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GENERALIZED 'USEFUL' INFORMATION GENERATING FUNCTIONS

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ABSTRACT. In the present paper, one new generalized 'useful' information generating function and two new relative 'useful' information generating functions have been defined with their particular and limiting cases. It is interesting to note that differentiations of these information generating functions at t=0 or t=1 give some known and unknown generalized measures of useful information and 'useful' relative information. The information generating functions facilitates to compute various measures and that has been illustrated by applying these information generating functions for Uniform, Geometric and Exponential probability distributions.

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1. Introduction

The moment generating function of a probability distribution is a convenient mean of calculating the moments of the distribution and an effective embodiment of properties of the distribution for various analytical processes.The successive derivatives of the moment generating function at point 0 gives successive moments of a probability distribution. In the same way by differentiating information generating function at point 0 or 1, we can derive the measures of information which are otherwise difficult to characterize and compute .

The concept of information generating function of a probability distribution was defined by Golomb [4] for Shannon's [14] entropy as given below:

$$I(t) = -\sum_{i \in N} p_i^t, t \geq 1, \tag{1}$$

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where $\{p_i\}$ is a complete probability distribution with $i \in N$, N is a discrete sample space and t is a real or complex variable. Further it may be noted that

$$\frac{\partial I(t)}{\partial t}\mid_{t=1}= H(P) = -\sum_{i \in N} p_i log p_i, \tag{2}$$

where H (P) is well-known Shannon's entropy , p_i is probability of occurrence the event E_i for each $i \in N$.

Later on Guiasu and Reischer [5] introduced the relative information generating function whose derivatives give well known statistical indices as the Kullback-Leibler divergence between two probability distributions and Watanabe's measure of interdependence. It contains Golomb's information generating function as a particular case and includes both binomial and Poisson distributions which were not covered in Golomb's work.

Hooda and Singh [9] defined an information improvement generating function whose derivatives at point 1 gives Theil's [16] measure of information improvement which has wide applications in Economics. It contains Guiasu and Reischer's [5] relative information generating function and Golomb's [4] information generating function as particular cases.

The quantity (2) measures average information, but does not take into account the qualitative information of the events. Belis and Guiasu [2] introduced a quantitative -qualitative measure of information

$$H(P;U) = -\sum_{i \in N} u_i p_i log p_i, \tag{3}$$

where $u_i > 0$ is the utility attached to the i^{th} event which occur with probability p_i.The measure (3) was called useful information by Longo [11].

Later on Bhaker and Hooda [3] gave mean value characterization of the following 'useful' information measures:

$$H(P;U) = -\sum_{i \in N} u_i p_i log p_i / \sum_{i \in N} u_i p_i \tag{4}$$

and

$$H_\alpha(P;U) = \frac{1}{1-\alpha} log \sum_{i \in N} u_i p_i^\alpha / \sum_{i \in N} u_i p_i. \tag{5}$$

Hooda and Bhaker [7] defined the following 'useful' information generating function:

$$I(P;U,t) = -\sum_{i \in N} p_i^t / \sum_{i \in N} u_i p_i, \tag{6}$$

where $P = (p_1, p_2, \cdots, p_n)$ *and* $U = (u_1, u_2, \cdots, u_n)$ are respectively probability and utility distributions and t is a real or complex variable.

Since $0 < p_i \leq 1\ \forall i$ and $< u_i >$ is bounded for an experiment, moreover, (4) being positive term series is absolutely convergent $\forall\ i \geq 1$ and also it converges uniformly, therefore each term of the series possesses continuous derivative. Thus, the derivative of (4) w.r.t t at t =1 is possible and given by

$$\frac{\partial}{\partial t} I(P;U,t)\ |_t = 1 = -\sum_{i \in N} u_i p_i log p_i / \sum_{i \in N} u_i p_i = H(P;U), \tag{7}$$

which is (4) and in case the utilities are ignored or $u_i = 1$ for each i, (4) reduces to (1).

2. Generalized 'Useful' information generating function

Suppose

$$P = \{(p_1, p_2, \cdots, p_n)\ 0 < p_i \leq 1,\ \sum_{i=1}^{n} p_i = 1\}$$

be a discrete probability distribution of a set of events $E = \{E_1, E_2, \cdots, E_n\}$ of a discrete infinite sample space N on the basis of an experiment having utility distribution $U = \{(u_1, u_2, \cdots, u_n)\ u_i > 0, \forall i\}$, where N is a discrete sample space. Following Hardy, Littlewood, and Polya [6], we have the following weighted mean of order $\alpha - 1$ of p_i and u_i with weights $(u_i p_i)^{\beta_i}$:

$$M_{\alpha,\beta_i}(P;U) = \left[\sum_{i=1}^{n} u_i^{\beta_i} p_i^{\alpha+\beta_i-1} / \sum_{i=1}^{n} (u_i p_i)^{\beta_i}\right]^{\frac{1}{\alpha-1}},\ \alpha > 0, \alpha \neq 1\ and\ \beta_i \geq 1 \tag{8}$$

for which we have the generalized 'useful' information generating function given by

$$I_{\alpha,\beta_i}(P;U,t) = [M_{\alpha,\beta_i}(P;U)]^{-t} = \left[\sum_{i=1}^{n} u_i^{\beta_i} p_i^{\alpha+\beta_i-1} / \sum_{i=1}^{n} (u_i p_i)^{\beta_i}\right]^{\frac{-t}{\alpha-1}}, \tag{9}$$

where t is a real or complex variable. On differentiating (9) w.r.t. t at t = 0 respectively, we have

$$H_{\alpha}^{\beta_i}(P;U) = (1-\alpha)^{-1} log \left[\sum_{i=1}^{n} u_i^{\beta_i} p_i^{\alpha+\beta_i-1} / \sum_{i=1}^{n} (u_i p_i)^{\beta_i}\right], \tag{10}$$

which is the generalized 'useful' information measure of order α and type $\{\beta_i\}$. When $\beta_i = \beta$ for each i, (10) reduces to

$$H_{\alpha}^{\beta}(P;U) = (1-\alpha)^{-1} log \left[\sum_{i=1}^{n} u_i^{\beta} p_i^{\alpha+\beta-1} / \sum_{i=1}^{n} (u_i p_i)^{\beta}\right], \tag{11}$$

which is the generalized 'useful' information measure of order α and type β characterized by Hooda and Singh [9].

Particular Cases

(i) If utilities in (11) are ignored i.e. $u_i = 1$ for each i, it reduces to Aczel and Daroczy's [1] generalized entropy order α and type β.

(ii) Further if we set $\beta = 1$,we get Renyi's [13] entropy of order α and Shannon's [14] entropy when $\alpha \to 1$.

(iii) When $\beta = 1$, (11) reduces to (5) and further, it reduces to (4) in case $\alpha \to 1$.

Examples. Next we consider information generating functions for uniform, geometric and exponential distributions as particular examples and derive the 'useful' information measures.

(a) For the uniform probability distribution $\left(\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}\right)$ and uniform utility distribution $(u, u, \cdots, u)$, we have $I_{\alpha,\beta}(P; U, t) = N^t$ and

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P; U, t) \mid_{t=0} = logN, \tag{12}$$

which is the same if utilities are ignored i.e. in case of uniform distributions utilities play no role in evaluation of the generalized 'useful' information measure. Moreover, (12) is independent of α and β. Renyi's entropy of any order and Shannon's entropy will have the same value.

(b)Consider the geometric distribution $(q, qp, qp^2, \cdots)$, $p+q = 1$ and geometric utility distribution $(v, vu, vu^2, \cdots)$. It is the most general case when utility also follows geometric distribution. Then we have

$$I_{\alpha,\beta}(P; U, t) = \left[q^{\alpha-1}(1 - u^{\beta}p^{\beta})/(1 - u^{beta}p^{\alpha+\beta-1})\right]^{\frac{t}{1-\alpha}} \tag{13}$$

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P; U, t) \mid_{t=0} = (1 - \alpha)^{-1} \log \left[q^{\alpha-1}(1 - u^{\beta}p^{\beta})/(1 - u^{\beta}p^{\alpha+\beta-1})\right] \tag{14}$$

In case utilities are ignored in (13) and (14), we get the result due to Mathur and Kashyap [12].

(c) For the exponential probability distribution with mean $\frac{1}{\lambda}$ and exponential utility distribution with mean $\frac{1}{\mu}$ we consider

$p(x) = \lambda e^{-\lambda x}$, $\lambda > 0, 0 \leq x < \infty$ and $u(y) = \mu e^{-\mu y}$, $\mu > 0, 0 \leq y < \infty$ and have

$$I_{\alpha,\beta}(P; U, t) = \left[\lambda^{\alpha-1}\beta/(\alpha + \beta - 1)\right]^{\frac{t}{1-\alpha}}, \ \alpha{\neq}1, \ \beta{\geq}1, \ \alpha > 0 \tag{15}$$

and

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P; U, t) \mid_{t=0} = (1 - \alpha)^{-1} log \left[\lambda^{\alpha-\beta}\beta/(\alpha + \beta - 1)\right]. \tag{16}$$

It may be noted that (16) comes out to be independent of utility distribution whatever form it may. Setting $\beta = 1$ in (15) and (16), we get results for Renyi's entropy and further letting $\alpha \to 1$, we get results for Shannon's entropy.

3. Two generalized 'Useful' relative information functions.

Suppose

$$P = \{(p_1, p_2, \cdots, p_n)\ , 0 < p_i \leq 1,\ \sum_{i=1}^{n} p_i = 1\}$$

be a discrete probability distribution whose predicted probability distribution is

$$Q = \{(q_1, q_2, \cdots, q_n),\ 0 < q_i \leq 1,\ \sum_{i=1}^{n} q_i = 1\}$$

and $U = \{(u_1, u_2, \cdots, u_n)\ , 0 < u_i, \forall i, \}$ is utility distribution of a discrete sample space N. Let us consider the measure of 'useful' relative information characterized by Bhaker and Hooda [3]:

$$H(P/Q;U) = \frac{\sum\limits_{i=1}^{n} u_i p_i log(p_i/q_i)}{\sum\limits_{i=1}^{n} u_i p_i} \tag{17}$$

Next we define

$$I(P/Q;U,t) = 2^{tH(P/Q;U)} \tag{18}$$

Differentiating (18) w.r.t. t at t=0, we have

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P;U,t) \mid_{t=0} = \frac{\sum\limits_{i=1}^{n} u_i p_i log(p_i/q_i)}{\sum\limits_{i=1}^{n} u_i p_i} = H(P/Q;U) \tag{19}$$

which is (17). Following Hardy, Littlewood, and Polya [6], we have the following weighted mean of order $\alpha - 1$ of p_i, q_i and u_i with weights $(u_i p_i)^{\beta} i$:

$$M_{\alpha,\beta_i}(P/Q;U) = \left[\sum_{i=1}^{n} {u_i}^{\beta_i}\, {p_i}^{\alpha+\beta_i-1} q_i^{1-\alpha} / \sum_{i=1}^{n} (u_i p_i)^{\beta_i}\right]^{\frac{1}{\alpha-1}}\ ,\ \alpha > 0, \alpha \neq 1 \tag{20}$$

and $\beta_i \geq 1$ for which we have generalized 'useful' relative information generating function given below:

$$I_{\alpha,\beta_i}(P/U;t) = [M_{\alpha,\beta_i}(P/Q;U)]^t = \left[\sum_{i=1}^{n} {u_i}^{\beta_i} {p_i}^{\alpha+\beta_i-1} q_i^{1-\alpha} / \sum_{i=1}^{n} (u_i p_i)^{\beta_i}\right]^{\frac{t}{\alpha-1}} \tag{21}$$

where t is a real or complex variable. Differentiating (21) w.r.t. t at t = 0, we have

$$H_\alpha^{\beta_i}(P/Q;U) = (\alpha-1)^{-1}log\left[\sum_{i=1}^{n} u_i^{\beta_i}\, p_i^{\alpha+\beta_i-1} q_i^{1-\alpha} \Big/ \sum_{i=1}^{n}(u_i p_i)^{\beta_i}\right]. \quad (22)$$

The measure (22) is called the generalized 'useful' relative information measure of order α and type β_i. when $\beta_i = \beta$ for each i, (21) and (22) respectively reduce to the following:

$$I_{\alpha,\beta}(P/U;t) = [M_{\alpha,\beta}(P/Q;U)]^t = \left[\sum_{i=1}^{n} u_i^{\beta}\, p_i^{\alpha+\beta-1} q_i^{1-\alpha} \Big/ \sum_{i=1}^{n}(u_i p_i)^{\beta}\right]^{\frac{t}{\alpha-1}}, \quad (23)$$

and

$$H_\alpha^{\beta}(P/Q;U) = (\alpha-1)^{-1}log\left[\sum_{i=1}^{n} u_i^{\beta}\, p_i^{\alpha+\beta-1} q_i^{1-\alpha} \Big/ \sum_{i=1}^{n}(u_i p_i)^{\beta}\right] \quad (24)$$

which is a new generalized 'useful' relative information measure of order α and type β.

Particular Cases

(i) If utilities are ignored or $u_i = 1$ for each i in (23), we have

$$I_{\alpha,\beta}(P/Q;t) = \left[\sum_{i=1}^{n} p_i^{\alpha+\beta-1} q_i^{1-\alpha} \Big/ \sum_{i=1}^{n} p_i^{\beta}\right]^{\frac{t}{\alpha-1}}, \quad (25)$$

Further, on differentiating (25) w.r.t. t at t = 0, we have

$$H_\alpha^{\beta}(P/Q) = (\alpha-1)^{-1}log\left[\sum_{i=1}^{n} p_i^{\alpha+\beta-1} q_i^{1-\alpha} \Big/ \sum_{i=1}^{n} p_i^{\beta}\right] \quad (26)$$

which is the generalized measure of relative information characterized and studied by Sharma [15].

(ii) If we set $\beta = 1$ in (25), we get

$$I_\alpha(P/Q;t) = \left[\sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha}\right]^{\frac{t}{\alpha-1}}, \quad (27)$$

which is generalized relative information generating function of order α. Further, if $\alpha \to 1$ in (27), it reduces to

$$I(P/Q;t) = 2^{t\sum_{i=1}^{n} p_i log(p_i/q_i)}, \quad (28)$$

On differentiation (28) at t= 0, we get Kullback- Liebler's [10] measure of relative information given by

$$H(P/Q) = \sum_{i=1}^{n} p_i log(p_i/q_i), \tag{29}$$

(iii)In case $\beta = 1$, (23) reduces to

$$I_\alpha(P/U;t) = [M_\alpha(P/Q;U)]^t = \left[\sum_{i=1}^{n} u_i p_i{}^\alpha q_i{}^{1-\alpha} / \sum_{i=1}^{n}(u_i p_i)\right]^{\frac{t}{\alpha-1}}, \tag{30}$$

(30) is the generalized 'useful' relative information generating function of order α and on differentiation at t = 0 it gives

$$H_\alpha(P/Q;U) = (\alpha-1)^{-1} log\left[\sum_{i=1}^{n} u_i p_i{}^\alpha q_i{}^{1-\alpha} / \sum_{i=1}^{n}(u_i p_i)\right], \tag{31}$$

which is the generalized measure of 'useful' relative information characterized by Bhaker and Hooda [3]. When $\alpha \to 1$ (31) reduces to (17). Following Hardy, Littlewood, and Polya [6], we can also have another weighted mean of order $\alpha - 1$ of p_i, q_i, and u_i, as given below:

$$M_{\alpha\beta_i}(P/Q;U) = \left[\frac{\sum_{i=1}^{n} u_i^{\beta_i} p_i{}^{\alpha\beta_i} q_i{}^{1-\alpha}}{\sum_{i=1}^{n} (u_i p_i)^{\beta_i}}\right]^{\frac{1}{1-\alpha}}, \quad \alpha{\neq}1,\ \beta{\geq}1,\ \alpha > 0 \tag{32}$$

for which we have generalized 'useful' relative information generating function given below

$$I_{\alpha\beta_i}(P/Q;U,t) = [M_{\alpha\beta_i}(P/Q;U)]^{-t} = \left[\frac{\sum_{i=1}^{n} u_i^{\beta_i} p_i^{\alpha\beta_i} q_i^{1-\alpha}}{\sum_{i=1}^{n} (u_i p_i)^{\beta_i}}\right]^{\frac{t}{\alpha-1}}, \tag{33}$$

where t is a real or complex variable. Differentiating (33) w.r.t t at t = 0, we have

$$\frac{\partial}{\partial t} I_{\alpha\beta_i}(P/Q;U)\,|_{t=0} = (\alpha-1)^{-1} log\left[\frac{\sum_{i=1}^{n} u_i^{\beta_i} p_i^{\alpha\beta_i} q_i^{1-\alpha}}{\sum_{i=1}^{n} (u_i p_i)^{\beta_i}}\right] = H_\alpha^{\beta_i}(P/Q;U) \tag{34}$$

(34) is a new measure and is called as the generalized useful relative measure of order α and type β_i. when $\beta_i = \beta$ for each i,(34) reduce to

$$I_{\alpha,\beta}(P/Q;U,t) = \left[\frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{\alpha\beta} q_i^{1-\alpha}}{\sum_{i=1}^{n} (u_i p_i)^{\beta}}\right]^{\frac{t}{\alpha-1}} \tag{35}$$

and on differentiation w.r.t. t at t = 1 it gives

$$H_{\alpha}^{\beta}(P/Q;U) = (\alpha-1)^{-1} log \frac{\sum_{i=1}^{n} u_i^{\beta} {p_i}^{\alpha\beta} q_i^{1-\alpha}}{\sum_{i=1}^{n} (u_i p_i)^{\beta}} \tag{36}$$

which is also a new measure and can be called the generalized measure of 'useful' relative information of order α and β.

Particular Cases

(i) If utilities are ignored or $u_i = 1$ for each i in (35), we have

$$I_{\alpha,\beta}(P/Q;t) = \left[\frac{\sum_{i=1}^{n} {p_i}^{\alpha\beta} {q_i}^{1-\alpha}}{\sum_{i=1}^{n} {p_i}^{\beta}}\right]^{\frac{t}{\alpha-1}} \tag{37}$$

Further, on differentiating (37) w.r.t. t at t=0, we get

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P/Q;t) \mid_{t=0} = (\alpha-1)^{-1} log \left[\frac{\sum_{i=1}^{n} {p_i}^{\alpha\beta} {q_i}^{1-\alpha}}{\sum_{i=1}^{n} (p_i)^{\beta}}\right] = H_{\alpha}^{\beta}(P/Q) \tag{38}$$

(ii) If we set $\beta = 1$ in (37), we get

$$I_{\alpha}(P/Q;t) = \left[\sum_{i=1}^{n} {p_i}^{\alpha} {q_i}^{1-\alpha}\right]^{\frac{t}{\alpha-1}}, \tag{39}$$

which is generalized relative information generating function of order α.

Further, if $\alpha \to 1$ in (39), it reduces to the following information generating function for complete discrete Probability distribution P and Q:

$$I(P/Q;t) = 2^{t\sum_{i=1}^{n} p_i log(p_i/q_i)}, \tag{40}$$

On differentiation (40) at t= 0, again we get Kullback- Lieblers [10] measure of relative information given by

$$H(P/Q) = \sum_{i=1}^{n} p_i log(p_i/q_i), \tag{41}$$

(iii) In case $\beta = 1$, (35) reduces to

$$I_\alpha(P/Q;U,t) = \left[\frac{\sum_{i=1}^{n} u_i p_i{}^{\alpha} q_i{}^{1-\alpha}}{\sum_{i=1}^{n}(u_i p_i)}\right]^{\frac{t}{\alpha-1}}, \tag{42}$$

which is the useful relative information generating function of order α On differentiation (42) gives

$$\frac{\partial}{\partial t} I_\alpha(P/Q;U)\mid_{t=0} = (\alpha-1)^{-1} log\left[\frac{\sum_{i=1}^{n} u_i p_i{}^{\alpha} q_i{}^{1-\alpha}}{\sum_{i=1}^{n}(u_i p_i)}\right] = H_\alpha(P/Q;U) \tag{43}$$

which is the generalized relative information characterized by Bhaker and Hooda [3]. Further, if $\alpha \to 1$, (43) becomes $H(P/Q;U) = \frac{\sum_{i=1}^{n} u_i p_i log(p_i/q_i)}{\sum_{i=1}^{n}(u_i p_i)}$, which is (17).

4. Applications

In this section we study the application of new generalized 'useful' relative information generating function in deriving the 'useful' information generating functions for Uniform and Exponential distribution and corresponding 'useful' relative information measures are also derived from these.

Example1. For the uniform probability distribution $\left(\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}\right)$ after experiment, predicted probability distribution $\left(\frac{1}{M}, \frac{1}{M}, \cdots, \frac{1}{M}\right)$ before experiment and the utility distribution $(u, u, \cdots, u)$ of an experiment, also fixing $\beta = 1$ after putting in (23) we have

$$I_{\alpha,\beta}(P;U,t) = \left(\frac{M}{N}\right)^t \tag{44}$$

and

$$\frac{\partial}{\partial t} I_{\alpha,\beta}(P;U,t)\mid_{t=0} = log\left(\frac{M}{N}\right), \tag{45}$$

which is the same result as if utilities are ignored. Thus in case of uniform probability distribution, utilities play no role in the generalized measure of 'useful' relative information.

Example 2. For the actual and predicted exponential density distributions with mean $1/\lambda$ and $1/\mu$ respectively and utility distribution with mean $1/\gamma$, $p(x) = \lambda e^{-\lambda x}$, $\lambda > 0$, $0 \leq x < \infty$ and $q(y) = \mu e^{-\mu y}$, $\mu > 0$, $0 \leq y < \infty$ and $u(z) = \gamma e^{-\gamma y}$, $\gamma > 0$, $0 \leq z < \infty$ On substituting these in (35) and integrating with $\beta = 1$ and $\alpha \to 1$, we get

$$I_\alpha((p(x), q(y); u(z), t)) = \left[\lambda^{\alpha-1}\mu^{1-\alpha}(1-\alpha)\alpha\right]^{\frac{t}{\alpha-1}} \tag{46}$$

and

$$\frac{\partial}{\partial t} I_\alpha((p(x), q(y); u(z), t)) \mid_{t=0} = (\alpha-1)^{-1} log\left[\lambda^{\alpha-1}\mu^{1-\alpha}/(1-\alpha)\alpha\right], \tag{47}$$

which is a generalized measure of useful information of order α given by (43). Further, when $\alpha \to 1$, in (46) and (47), we get the results obtained by Hooda and Bhakar [7].

Similarly, if we consider the actual and predicted Gamma probability distribution with mean p/α and p/μ respectively and utility distribution with mean p/γ, we have

$$p(x) = \frac{\alpha^p e^{-\alpha x} x^{p-1}}{\Gamma(p)}; \; 0 < x < \infty, \alpha > 0,$$
$$q(y) = \frac{\mu^p e^{-\mu x} y^{p-1}}{\Gamma(p)}; \; 0 < y < \infty, \mu > 0,$$
$$u(z) = \frac{\gamma^p e^{-\gamma x} z^{p-1}}{\Gamma(p)}; \; 0 < z < \infty, \gamma > 0.$$

On substituting these in (35) and integrating with $\beta = 1$ and $\alpha \to 1$, we get the result obtained by Hooda and Bhakar [8].

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