several cases based on Lemma 3 and Table [1]-[4] in [3].

Case 1. G can't be a Frobenius group or a 2-Frobenius group.

Subcase 1.1 If G is a Frobenius group with Frobenius kernel H and complement K, then $|H| = m_1$, $|K| = m_2$. There exists a primitive prime divisor r of $3^{2p}-1$ by Lemma 5. Set $S_r \in Syl_r(H)$, of course $|S_r| \mid (3^p+1)/4$ and $S_r \trianglelefteq G$. $|S_r| \equiv 1 \pmod{m_2}$ by Lemma 1, which is impossible.

Subcase 1.2 If G is a 2-Frobenius group , there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H is a nilpotent π_1 group, $|K/H| = m_2$, $|G/K| \mid (|K/H|-1) = (3(3^{p-1}-1))/2$. Hence $(3^p+1) \mid |H|$. Similarly to Subcase 1.1, we can show it's impossible.

From Subcase 1.1, Subcase 1.2 and Lemma 3 we have the following properties:

1. There is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that K/H is a simple group, H and G/K are π_1 group and H is nilpotent.

2. The odd order component of G is one of those of K/H, consequently $t(K/H) \ge 2$. Hence K/H may be one of the simple groups listed in Table [1]-[4] in [3].

Case 2. $K/H \not\cong E_7(2)$, $E_7(3)$, $A_2(2)$, $A_2(4)$, ${}^2A_5(2)$, ${}^2E_6(2)$, ${}^2F_4(2)'$ or one of the sporadic simple groups.

Any odd order component of $E_7(2)$, $E_7(3)$, $A_2(2)$, $A_2(4)$, ${}^2A_5(2)$, or one of the sporadic simple groups(except Suz and F_{22}) can't be written into the form $(3^p-1)/2$ for $p \ge 3$. Though ${}^2E_6(2)$, Suz, F_{22} or ${}^2F_4(2)'$ has an order components 13 which can be written into the form $(3^p-1)/2$ and $p=3$, the order of ${}^2E_6(2)$, Suz, F_{22} or ${}^2F_4(2)'$ can't divides $|B_3(3)|$ or $C_3(3)$. So $K/H \not\cong E_7(2)$, $E_7(3)$, $A_2(2)$, $A_2(4)$, ${}^2A_5(2)$, ${}^2E_6(2)$, ${}^2F_4(2)'$ or one of the sporadic simple groups.

Case 3. $K/H \not\cong A_n$. If $K/H \cong A_n$ then A_n has an odd component equal to $(3^p-1)/2$. Thus $|A_{(3^p-1)/2}| \mid |A_n| \mid |M|$. By Lemma 6, there exists at least six primes p_i satisfying $(3^p+1)/4 < p_i < (3^p-1)/2$ for $p \ge 5$,On the other hand there exists at most one prime divisors of M between $(3^p+1)/4$ and $(3^p-1)/2$, a contradiction. By calculation we have that $p \ne 3$.

Case 4 $K/H \not\cong A_n(q)$ or ${}^2A_n(q)$.

Subcase 4.1 If $K/H \cong A_1(q)$, then $(3^p-1)/2 = q$, $q\pm1$ or $(q\pm1)/2$. Whenever in any case we have that $q \le 3^p$, hence $|K/H| < 3^{3(p)}$. Assume $q = r^f$, we have that $|G/K| < 3^p$ since $2^{3^p/2} > 3^p$ and $|G/K| \mid |Out(K/H)| = 2f$. If $p \geq 14$ then there exists a hall factor g of $|G| = 3^{p^2)} \prod_{i=1}^{p}(3^{2i}-1)$ satisfying that $g > 3^{4p}$ and for any prime number $r' \mid g$ we have that $g_{r'} < (3^p-1)/2$ by Lemma 10. Clearly $(g,\ |H|) \neq 1$. Let prime p' satisfy $p' \mid (g,\ |H|)$ and $S_{p'} \in Syl_{p'}(G)$. $S_{p'}$ is a normal π_1−subgroup of G and $|S_{p'}| < (3^p-1)/2$, which contradicts Lemma 1.

By trivial calculation, we can show that p can't be 3, 5, 7, 11 or 13.

Subcase 4.2 If $K/H \cong A_{p'}(q)$, $q-1 \mid p'-1$ then $(3^p-1)/2 = (q^{p'}-1)/(q-1)$. Thus $q^{p'+1} \geq 3^p$.

If $p' > 7$ then $q^{p'(p'+1)/2} > 3^{3p}$, which implies that q is a power of 3 by Lemma 8 and Lemma 9. It follows that $3^p - 3 = 2(q^{p'-1} + q^{p'-2} + \cdots + q)$, consequently $q = 3$ and $p = p'$, furthermore $3^{p-1}+1 \mid |H|$ since $|G/K||Out(K/H)| = 4$. Similarly to Subcase 1.1, we can get a contradiction. By trivial calculation we can show that p' can't be 3 or 5.

Similarly, we can show that $K/H \ncong {}^2A_n(q)$.

Subcase 4.3. If $K/H \cong A_{p'-1}(q)$ then $(q^{p'}-1)/(q-1)(p',\ q-1) = (3^p-1)/2$.

Similarly to *Subcase 4.2*, we can prove that p' can't be greater than 11. And similarly to *Subcase 4.1*, it is easy to prove that p' can't be 3, 5, 7 or 11.

Case 5. $K/H \ncong {}^2D_n(q)$.

Subcase 5.1. If $K/H \cong {}^2D_{p'}(3)(5 \leq p' \neq 2^k+1)$, then $(3^p-1)/2 = (3^{p'}+1)/42 \cdot 3^p - 3^{p'} = 3$, a contradiction. Similarly, we can prove that $K/H \ncong {}^2D_n(3)(9 \leq n = 2^k+1$ isn't a prime); $K/H \ncong {}^2D_{p'}(3)(5 \leq p' = 2^k+1)$.

Subcase 5.2. If $K/H \cong {}^2D_{p'+1}(2)(p' \neq 2^m-1)$, then $(3^p-1)/2 = 2^{p'}-1$, $3^p = 2^{p'+1}-1$, which contradicts Lemma 4. Similarly we can prove $K/H \ncong {}^2D_{p'+1}(2)(3 \leq p' = 2^k-1)$.

Subcase 5.3. Similarly to Subcase 4.2, we can prove that $K/H \ncong {}^2D_n(q)(2 \leq n = 2^k)$.

Case 6. $K/H \neq E_6(q)$, $E_8(q)$, $F_4(q)$, ${}^2F_4(q)$ or ${}^2E_6(q)$. If $K/H \cong E_6(q)$ then $(q^6+q+1)/(3,\ q-1) = (3^p-1)/2$. Hence $q^9 > 3^p$, $q^{36} > 3^{3(p+1)}$, q is a power of 3 by Lemma 8 and Lemma 9, furthermore $2(q^6+q^3) = 3^p-3$, a contradiction.

Similarly, we can prove that $K/H \neq E_8(q)$, $F_4(q)$, ${}^2F_4(q)$ or ${}^2E_6(q)$.

Case 7. $K/H \ncong G_2(q)$; $K/H \ncong {}^3D_4(q)$; $K/H \ncong {}^2G_2(q)(q = 3^{2k+1})$.

Subcase 7.1. If $K/H \cong G_2(q)(3 \mid q)$, then $(3^p-1)/2 = q^2 \pm q+1$, $3^p-3 = q^2 \pm q$, a contradiction. Similarly we have that $K/H \ncong {}^2G_2(q)(q = 3^{2k+1})$.

Subcase 7.2. Similarly to Subcase 4.1, we can prove that $K/H \ncong G_2(q)(3 \mid q \pm 1)$ or ${}^3D_4(q)$.

Case 8. $K/H \ncong {}^2B_2(q)(4 \leq q = 2^{2k+1})$. If $K/H \cong {}^2B_2(q)(4 \leq q = 2^{2k+1})$, then $(3^p-1)/2 = q \pm \sqrt{2q}+1$ or $q-1$. Clearly $(3^p-1)/2 \neq q-1$

If $(3^p-1)/2 = q+\sqrt{2q}+1$, then $3(3^{p-1}-1) = 2^{k+2}(2^k+1)$, $2^k \mid p-1$ by Lemma 7, furthermore, $2^{k+2}(2^k+1) = 3(3^{p-1}-1) > 3^{p-1} > 3^{2^k} > 2^{2^k} > 2^{2k+3} > 2^{k+2}(2^k+1)$ for $k \geq 4$, a contradiction. By calculation we can prove that k can't be 1, 2 or 3.

Similarly we have $(3^p-1)/2 \neq q-\sqrt{2q}+1$.

Case 9. $K/H \ncong D_n(q)$.

Subcase 9.1. If $K/H \cong D_{p'}(3)$, then $(3^p-1)/2=(3^{p'}-1)/2$, hence $p=p'$, $|G/K|\cdot|H|=3^p(3^P+1)$, which implies that $|H|=3^p(3^p+1)$ or $3^p(3^p+1)/2$ since $|G/K| \mid |Out(K/H)=2$, similarly to Subcase 1.1, we can get a contradiction.

Subcase 9.2. If $K/H \cong D_{p'+1}(3)$, then $(3^p-1)/2=(3^{p'}-1)/2$, $p=p'$, hence, $|D_{p+1}(3)| \mid |B_p(3)|$, which is impossible.

Subcase 9.3. If $K/H \cong D_{p'}(5)(p' \geq 5)$, then $(5^{p'}-1)/4=(3^p-1)/2$, $5^{p'}>3^p$. Furthermore, $5^{p'(p'-1)}>3^{3p}$, which contradicts Lemma 8.

Case 10. From *Case 1* to Case 10 and Lemma 3 we have K/H isomorphic to one of $B_n(q)$ and $C_n(q)$.

Similarly to Subcase 4.2 we can show that $K/H \ncong B_n(q)$ or $C_n(q)(4 \leq n = 2^m)$, and similarly to Subcase 5.2, we can prove that $K/H \ncong C_{p'}(2)$. Hence $K/H \cong B_{p'}(3)$ or $C_{p'}(3)$, it follows that $(3^{p'}-1)/2=(3^p-1)/2$ and $p=p'$, furthermore $K/H=1$, $H=1$, so $G \cong B_p(3)$ or $C_p(3)$. □

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