

## 쇼케이 적분 기준을 통한 구간치 필요측도에 관한 연구

### A study on interval-valued necessity measures through the Choquet integral criterian

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#### Abstract

Y. Réballé[Fuzzy Sets and Systems, vol.157, pp.3025–2039, 2006] discussed the representation of necessity measure through the Choquet integral criterian. He also considered a decision maker who ranks necessity measures related with Choquet integral representation. Our motivation of this paper is that a decision maker have an "ambiguity" necessity measure to present preferences. In this paper, we discuss the representation of interval-valued necessity measures through the Choquet integral criterian.

**Key Words :** non-additive measures, necessity measures, Choquet integrals.

#### 1. Introduction

Murofushi and Sugeno[10] have been studying Choquet integrals which allow to define necessity measures and risk measure. Y. Réballé[11] discussed the representation of necessity measure through the Choquet integral criterian. He also considered a decision maker who ranks necessity measures related with Choquet integral representation. Another researchers have been studying topics related with Choquet integrals, for examples, preference representation theorem(Y. Réballé [11]), integral representation(D. Schmeidler[12]), subjective probability and expected utility without additivity(D. Schmeidler[13]), interval-valued Choquet price functionals(L. Jang[8]), applications in pricing risks(L. Jang[9]), etc.

Motivation of this paper is that a decision maker have ambiguity necessity measures to present risky prospects in the sense of mathematical theory. We note that ambiguity measures can be represented by interval-valued necessity measure. We can see that this idea is similar to the concept of the Choquet integral of an interval-valued measurable function (see [1, 2, 5–9]).

In this paper, we discuss the representation of interval-valued necessity measures through the Choquet integral criterian.

#### 2. Definitions and Preliminaries

In this section we list the set-theoretical arithmetic operations on the set of subintervals of an unit interval  $I=[0,1]$  in  $\mathbb{R}$ . We denote  $[\bar{I}]$  by

$$[\bar{I}]=\{\bar{a}=[a^-, a^+]|a^-, a^+\in I \text{ and } a^- \leq a^+\}.$$

For any  $a\in I$ , we define  $a=[a, a]$ . Obviously,  $a\in [\bar{I}]$ .

**Definition 2.1** ([7–9]) If  $\bar{a}, \bar{b}\in [\bar{I}], k\in I$ , then we define

- (1)  $\bar{a}+\bar{b}=[a^-+b^-, a^++b^+]$ ,
- (2)  $k\bar{a}=[ka^-, ka^+]$ ,
- (3)  $\bar{a}\wedge\bar{b}=[a^-\wedge b^-, a^+\wedge b^+]$ ,
- (4)  $\bar{a}\vee\bar{b}=[a^-\vee b^-, a^+\vee b^+]$ ,
- (5)  $\bar{a}\leq\bar{b}$  if and only if  $a^- \leq b^-$  and  $a^+ \leq b^+$ ,
- (6)  $\bar{a}<\bar{b}$  if and only if  $\bar{a}\leq\bar{b}$  and  $\bar{a}\neq\bar{b}$ ,
- (7)  $\bar{a}\subset\bar{b}$  if and only if  $b^- \leq a^-$  and  $a^+ \leq b^+$ .

**Theorem 2.2** ([7–9]) Let  $\bar{a}, \bar{b}\in [\bar{I}]$ . Then the followings hold.

- (1) idempotent law:  $\bar{a}\wedge\bar{a}=\bar{a}, \bar{a}\vee\bar{a}=\bar{a}$ ,
- (2) commutative law:  $\bar{a}\wedge\bar{b}=\bar{b}\wedge\bar{a}, \bar{a}\vee\bar{b}=\bar{b}\vee\bar{a}$ ,
- (3) associative law:  $(\bar{a}\wedge\bar{b})\wedge\bar{c}=\bar{a}\wedge(\bar{b}\wedge\bar{c}), (\bar{a}\vee\bar{b})\vee\bar{c}=\bar{a}\vee(\bar{b}\vee\bar{c})$ ,
- (4) absorption law:  $\bar{a}\wedge(\bar{a}\vee\bar{b})=\bar{a}\vee(\bar{a}\wedge\bar{b})=\bar{a}$ ,
- (5) distributive law:  $\bar{a}\wedge(\bar{b}\vee\bar{c})=(\bar{a}\wedge\bar{b})\vee(\bar{a}\wedge\bar{c}), \bar{a}\vee(\bar{b}\wedge\bar{c})=(\bar{a}\vee\bar{b})\wedge(\bar{a}\vee\bar{c})$ ,

It is easily to see that  $([\bar{I}], d_H)$  is a metric where  $d_H$  is the Hausdorff metric defined by

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$$d_H(A, B) = \max \{ \vee_{a \in A} \wedge_{b \in B} |a - b|, \vee_{b \in B} \wedge_{a \in A} |a - b| \}$$

for all  $A, B \in [\mathbb{I}]$ . Clearly, we have the following theorem for multiplication and Hausdorff metric on  $[\mathbb{I}]$ .

**Theorem 2.3** ([7-9]) (1) If we define

$$\bar{a} \cdot \bar{b} = \{x \cdot y \mid x \in \bar{a}, y \in \bar{b}\}$$

for  $\bar{a}, \bar{b} \in [\mathbb{I}]$ , then  $\bar{a} \cdot \bar{b} = [a^- \cdot b^-, a^+ \cdot b^+]$ .

(2) If  $d_H: [\mathbb{I}] \times [\mathbb{I}] \rightarrow [0, \infty)$  is the above Hausdorff metric, then  $d_H(\bar{a}, \bar{b}) = \max \{|a^- - b^-|, |a^+ - b^+|\}$ .

Let  $\Omega$  be a non-empty set and  $\mathfrak{I}(\Omega)$  a non-empty family of subsets of  $\Omega$ . A function  $X: \Omega \rightarrow I$  is said to be  $\mathfrak{I}(\Omega)$ -measurable if for every  $\alpha \in (0, 1)$ ,  $\{w \in \Omega \mid X(w) \geq \alpha\} \in \mathfrak{I}(\Omega)$ .

Let  $B(\Omega, \mathfrak{I}(\Omega))$  be the set of  $\mathfrak{I}(\Omega)$ -measurable functions. We remark that  $B(\Omega, \mathfrak{I}(\Omega))$  is not convex (see [11]). We also list non-additive measures, possibility measures, and necessity measures.

**Definition 2.4** ([3, 7-9, 10-13]) A set function  $\mu$  on  $\mathfrak{I}(\Omega)$  is called a non-additive measure if  $\mu(\emptyset) = 0$  and  $\mu(A) \leq \mu(B)$  whenever  $A, B \in \mathfrak{I}(\Omega)$  and  $A \subset B$ .

**Definition 2.5** ([11, 14]) (1) A set function  $\mu$  on  $\mathfrak{I}(\Omega)$  is called a possibility measure if  $\mu(\emptyset) = 0$  and  $\mu(X) = 1$  and  $\mu(\bigcup_i A_i) \leq \max_i \mu(A_i)$  for all collections  $\{A_i\} \subset \mathfrak{I}(\Omega)$ .

(2) A set function  $\nu$  on  $\mathfrak{I}(\Omega)$  is called a necessity measure if  $\nu(A) = 1 - \mu(A^c)$  for all  $A \in \mathfrak{I}(\Omega)$  and  $A^c = \{w \in \Omega \mid w \notin A\}$ .

We note that every possibility measure and necessity measure is a non-additive measure. Let us discuss the following Choquet integral.

**Definition 2.4** ([3, 7-9, 10-13]) Let  $\mu$  be a non-additive measure on  $\mathfrak{I}(\Omega)$  and  $X \in B(\Omega, \mathfrak{I}(\Omega))$ . The Choquet integral of  $X$  with respect to  $\mu$  is defined by

$$(C) \int f d\mu = \int_0^1 \mu_X(\alpha) d\alpha$$

where  $\mu_X(\alpha) = \mu(\{w \in \Omega \mid X(w) > \alpha\})$  and the integral on the right hand side is Lebesgue integral.

For the case of characteristic function, one has  $(C) \int I_A d\mu = \mu(A)$  where  $A \in \mathfrak{I}(\Omega)$  and  $I_A$  is the characteristic function of  $A$ . We note that if  $\Omega$  is a finite set and  $X \in B(\Omega, \mathfrak{I}(\Omega))$ , then there is a unique decomposition of  $X$  in the following manner

$$X = \sum_{i=1}^n \alpha_i I_{A_i}, \text{ where } \alpha_1, \dots, \alpha_n > 0 \text{ and } \sum_{i=1}^n \alpha_i \leq 1 \text{ and } A_1 \supset A_2 \supset \dots \supset A_n \neq \emptyset \text{ and } A_i \in \mathfrak{I}(\Omega) \text{ for all}$$

$i = 1, 2, \dots, n$ , possibly  $A_1 = \Omega$  if  $\Omega \in \mathfrak{I}(\Omega)$ . The computation of the Choquet integral of  $X$  with respect to  $\mu$  gives

$$(C) \int X d\mu = \sum_{i=1}^n \alpha_i \mu(A_i).$$

**Definition 2.5** ([3, 7-9, 10-13]) Let  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$ . We say that  $X$  and  $Y$  are comonotonic, in symbol  $X \sim Y$  if

$$X(w) < X(w') \Rightarrow Y(w) \leq Y(w')$$

for all  $w, w' \in \Omega$ .

**Definition 2.6** ([11]) We say that the Choquet integral has comonotonic affinity if  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $\alpha \in (0, 1)$  and  $\alpha X + (1 - \alpha) Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $X \sim Y$ , then

$$(C) \int (\alpha X + (1 - \alpha) Y) d\mu = \alpha (C) \int X d\mu + (1 - \alpha) (C) \int Y d\mu.$$

Now, we introduce the following basic properties of the comonotonicity and the Choquet integral.

**Theorem 2.7** ([3, 10-13]) Let  $X, Y, Z \in B(\Omega, \mathfrak{I}(\Omega))$ . Then we have the following.

- (1)  $X \sim X$ ,
- (2)  $X \sim Y \Rightarrow Y \sim X$ ,
- (3)  $X \sim a$  for all  $a \in I$ ,
- (4)  $X \sim Y$  and  $X \sim Z \Rightarrow X \sim Y + Z$ .

**Theorem 2.8** ([3, 10-13]) Let  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$ . Then we have the following.

- (1) If  $X \leq Y$ , then  $(C) \int X d\mu \leq (C) \int Y d\mu$ .
- (2) If  $A \subset B$  and  $A, B \in \mathfrak{I}(\Omega)$ , then

$$(C) \int_A X d\mu \leq (C) \int_B X d\mu.$$

- (3) If  $X \sim Y$  and  $a, b \in I$ , then

$$(C) \int (aX + bY) d\mu = a(C) \int X d\mu + b(C) \int Y d\mu.$$

- (4) If  $(X \vee Y)(w) = X(w) \vee Y(w)$  and  $(X \wedge Y)(w) = X(w) \wedge Y(w)$  for all  $w \in \Omega$ , then

$$(C) \int X \vee Y d\mu \geq (C) \int X d\mu \vee (C) \int Y d\mu$$

and

$$(C) \int X \wedge Y d\mu \leq (C) \int X d\mu \wedge (C) \int Y d\mu.$$

### 3. Non-additive interval-valued measures

**Definition 3.1** An interval-valued set function

$\bar{\mu}: \mathfrak{I}(\Omega) \rightarrow [\underline{I}]$  is a non-additive interval-valued measure if  $\bar{\mu}(\emptyset) = \bar{0}$  and  $\bar{\mu}(A) \leq \bar{\mu}(B)$ , whenever  $A, B \in \mathfrak{I}(\Omega)$  and  $A \subset B$ .

It is easily to see that for each  $\bar{\mu}$ , there are uniquely two non-additive measures  $\mu^-$  and  $\mu^+$  on  $\mathfrak{I}(\Omega)$  such that  $\bar{\mu} = [\mu^-, \mu^+]$ .

**Definition 3.2** The Choquet integral with respect to  $\bar{\mu} = [\mu^-, \mu^+]$  of  $X \in B(\Omega, \mathfrak{I}(\Omega))$  is defined by

$$(C) \int X d\bar{\mu} = [(C) \int X d\mu^-, (C) \int X d\mu^+].$$

Then we have the following characterization of the functional which is representable as the Choquet integral with respect to  $\bar{\mu}$ . We recall that a mapping  $l: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow I$  is said to be comonotonic affine functional if for all  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $\alpha \in (0,1)$ ,

$$l(\alpha X + (1-\alpha)Y) = \alpha l(X) + (1-\alpha)l(Y)$$

and it is said to be monotone if for all  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $X \leq Y$ ,  $l(X) \leq l(Y)$ .

**Definition 3.3** A mapping  $T: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow [\underline{I}]$  is said to be comonotonic affine interval-valued functional if for all  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $\alpha \in (0,1)$ ,

$$T(\alpha X + (1-\alpha)Y) = \alpha T(X) + (1-\alpha)T(Y)$$

and it is said to be monotone if for all  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $X \leq Y$ ,  $T(X) \leq T(Y)$ .

Note that a mapping  $T = [l_1, l_2]: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow [\underline{I}]$  is a monotone comonotonic affine interval-valued functional if and only if  $l_i: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow I$  is a monotone comonotonic affine interval-valued functional for  $i=1,2$ .

**Theorem 3.4** If  $T = [l_1, l_2]: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow [\underline{I}]$  is a monotone and comonotonic affine interval-valued functional, then there exists a monotone non-additive interval-valued measure  $\bar{\mu} = [\mu^-, \mu^+]$  uniquely defined by

$$\forall A \in \mathfrak{I}(\Omega), \bar{\mu}(A) = [\mu^-(A), \mu^+(A)]$$

such that

$$T(X) = (C) \int X d\bar{\mu}, \quad \forall X \in B(\Omega, \mathfrak{I}(\Omega)).$$

Conversely, if  $\bar{\mu} = [\mu^-, \mu^+]$  is a non-additive interval-valued measure on  $\mathfrak{I}(\Omega)$ , then  $(C) \int (\cdot) d\bar{\mu}$  is a monotone and comonotonic affine interval-valued functional.

**Proof.**  $(\Rightarrow)$  From the above note, we obtain  $l_i: B(\Omega, \mathfrak{I}(\Omega)) \rightarrow I$  is a monotone comonotonic affine interval-valued functional for  $i=1,2$ . By Theorem 2.1[11], there exist two non-additive measures  $\mu_1, \mu_2$

such that

$$l_i(X) = (C) \int X d\mu_i, \quad \forall X \in B(\Omega, \mathfrak{I}(\Omega)), \text{ for } i=1,2.$$

Thus for all  $X \in B(\Omega, \mathfrak{I}(\Omega))$ ,

$$\begin{aligned} T(X) &= [l_1(X), l_2(X)] \\ &= [(C) \int X d\mu_1, (C) \int X d\mu_2] \\ &= (C) \int X d\bar{\mu}. \end{aligned}$$

$(\Leftarrow)$  Let  $\bar{\mu} = [\mu^-, \mu^+]$  be a non-additive interval-valued measure. If  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$  and  $X \leq Y$ , by Theorem 2.4 (1),

$$\begin{aligned} (C) \int X d\bar{\mu} &= [(C) \int X d\mu^-, (C) \int X d\mu^+] \\ &\leq [(C) \int Y d\mu^-, (C) \int Y d\mu^+] \\ &= (C) \int Y d\bar{\mu}. \end{aligned}$$

Thus  $(C) \int (\cdot) d\bar{\mu}$  is monotone. If  $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$ ,  $X \sim Y$  and  $\alpha \in (0,1)$ , then

$$\begin{aligned} (C) \int [\alpha X + (1-\alpha)Y] d\bar{\mu} &= [(C) \int [\alpha X + (1-\alpha)Y] d\mu^-, \\ &\quad (C) \int [\alpha X + (1-\alpha)Y] d\mu^+] \\ &= [\alpha (C) \int X d\mu^- + (1-\alpha) \int Y d\mu^-, \\ &\quad \alpha (C) \int X d\mu^+ + (1-\alpha) (C) \int Y d\mu^+] \\ &= \alpha [(C) \int X d\bar{\mu}, (C) \int X d\bar{\mu}] \\ &\quad + (1-\alpha) [(C) \int Y d\bar{\mu}, (C) \int Y d\bar{\mu}] \\ &= \alpha \int X d\bar{\mu} + (1-\alpha) \int Y d\bar{\mu}. \end{aligned}$$

Thus  $(C) \int (\cdot) d\bar{\mu}$  is comonotonic affine.

#### 4. Interval-valued necessity measures

In this section, we our concern is to rank interval-valued necessity measures. Let  $\wp(\Omega)$  be the power set of  $\Omega$ .

**Definition 4.1** (1) An interval-valued set function  $\bar{\nu}: \wp(\Omega) \rightarrow [\underline{I}]$  is called an interval-valued possibility measure if

$$\bar{\nu}(\emptyset) = \bar{0}, \bar{\nu}(\Omega) = 1 \text{ and } \bar{\nu}(\bigcup_i A_i) = \max_i \bar{\nu}(A_i)$$

for all collections  $\{A_i\} \subset \mathfrak{I}(\Omega)$ .

(2) An interval-valued set function  $\bar{\nu}$  on  $\wp(\Omega)$  is called an interval-valued necessity measure if  $\bar{\nu}(A) = 1 - \bar{\mu}(A^c)$  for all  $A \in \wp(\Omega)$  and  $A^c = \{w \in \Omega | w \notin A\}$ .

Let  $\Omega$  be a finite non-empty set and  $\wp(\Omega)$  a non-empty family of subsets of  $\Omega$ .  $A^u = \{B \mid A \subset B \subset \Omega\}$  stands for the upset generated by  $A$ . Then these sets of subsets of  $\Omega$  are known as filters(see [11]), we denote the set of filters by  $F(\Omega)$ . We recall that a family  $F$  of subsets of  $\Omega$  is said to be a filter if

- (i)  $\emptyset \notin F, \Omega \in F$ ,
- (ii)  $A, B \in F \Rightarrow A \cap B \in F$ ,
- (iii)  $A \in F, A \subset B \Rightarrow B \in F$ .

From Definition 4.1(2) and Proposition 2.1([11]), we obtain the following theorem.

**Theorem 4.2** An interval-valued set function  $\bar{\nu} = [\nu^-, \nu^+] : \wp(\Omega) \rightarrow [\bar{I}]$  is an interval-valued necessity measure if and only if  $\nu^-, \nu^+$  are necessity measures on  $\wp(\Omega)$ .

**Definition 4.3** Interval-valued necessity measures  $\bar{\nu}, \bar{\eta}$  are said to be agree if  $\nu^-, \eta^-$  ( $\nu^+, \eta^+$ , resp.) are agree, that is, there is no subsets  $A, B \in \wp(\Omega)$  such that

$$\begin{aligned} \nu^-(A) &> \nu^-(B) \text{ and } \eta^-(A) < \eta^-(B) \\ (\nu^+(A) &> \nu^+(B) \text{ and } \eta^+(A) < \eta^+(B), \text{ resp.).} \end{aligned}$$

From Definition 4.1 and Definition 4.3, clearly we have the following theorem.

**Theorem 4.4** Let  $\bar{\nu}, \bar{\eta}$  be interval-valued necessity measures and  $\alpha \in (0,1)$ . Then, one has

- (1)  $\alpha \bar{\nu} + (1-\alpha) \bar{\eta}$  is an interval-valued necessity measure.
- (2)  $\bar{\nu}, \bar{\eta}$  are agree if and only if  $\alpha \bar{\nu} + (1-\alpha) \bar{\eta}$  is agree.

We recall that if  $\bar{\nu} = [\nu^-, \nu^+] : \wp(\Omega) \rightarrow [\bar{I}]$  is an interval-valued measure, then there is a unique decomposition of  $\bar{\nu}$  over unanimity games known as Möbius transforms of  $\nu^-$  and  $\nu^+$ (see [11]):

$$\bar{\nu} = \left[ \sum_{j=1}^n \alpha_j u_{A_j}^-, \sum_{k=1}^m \beta_k u_{B_k}^+ \right]$$

where  $\alpha_1, \dots, \alpha_n > 0, \beta_1, \dots, \beta_m > 0, \sum_{j=1}^n \alpha_j = 1, \sum_{k=1}^m \beta_k = 1$ ,  $\Omega \supset A_1 \supset \dots \supset A_n \neq \emptyset, \Omega \supset B_1 \supset \dots \supset B_m \neq \emptyset, u_A^-$  ( $u_A^+$ , resp.) denote a unanimity game associated with  $\nu^-$  ( $\nu^+$ , resp.), that is, elementary belief function with support  $A$  defined by,

$$\forall A \subset \Omega, u_A(B) = \begin{cases} 1 & \text{if } A \subset B \\ 0 & \text{otherwise} \end{cases}$$

or otherwise put,  $\bar{\nu}$  can be expressed as follows,

$$\bar{\nu} = \left[ \sum_{j=1}^n \alpha_j I_{A_j}^u, \sum_{k=1}^m \beta_k I_{B_k}^u \right]$$

where  $\alpha_1, \dots, \alpha_n > 0, \beta_1, \dots, \beta_m > 0, \sum_{j=1}^n \alpha_j = 1, \sum_{k=1}^m \beta_k = 1, \emptyset \neq A_1^u \subset \dots \subset A_n^u$  and  $\emptyset \neq B_1^u \subset \dots \subset B_m^u$ .

As a consequence of Proposition 2.1([11]) and Proposition 2.2([11]) with  $\nu^-, \nu^+$ , given non-additive measures  $\mu^-, \mu^+$  defined on  $\wp(\Omega)$ , we can obtain the Choquet integral of an interval-valued necessity measure  $\bar{\nu}$  with respect to an interval-valued nonadditive measure  $\bar{\mu}$  as following:

$$(C) \int \bar{\nu} d\bar{\mu} = \left[ \sum_{j=1}^n \alpha_j \mu^-(A_j^u), \sum_{k=1}^m \beta_k \mu^+(B_k^u) \right].$$

By using the above Choquet integral of an interval-valued necessity measure with respect to an interval-valued nonadditive measure, we can discuss that this object is the criterion which is used to rank interval-valued necessity measures in order to obtain a weak integral representation, that is for all interval-valued necessity measures  $\bar{\nu}, \bar{\eta}$ :

$$\bar{\nu} \geq \bar{\eta} \Leftrightarrow (C) \int \bar{\nu} d\bar{\mu} \geq (C) \int \bar{\eta} d\bar{\mu}.$$

In the future, by using the above weak integral representation, we can study the integral representation of interval-valued preferences which are like ambiguity preferences.

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