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# Fuzzy Semi-Weakly r-Semicontinuous 함수에 관한 연구

# Fuzzy Semi-Weakly r-Semicontinuous Mappings

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**요 약**

Fuzzy semi-weakly r-semicontinuous 함수의 개념을 소개하며 특성을 조사한다. 본 논문에서 소개된 함수와 fuzzy r-semicontinuity와 fuzzy weakly r-semicontinuity의 관계를 밝힌다.

## Abstract

In this paper, we introduce the concept of fuzzy semi-weakly r-semicontinuous mappings on a fuzzy topological space and study characterizations for such mappings. And we investigate the relationships among fuzzy r-semicontinuity, fuzzy semi-weakly r-semicontinuity and fuzzy weakly r-semicontinuity.

**Key Words** : fuzzy semi-weakly r-semicontinuous, fuzzy weakly r-semicontinuous, fuzzy S-weakly r-continuous, fuzzy r-irresolute.

## 1. 서 론

Chang [1] defined fuzzy topological spaces using fuzzy sets introduced by Zadeh [10]. In [3, 4], Chattopadhyay, Hazra and Samanta introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces.

Lee and Kim [8] introduced and studied the concept of fuzzy weakly $r$-semicontinuous mappings in fuzzy topological spaces defined by Chattopadhyay, which is a generalized concept of fuzzy weakly semicontinuous mappings defined in the Chang's fuzzy topological spaces.

In this paper, we introduce and study the concept of fuzzy semi-weakly $r$-semicontinuous mappings on the fuzzy topological space which is a generalization of fuzzy $r$-irresolute mappings. In particular, we investigate the relationships among fuzzy $r$-semicontinuity, fuzzy weakly $r$-semicontinuity and fuzzy semi-weakly $r$-semicontinuity.

## 2. Preliminaries

Let $I$ be the unit interval $[0,1]$ of the real line. A member $\mu$ of $I^X$ is called a *fuzzy set* of $X$. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, $\mu^c$ denotes the complement $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* $x_\alpha$ in $X$ is a fuzzy set $x_\alpha$ is defined as follows

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

A fuzzy point $x_\alpha$ is said to belong to a fuzzy set $A$ in $X$, denoted by $x_\alpha \in A$, if $\alpha \le A(x)$ for $x \in X$.

A fuzzy set $A$ in $X$ is the union of all fuzzy points which belong to $A$.

Let $f: X \to Y$ be a mapping and $\alpha \in I^X$ and $\beta \in I^Y$. Then $f(\alpha)$ is a fuzzy set in $Y$, defined by

$$f(\alpha)(y) \begin{cases} \sup_{z \in f^{-1}(y)} \alpha(z), & \text{if } f^{-1}(y) \neq \varnothing, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$ and $f^{-1}(\beta)$ is a fuzzy set in $X$, defined by $f^{-1}(\beta)(x) = \beta(f(x))$, $x \in X$.

A *fuzzy topology* [3, 4] on $X$ is a map $T: I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \ge T(\mu_1) \wedge T(\mu_2)$ for $\mu_1, \mu_2 \in I^X$.

(3) $T(\vee \mu_i) \ge \wedge T(\mu_i)$ for $\mu_i \in I^X$.

The pair $(X, T)$ is alled a *fuzzy topological space.*

And $\mu \in I^X$ is said to be *fuzzy $r$-open* (resp., *fuzzy $r$-closed*) if $T(\mu) \ge r$ (resp., $T(\mu^c) \ge$ r).

The *$r$-closure* and the *$r$-interior* of $A$, denoted by

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cl($A$, $r$) and int($A$, $r$), respectively, are defined as

cl($A$, $r$)= $\cap\{B\in I^X$: $A\subseteq B$ and $B$ is fuzzy $r$-closed$\}$,

int($A$, $r$)= $\cup\{B\in I^X$: $B\subseteq A$ and $B$ is fuzzy $r$-open$\}$.

**Definition 2.1 ([6]).** Let $A$ be a fuzzy set in an FTS $(X,T)$ and $r\in(0,1]=I_0$. Then $A$ is said to be *fuzzy $r$-semiopen* if there is a fuzzy $r$-open set $B$ in $X$ such that $B\subseteq A\subseteq \mathrm{cl}(B,r)$.

Let $A\in I^X$ in an FTS $(X,T)$ and $r\in(0,1]=I_0$. The *fuzzy $r$-semi-closure* and the *fuzzy $r$-semi-interior* of $A$, denoted by scl($A$, $r$) and sint($A$, $r$), respectively, are defined as

scl($A$, $r$)= $\cap\{B\in I^X$: $A\subseteq B$ and $B$ is fuzzy $r$-semiclosed$\}$,

sint($A$, $r$)= $\cup\{B\in I^X$: $B\subseteq A$ and $B$ is fuzzy $r$-semiopen$\}$.

**Definition 2.2 ([6, 7, 8, 9]).** Let $f:X\rightarrow Y$ be a mapping from FTS's $X$ and $Y$. Then $f$ is said to be

(1) *fuzzy $r$-irresolute* [7] if for each fuzzy $r$-semiopen set $B$ of $Y$, $f^{-1}(B)$ is a fuzzy $r$-semiopen set in $X$,

(2) *fuzzy $r$-semicontinous* [6] if for each fuzzy $r$-semiopen set $B$ of $Y$, $f^{-1}(B)$ is a fuzzy $r$-semiopen set in $X$,

(3) *fuzzy weakly $r$-semicontinuous* [8] if for each fuzzy $r$-open set $B$ of $Y$, $f^{-1}(B)\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r)$,

(4) *fuzzy $S$-weakly $r$-continuous* [9] if $f^{-1}(B)\subseteq \mathrm{sint}(f^{-1}(\mathrm{cl}(B,r)),r)$ for each fuzzy $r$-open set $B$ of $Y$.

## 3. Main Results

**Definition 3.1.** Let $f:X\rightarrow Y$ be a mapping from FTS's $X$ and $Y$ and $r\in(0,1]=I_0$. Then $f$ is said to be *fuzzy semi-weakly $r$-semicontinuous* if $f^{-1}(A)\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(A,\ r)),\ r)$ for each fuzzy $r$-semiopen set $A$ of $Y$.

**Remark 3.2.** Every fuzzy $r$-semicontinuous mapping is fuzzy semi-weakly $r$-semicontinuous but the converse is not always true.

**Example 3.3.** Let $X$=$I$ and let $A_1$ and $A_2$ be fuzzy sets of $X$ defined as

$$A_1(x)=-\frac{1}{4}x+1,\quad \text{for } x\in I,$$

$$A_2(x)=-\frac{1}{2}x+1,\quad \text{for } x\in I.$$

Define a fuzzy topology $T:I^X\rightarrow I$ by

$$T(\sigma)=\begin{cases}1, & \text{if } \sigma=\tilde{0},\tilde{1},\\ \frac{1}{2}, & \text{if } \sigma=A_1,\\ 0, & otherwise;\end{cases}$$

and a fuzzy topology $U$: $I^X\rightarrow I$ by

$$U(\sigma)=\begin{cases}1, & \text{if } \sigma=\tilde{0},\tilde{1},\\ \frac{2}{3}, & \text{if } \sigma=A_2,\\ 0, & otherwise.\end{cases}$$

Consider the identity mapping $f:(X,\ T)\rightarrow(X,\ U)$. We know that every fuzzy set $B$ containing $A_2$ is fuzzy $\frac{1}{2}$-semiopen in the FTS $(X,\ U)$ and $\mathrm{scl}(B,\ \frac{1}{2})=\tilde{1}$. Hence the mapping $f$ is a fuzzy semi-weakly $\frac{1}{2}$-semicontinuous mapping but it is not fuzzy $\frac{1}{2}$-semicontinuous.

**Remark 3.4.** Every fuzzy semi-weakly $r$-semi continuous mapping is fuzzy weakly $r$-semicontinuous but the converse is not always true.

**Example 3.5.** Let $X$=$I$ and let $A_1$, $A_2$ and $A_3$ be fuzzy sets of $X$ defined as

$$A_1(x)=\frac{1}{10},\quad \text{for } x\in I,$$

$$A_2(x)=\frac{3}{10},\quad \text{for } x\in I,$$

$$A_3(x)=\frac{8}{10},\quad \text{for } x\in I.$$

Define a fuzzy topology $T:I^X\rightarrow I$ by

$$T(\sigma)=\begin{cases}1, & \text{if } \sigma=\tilde{0},\tilde{1},\\ \frac{2}{3}, & \text{if } \sigma=A_1,\ A_3,\\ 0, & otherwise.\end{cases}$$

and a fuzzy topology $U$: $I^X\rightarrow I$ by

$$U(\sigma)=\begin{cases}1, & \text{if } \sigma=\tilde{0},\tilde{1},\\ \frac{1}{2}, & \text{if } \sigma=A_1,\ A_2,\\ 0, & otherwise;\end{cases}$$

Consider the identity mapping $f:(X,\ T)\rightarrow(X,\ U)$. Then obviously $f$ is fuzzy weakly $\frac{1}{2}$-semicontinuous. Consider a fuzzy semiopen set $B$ in $(X,\ U)$ defined as $B(x)=\frac{1}{4}$ for $x\in I$. Then $\mathrm{sint}(f^{-1}(\mathrm{scl}(B,\ \frac{1}{2})),\ \frac{1}{2})=\mathrm{sint}(A_2,\ \frac{1}{2})=\tilde{1}-A_3$ in the FTS $(X,\ T)$ and $f^{-1}(B)=B>\tilde{1}-A_3$. Hence the mapping $f$ is not a fuzzy semi-weakly $\frac{1}{2}$-semicontinuous mapping.

Now the following implications are obtained.

fuzzy $r$-irresolute ⇒ fuzzy weakly $r$-semicont. ⇒ fuzzy semi-weakly $r$-semicont. ⇒ fuzzy weakly $r$-semicont. ⇒ fuzzy S-weakly $r$-cont.

**Theorem 3.6.** Let $f:(X,T)\to(X,U)$ be a mapping on FTS's $(X,T)$ and $(Y,U)$ $(r\in I_0)$. Then $f$ is a fuzzy semi-weakly $r$-semicontinuous mapping if and only if for every fuzzy point $x_\alpha$ and each fuzzy $r$-semiopen set $V$ containing $f(x_\alpha)$, there exists a fuzzy $r$-semiopen set $U$ containing $x_\alpha$ such that $f(U)\subseteq \mathrm{scl}(V,r)$.

**Proof.** Suppose $f$ is a fuzzy semi-weakly $r$-semi-continuous mapping. Let $x_\alpha$ be a fuzzy point in $X$ and $V$ a fuzzy $r$-semiopen set containing $f(x_\alpha)$. Then there exists a fuzzy $r$-semiopen set $B$ such that $f(x_\alpha)\in B\subseteq V$. From the hypothesis, it follows

$$f^{-1}(B)\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r)\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(V,r)),r).$$

Set $U=\mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r)$. Since $U$ is a fuzzy $r$-semiopen set such that $f^{-1}(B)\subseteq U\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(V,r)),r)\subseteq f^{-1}(\mathrm{scl}(V,r))$, we have $f(U)\subseteq \mathrm{scl}(V,r)$.

For the converse, let $V$ be a fuzzy $r$-semiopen set in $Y$. For each $x_\alpha\in f^{-1}(V)$, there exists a fuzzy $r$-semiopen set $U_{x_\alpha}$ containing $x_\alpha$ such that $f(U_{x_\alpha})\subseteq \mathrm{scl}(V,r)$. This implies

$$f^{-1}(V)\subseteq \cup\{U_{x_\alpha}:\ x_\alpha\in f^{-1}(V)\}\subseteq f^{-1}(\mathrm{scl}(V,r)).$$

Since $\cup\{U_{x_\alpha}:\ x_\alpha\in f^{-1}(V)\}$ is a fuzzy $r$-semiopen set containing $f^{-1}(V)$, we have $f^{-1}(V)\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(V,r)),r)$. Hence $f$ is a fuzzy semi-weakly $r$-semi continuous function.

**Theorem 3.7.** ([7]) Let $A$ be a fuzzy set in an FTS $(X,T)$ and $r\in I_0$. Then we have

(1) $\tilde{1}-\mathrm{scl}(A,r)=\mathrm{sint}(\tilde{1}-A,r)$,

(2) $\tilde{1}-\mathrm{sint}(A,r)=\mathrm{scl}(\tilde{1}-A,r)$.

**Theorem 3.8.** Let $f:(X,T)\to(X,U)$ be a mapping on FTS's $(X,T)$ and $(Y,U)$ $(r\in I_0)$. Then the following statements are equivalent:

(1) $f$ is fuzzy semi-weakly $r$-semicontinuous.

(2) $\mathrm{scl}(f^{-1}(\mathrm{sint}(F,r)),r)\subseteq f^{-1}(F)$ for each fuzzy $r$-semiclosed set $F$ in $Y$.

(3) $\mathrm{scl}(f^{-1}(\mathrm{sint}(B,r)),r)\subseteq f^{-1}(\mathrm{scl}(B,r))$ for each fuzzy set $B$ in $Y$.

(4) $f^{-1}(\mathrm{sint}(B,r))\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r)$ for each fuzzy set $B$ in $Y$.

(5) $\mathrm{scl}(f^{-1}(V),r)\subseteq f^{-1}(\mathrm{scl}(V,r))$ for a fuzzy $r$-semiopen set $V$ in $Y$.

**Proof.** (1) ⇒ (2) Let $F$ be any fuzzy $r$-semiclosed set of $Y$. Then from the hypothesis and Theorem 3.7, it follows

$$\begin{aligned}f^{-1}(\tilde{1}-F)&\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(\tilde{1}-F,r)),r)\\&=\mathrm{sint}(f^{-1}(\tilde{1}-\mathrm{sint}(F,r)),r)\\&=\mathrm{sint}(\tilde{1}-f^{-1}(\mathrm{sint}(F,r)),r)\\&=\tilde{1}-\mathrm{scl}(f^{-1}(\mathrm{sint}(F,r)),r).\end{aligned}$$

Hence we have $\mathrm{scl}(f^{-1}(\mathrm{sint}(F,r)),r)\subseteq f^{-1}(F)$.

(2) ⇒ (3) For $B\in I^Y$, since $\mathrm{scl}(B,r)$ is a fuzzy $r$-semiclosed set in $Y$, from (2), it follows

$$\begin{aligned}\mathrm{scl}(f^{-1}(\mathrm{sint}(B,r)),r)&\subseteq \mathrm{scl}(f^{-1}(\mathrm{sint}(\mathrm{scl}(B,r),r))\\&\subseteq f^{-1}(\mathrm{scl}(B,r)).\end{aligned}$$

(3) ⇒ (4) For $B\in I^Y$, from Theorem 3.7 and (3),

$$\begin{aligned}f^{-1}(\mathrm{sint}(B,r))&=f^{-1}(\tilde{1}-\mathrm{scl}(\tilde{1}-B,r))\\&=\tilde{1}-(f^{-1}(\mathrm{scl}(\tilde{1}-B,r)))\\&\subseteq \tilde{1}-\mathrm{scl}(f^{-1}(\mathrm{sint}(\tilde{1}-B,r)),r)\\&=\mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r).\end{aligned}$$

This implies

$$f^{-1}(\mathrm{sint}(B,r))\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B,r)),r).$$

(4) ⇒ (5) Let $V$ be any fuzzy $r$-semiopen set of $Y$. From (4) and $\mathrm{scl}(\tilde{1}-V,r)=\tilde{1}-V$, it follows

$$\begin{aligned}\tilde{1}-f^{-1}(\mathrm{scl}(V,r))&=f^{-1}(\mathrm{sint}(\tilde{1}-V,r))\\&\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(\tilde{1}-V,r)),r)\\&=\mathrm{sint}(f^{-1}(\tilde{1}-V),r)\\&=\mathrm{sint}(\tilde{1}-f^{-1}(V),r)\\&=\tilde{1}-\mathrm{scl}(f^{-1}(V),r).\end{aligned}$$

Hence we have $\mathrm{scl}(f^{-1}(V),r)\subseteq f^{-1}(\mathrm{scl}(V,r))$.

(5) ⇒ (1) Let $V$ be a fuzzy $r$-semiopen set in $Y$. Then from $V\subseteq \mathrm{sint}(\mathrm{scl}(V,r),r)$, it follows

$$\begin{aligned}f^{-1}(V)&\subseteq f^{-1}(\mathrm{sint}(\mathrm{scl}(V,r),r))\\&=\tilde{1}-f^{-1}(\mathrm{scl}(\tilde{1}-\mathrm{scl}(V,r),r))\\&\subseteq \tilde{1}-\mathrm{scl}(f^{-1}(\tilde{1}-\mathrm{scl}(V,r)),r)\\&=\mathrm{sint}(f^{-1}(\mathrm{scl}(V,r)),r).\end{aligned}$$

Hence $f$ is a fuzzy semi-weakly $r$-semicontinuous mapping.

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