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Intuitionistic fuzzy k-ideals of a semiring

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Abstract

We introduce the concepts of intuitionistic fuzzy k-ideals and intuitionistic fuzzy prime k-ideals of a semiring. And we investigate some properties of them.

Keywords and phrases : intuitionistic fuzzy ideal, intuitionistic fuzzy k-ideal, intuitionistic fuzzy prime k-ideal.

1. Introduction

As a generalization of fuzzy sets introduced by Zadeh[17], Atanassov [4] introduced the notion of intuitionistic fuzzy sets in 1986. After that time, Çoker [8] introduced the concept of intuitionistic fuzzy topology by using intuitionistic fuzzy sets. In 1989, Biswas [6] introduced the notion of intuitionistic fuzzy subgroups and studied some of it's properties. In 2003, Banerjee and Basnet [5], Hur et al. [10,11] applied the concept of intuitionistic fuzzy sets to algebra. Since then, Hur et.al. [1,2,12-14] have applied one to group theory, and ring theory.

In this paper, we introduce the concepts of intuitionistic fuzzy k-ideals and intuitionistic fuzzy prime k-ideals of a semiring. And we investigate some properties of them.

2. Preliminaries

We will list some concepts and results needed in the later sections.

For sets X, Y and $Z, f = (f_1, f_2) : X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ *and* $f_2 : X \to Z$ are mappings.

Throughout this paper, we will denote the unit interval[0,1] as I.

Definition 2.1 [4,8]. Let X be a nonempty set. A complex mapping $A = (\mu_A,\ \nu_A) :\ X \to I \times I$ is called *intuitionistic fuzzy set* (in short, IFS) in X if $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each $x \in X$ to A, respectively. In particular, $0_\sim$ and $1_\sim$ denote the *intuitionistic fuzzy empty set* and *intuitionistic fuzzy whole set* in X defined by $0_\sim(x) = (0, 1)$ and $1_\sim(x) = (1, 0)$ for each $x \in X$, respectively.

We will denote the set of all IFSs in X as IFS(X).

Definition 2.2 [4]. Let X be a nonempty sets and let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs on X. Then

(1) $A \subset B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
(2) $A = B$ if and only if $A \subset B$ and $B \subset A$.
(3) $A^c = (\nu_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.

Definition 2.3 [10]. Let $(X, \cdot)$ be a groupoid and let $A, B \in$ IFS(X). Then the *intuitionistic fuzzy product* of A and B, $A \circ B$ is defined as follows : for each $x \in X$,

$$\mathbb{A} \circ B(x) = \begin{cases} \bigvee_{x=yz}[\mu_A(y) \wedge \mu_B(z)], & \\ \bigwedge_{x=yz}[\nu_A(y) \vee \nu_B(y)]) & \text{if} \quad \text{x=yz}, \\ (0, 1) & \text{otherwise} \end{cases}$$

It is clear that $A \circ B \in$ IFS(X), i.e., (IFS(X), $\circ$) is a groupoid.

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3. Intuitionistic fuzzy k- ideal

A *semiring* is defined by an algebra $(S, +, \cdot)$ such that $(S, +)$ and $(S, \cdot)$ are semigroups connected by $a(b+c) = ab + ac$ and $(b+c)a = ba + ca$ for all $a, b, c \in S$. A semiring may have an identity 1, defined by $1 \cdot a = a \cdot 1 = a$ and a zero 0 (which is an absorbing zero also), defined by $0 + a = a + 0 = a$ and $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$(see[7]).

A subset $J\ (\neq \emptyset)$ of a semiring S is called *a left ideal* of S, if $a + b \in J, sa \in J$ for all $a, b \in J$ and all $s \in S$. Right ideal is defined dually and a *two sided ideal* or simply an *ideal* is both a left and a right ideal(see[7]).

Definition 3.1 [15]. Let A be a nonempty intuitionistic fuzzy set in a semiring S. Then A is called an:

(1) *intuitionistic fuzzy left ideal* (in short, $IFLI$) of S if

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y),\ \nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$$

and

$$\mu_A(xy) \geq \mu_A(y),\ \nu_A(xy) \leq \nu_A(y),\ \text{for any } x, y \in S.$$

(2) *intuitionistic fuzzy right ideal* (in short, $IFRI$) of S if

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y),\ \nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$$

and

$$\mu_A(xy) \geq \mu_A(x),\ \ \nu_A(xy) \leq \nu_A(x).$$

(3) *intuitionistic fuzzy ideal* (in short, IFI) of S if it is both an IFLI and an IFRI of S.

We will denote the set of all IFIs[resp. IFLIs and IFRIs] of S as IFI(S) [resp. IFRI(S) and IFRI(S)], respectively.

It is clear that $A \in$ IFI(S) if and only if for any $x, y \in S$.

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y),\ \nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$$

and

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y),\ \nu_A(xy) \leq \nu_A(x) \vee \nu_A(y).$$

Moreover, it is clear that if S is a semiring with zero 0 and $A \in$ IFI(S), then $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$ for each $x \in S$.

A *left k-ideal* J of a semiring S is a left ideal such that if $a \in J$ and $x \in S$, and if $a + x \in J$ or $x + a \in J$ then $x \in J$.

Right k-ideal is defined dually, and *two sided k- ideal* or simply a *k- ideal* is both a left and a right k- ideal (See[7]).

Definition 3.2 Let A be a nonempty intuitionistic fuzzy set in a semiring S satisfying the following conditions : for any $x, y \in S$,

$$\mu_A(x) \geq [\mu_A(x+y) \vee \mu_A(y+x)] \wedge \mu_A(y)$$

and

$$\nu_A(x) \leq [\nu_A(x+y) \wedge \nu_A(y+x)] \vee \nu_A(y).$$

If S is additively commutative, then the conditions reduce to

$$\mu_A(x) \geq \mu_A(x+y) \wedge \mu_A(y) \text{ and } \nu_A(x) \leq \nu_A(x+y) \vee \nu_A(y) \text{ for any } x, y \in S.$$

Then A is called an :

(1) *intuitionistic fuzzy left k-ideal*(in short, $IFLKI$)of S if $A \in$ IFLI(S).

(2) *intuitionistic fuzzy right k-ideal*(in short, $IFRKI$)of S if $A \in$ IFRI(S).

(3) *intuitionistic fuzzy k-ideal*(in short, $IFKI$)of S if $A \in$ IFI(S).

We will denote the set of all IFKIs[resp. IFLKIs and IFRKIs] of S as IFKI(S) [resp. IFKI(S) and IFKI(S)].

Example 3.2. (1) In a ring, every intuitionistic fuzzy ideal is an intuitionistic fuzzy k-ideal.

(2) Let A be an intuitionistic fuzzy set in the semiring $\mathbb{N}$ defined by : for any $x \in \mathbb{N}$,

$$A(x) = (0.3, 0.6) \quad \text{if } x \text{ is odd},$$
$$= (0.5, 0.4) \quad \text{if } x \text{ is non-zero even},$$
$$= (1, 0) \quad \text{if } x = 0.$$

where $\mathbb{N}$ denotes the semiring of non-negative integers under the usual operations. Then $A \in$ IFKI($\mathbb{N}$).

(3) Let B be an intuitionistic fuzzy set in $\mathbb{N}$ denoted by : for any $x \in \mathbb{N}$,

$$B(x) = (1, 0) \quad \text{if } x \geq 7,$$
$$= (0.5, 0.4) \quad \text{if } 5 \leq x < 7,$$
$$= (0, 1) \quad \text{if } 0 \leq x < 5.$$

Then it can be easily verified that $B \in$ IFI($\mathbb{N}$) but $B \notin$ IFKI($\mathbb{N}$).

Result 3.A[10, Proposition 3.8]. *Let A be a nonempty subset of a semigroup S. Where χ_A denotes the characteristic function of A.*

(1) *A is a left [resp. right] ideal of S of and only if $(\chi_A, \chi_{A^c}) \in IFLI(S)$ [resp. $IFRI(S)$.]*

(2) *A is an ideal of S if and only if $(\chi_A, \chi_{A^c}) \in IFI(S)$.*

The following is the immediate result of Result 3.A and Definition 3.2 :

Propositi 3.3. *Let A be a nonempty subset of a semiring S. Then A is a k-ideal of S if and only if $(\chi_A, \chi_{A^c}) \in IFKI(S)$.*

Proposition 3.4. *Let S be a semiring with zero 0 and let $A \in IFKI(S)$. If $x+y=0$, then $A(x)=A(y)$ for any $x, y \in S$.*

Proof. Since $A \in$ IFKI(S),

$$\mu_A(x) \geq [\mu_A(x+y) \vee \mu_A(y+x)] \wedge \mu_A(y) = [\mu_A(0) \vee \mu_A(y+x)] \wedge \mu_A(y).$$

and

$$\nu_A(x) \leq [\nu_A(x+y) \wedge \nu_A(y+x)] \vee \nu_A(y) = [\nu_A(0) \wedge \nu_A(y+x)] \vee \nu_A(y).$$

Since $A \in$ IFI(S),

$\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$, for each $x \in S$.

Thus $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$. By the similar arguments, $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$. So $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.
Hence $A(x) = A(y)$. □

Proposition 3.5. *Let S be a semiring with zero 0, let $A \in IFKI(S)$ and let $S_A = \{x \in S : A(x) = A(0)\}$. Then S_A is a k-ideal of S.*

Proof. Let $x, y \in S_A$. Then

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y) \quad \text{[Since } A \in \text{IFI}(S)\text{]}$$
$$= \mu_A(0) \quad \text{[By the definition of } S_A\text{]}$$

and

$$\nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y) = \nu_A(0).$$

On the other hand, since $A \in$ IFI(S),

$\mu_A(0) \geq \mu_A(x+y)$ and $\nu_A(0) \leq \nu_A(x+y)$.

Thus $A(x+y) = A(0)$. So $x+y \in S_A$. Now let $s \in S$ and let $x \in S_A$.

Then

$$\mu_A(sx) \geq \mu_A(x) = \mu_A(0)$$

and

$$\nu_A(sx) \leq \nu_A(x) = \nu_A(0).$$

Similarly, $\mu_A(0 \geq \mu_A(sx)$ and $\nu_A(0) \leq \nu_A(sx)$.

Thus $A(sx) = A(0)$. So $sx \in S_A$. By the similar arguments, it can be shown that $xs \in S_A$. Hence S_A is an ideal of S.

Now suppose $x \in S, a \in S_A$ and $a+x \in S_A$ or $x+a \in S_A$.
Then

$$\mu_A(x) \geq [\mu_A(a+x) \vee \mu_A(x+a)] \wedge \mu_A(a) = \mu_A(0)$$

and

$$\nu_A(x) \leq [\nu_A(a+x) \wedge \nu_A(x+a)] \vee \nu_A(a) = \nu_A(0).$$

Similarly, we can see that $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$.

Thus $A(x) = A(0)$. So $x \in S_A$. Therefore S_A is a k-ideal of S. □

4. Intuitionistic fuzzy prime k-ideal of $\mathbb{N}$

An ideal P of a semiring S is said to be *prime* if and only if $AB \subset P$ for any two ideals A, B of S implies that either $A \subset P$ or $B \subset P$.

P is defined to be *prime k-ideal* if P is a k-ideal satisfying the above condition (See[6]).

Definition 4.1[14]. Let P be an IFI of a semiring S. Then P is said to be *prime* if P is not a constant mapping and for any $A, B \in$ IFI(S), $A \circ B \subset P$ implies either $A \subset P$ or $B \subset P$.

We will denote the set of all intuitionistic fuzzy prime ideals of S as IFPI(S).

Result 4.A[14, Proposition 3.3]. *Let S be a semiring with zero and let $P \in IFPI(S)$. Then S_P is a prime ideal of S.*

Definition 4.2. Let P be an intuitionistic fuzzy set in a semiring S. Then P is called an *intuitionistic fuzzy prime k-ideal* (in short, $IFPKI$) of S if $P \in$ IFPI(S)∩IFKI(S).

We will denote the set of all intuitionistic fuzzy prime k-ideals of S as IFPKI(S).

The following is the immediate result of Result 4.A and Proposition 3.5 :

Proposition 4.3. *Let S be a semiring with zero and let $P \in$ IFPKI(S). Then S_P is a prime k-ideal of S.*

Result 4.B[16]. *The semiring $\mathbb{N}$ has exactly the k-ideal*

$$(a) = \{na : n \in \mathbb{N}\} \text{ for each } a \in \mathbb{N}.$$

Consequently, the maximal k-ideal of $\mathbb{N}$ are given by (p) for each prime number p.

Proposition 4.4. *Let $A \in IFKI(\mathbb{N})$. Then there exists $a \in \mathbb{N}$ such that*
$\mathbb{N}_A = \{na : n \in \mathbb{N}\}$.

Proof. Since $A \in$ IFKI($\mathbb{N}$), by Proposition 3.5, $\mathbb{N}_A$ is a k-ideal of $\mathbb{N}$. Thus, by Result 4.B, there exists $a \in \mathbb{N}$ such that $\mathbb{N}_A = \{na : n \in \mathbb{N}\}$. Hence this completes the proof. □

Result 4.C[3]. *Let $a, b \in \mathbb{N}$ such that $a \neq 0$ and $b \neq 0$. If d is the greatest common divisor of a and b, then there exist $s, t \in \mathbb{N}$ such that $sa = tb + a$ or $tb = sa + d$.*

Proposition 4.5. *Let $A \in IFKI(\mathbb{N})$ with $\mathbb{N}_A = n\mathbb{N} \neq (0)$ $(n \in \mathbb{N})$. Then A takes almost r values, where r is the number of distinct divisors of n.*

Proof. Let $a \in \mathbb{N}$ with $a \neq 0$. Suppose d is the greatest common divisor of a and n. Then, by Result 4.C, there exist $s, t \in \mathbb{N}$ such that $ns = at+d$ or $at = ns+d$.

Case1 : $ns = at + d$. Then

$$\mu_A(at+d) = \mu_A(ns) \geq \mu_A(n) \text{ [Since } A \in \text{IFI}(\mathbb{N})] = \mu_A(0) \text{ [Since } n \in n\mathbb{N} = \mathbb{N}_A] \geq \mu_A(at)$$

and

$$\nu_A(at+d) = \nu_A(ns) \leq \nu_A(n) = \nu_A(0) \leq \nu_A(at).$$

Thus

$$\mu_A(d) \geq \mu_A(at+d) \wedge \mu_A(at) \text{ [Since } A \in \text{IFKI}(\mathbb{N})] = \mu_A(at) \geq \mu_A(a)$$

and

$$\nu_A(d) \leq \nu_A(at+d) \vee \nu_A(at) = \nu_A(at) \leq \nu_A(a)$$

Case2 : $at = ns + d$. Then, by the similar arguments of Case1,

$$\mu_A(d) \geq \mu_A(a) \text{ and } \nu_A(d) \leq \nu_A(a)$$

So, in both the cases, $\mu_A(d) \geq \mu_A(a)$ and $\nu_A(d) \leq \nu_A(a)$.
Since d is a divisor of a, there exits $r \in \mathbb{N}$ such that $a = dr$.
Then, since $A \in \text{IFI}(\mathbb{N})$,

$$\mu_A(a) = \mu_A(dr) \geq \mu_A(d)$$

and

$$\nu_A(a) = \nu_A(dr) \leq \nu_A(d)$$

Thus $\mu_A(a) = \mu_A(d)$ and $\nu_A(a) = \nu_A(d)$. So $A(a) = A(d)$.

Hence for each $0 \neq a \in \mathbb{N}$ there exits a divisor d of n such that $A(a) = A(d)$.

Suppose $a = 0$. Since $\mathbb{N}_A = n\mathbb{N}$, $A(a) = A(0) = A(n)$. This completes the proof. □

Result 4.D[9, Lemma 4.1]. *In $\mathbb{N}$, (P) is a prime k-ideal if and only if p is prime.*

Theorem 4.6. *Let A be a non null [i.e., $\mathbb{N}_A \neq (0)$] intuitionistic fuzzy prime k-ideal of $\mathbb{N}$. Then A has two distinct values. Conversely, if $A \in IFS(\mathbb{N})$ such that $A(n) = (1, 0)$ when $p|n$ and $A(n) = (\lambda, \mu)$ when $p \nmid n$, where p is a fixed prime and $(\lambda, \mu) \in [0, 1) \times (0, 1]$ and $\lambda + \mu \leq 1$, then A is a non null intuitionistic fuzzy prime k-ideal of $\mathbb{N}$.*

Proof. Since $A \in \text{IFKI}(\mathbb{N})$, by Proposition 4.4, there exists $a \in \mathbb{N}$ such that $\mathbb{N}_A = \{na : n \in \mathbb{N}\}$. Then, by the hypothesis, $\mathbb{N}_A = n\mathbb{N} \neq (0)$. Since $A \in \text{IFPKI}(\mathbb{N})$, by Proposition 4.3, $\mathbb{N}_A$ is a prime *k-ideal* of $\mathbb{N}$. Thus, by Result 4.D, P is a prime number. So, by Proposition 4.5, A has almost two values. But, A is not a constant function, since A is an intuitionistic fuzzy prime *k-ideal* of $\mathbb{N}$. Hence A has two distinct valuses.

Conversely, suppose A is an intuitionistic fuzzy set in $\mathbb{N}$ satisfying the given conditions. Let $x, y \in \mathbb{N}$.

Case1 : $A(x) = (\lambda, \mu)$ or $A(y) = (\lambda, \mu)$. Then

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y) \text{ and } \nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y).$$

Case2 : $A(x) = (1, 0)$ and $A(y) = (1, 0)$. Then $p|x$ and $p|y$.

Thus $p|(x + y)$. So $A(x + y) = (1, 0) = (\mu_A(x) \wedge \mu_A(y),\ \nu_A(x) \vee \nu_A(y))$. Hence

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y) \text{ and } \nu_A(x+y) \leq \nu_A(x) \vee \nu_A(y)$$

Case3 : $A(x) = (1.0)$. Then $p|x$. Thus $p|xy$. So $A(xy) = (1, 0) = A(x)$.
Hence

$$\mu_A(xy) \geq \mu_A(x) \text{ and } \nu_A(xy) \leq \nu_A(x).$$

Case4 : $A(x) = (\lambda, \mu)$, Then clearly

$$\mu_A(xy) \geq \mu_A(x) \text{ and } \nu_A(xy) \leq \nu_A(x).$$

Therefore $A \in \text{IFI}(\mathbb{N})$.

Now we will prove that

$$\mu_A(x) \geq \mu_A(x+y) \wedge \mu_A(y) \text{ and } \nu_A(x) \leq \nu_A(x+y) \vee \mu_A(y). \tag{4.6.1}$$

Case1 : $A(x + y) = (\lambda, \mu)$ or $A(y) = (\lambda, \mu)$. Then there is nothing to prove.

Case2 : $A(x + y) = (1, 0)$ and $A(y) = (1, 0)$. Then $p|(x + y)$ and $p|y$. Thus $p|x$. So $A(x) = (1, 0)$. Hecne (4.6.1) holds. Therefore $A \in \text{IFKI}(\mathbb{N})$.
By proposition 3.5, $\mathbb{N}_A$ is a k-ideal of $\mathbb{N}$. Next we prove that $\mathbb{N} = p\mathbb{N} \neq (0)$ is a prime k-ideal of $\mathbb{N}$. Let $x \in \mathbb{N}_A$. Then

$$A(x) = A(0) = (1, 0) \Leftrightarrow p|x \Leftrightarrow x = pn \text{ for some } n \in \mathbb{N} \Leftrightarrow x \in p\mathbb{N}.$$

Thus $\mathbb{N}_A = p\mathbb{N} \neq (0)$, where p is a fixed prime. So, by Result 4.D, $\mathbb{N}_A$ is a prime k-ideal of $\mathbb{N}$. Since A has two distinct values $(1, 0)$ and (λ, μ), A is not a constant function. Now assume that $A_1, A_2 \in \text{IFKI}(\mathbb{N})$ such that $A_1 \circ A_2 \subset A$ and $A_1 \not\subset A$ and $A_2 \not\subset A$. Then there exist $x, y \in \mathbb{N}$ such that

$$\mu_{A_1}(x) > \mu_A(x),\ \nu_{A_1}(x) < \nu_A(x)$$

and

$$\mu_{A_2}(y) > \mu_A(y),\ \nu_{A_2}(y) < \nu_A(y).$$

Thus $A(x) = A(y) = (\lambda, \mu)$. So $x, y \notin \mathbb{N}_A$. Since $\mathbb{N}_A$ is a prime k-ideal of commutative semiring $\mathbb{N}$, $xy \notin \mathbb{N}_A$. Then $A(xy) = (\lambda, \mu)$. So

$$\mu_{A_1 \circ A_2}(xy) \leq \mu_A(xy) = \lambda \text{ and } \nu_{A_1 \circ A_2}(xy) \geq \nu_A(xy) = \mu. \tag{4.6.2}$$

But $\mu_{A_1 \circ A_2}(xy) \geq \mu_{A_1}(x) \wedge \mu_{A_2}(y) > \lambda$ and $\nu_{A_1 \circ A_2}(xy) \leq \nu_{A_1}(x) \vee \nu_{A_2}(y) < \mu$.
This contradicts (4.6.2). Hence $A_1 \subset A$ or $A_2 \subset A$. Therefore $A \in \text{IFPKI}(\mathbb{N})$. This completes the

proof. □

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