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쇼케이 적분과 구간치 필요측도

Choquet integrals and interval-valued necessity measures

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요 약

Y. Réballé [11]교수는 쇼케이적분 기준에 의한 필요측도의 표현에 관해 조사한다. 또한 쇼케이적분 표현관 관련된 필요측도의 순위를 결정 연장을 생각한다. 이 논문에서, 우리는 결정연장이 쇼케이 기대효용에 따른 애매한(구간치로 명명함) 필요측도를 가지는 경우를 생각한다. 더욱이, 구간치 필요측도에 대한 단조 집합치 함수를 갖는 기호에 대한 약 쇼케이적분 표현과 필요측도에 대한 구간치 효용함수를 갖는 기호에 대한 강 쇼케이적분 표현에 대한 두 가지 정리를 증명한다.

Abstract

Y. Réballé [11] discussed the representation of necessity measure through the Choquet integral criterian. He also consider a decision maker who ranks necessity measures related with Choquet integral representation. In this paper, we consider a decision maker have an "ambiguity"(say, interval-valued) necessity measure according to their Choquet's expected utility. Furthermore, we prove two theorems which are weak Choquet integral representation of preferences with a monotone set function for interval-valued necessity measures and strong Choquet integral representation of preferences with an interval-valued utility function for necessity measures.

Key Words : non-additive measures, necessity measures, interval-valued necessity measures, Choquet integrals.

1. Introduction

In a previous work [11], the author investigated the representation of necessity measure through the Choquet integral criterian. We note that G. Choquet (1953, [3]) first have studied Choquet integrals and Murofush and Sugeno [10] have been studied Choquet integrals with respect to a fuzzy measure. Choquet integrals allow to define the utility and a risk measure of a measurable function, for example, a bounded random payment and an utility function.

Motivation of this paper is that a decision maker have interval-valued necessity measures according to their Choquet's expected utility. The concept of interval-valued Choquet integral are useful tools in order to get numerous applications, for examples, mathematical economics, information theory, expected utility theory, and risk analysis (see [5-8]).

In this paper, by using Choquet integrals with respected to an interval-valued necessity measure, we discuss two theorems which are weak Choquet integral representation of preferences with a monotone set function for interval-valued necessity measures and strong Choquet integral representation of preferences with an interval-valued utility function for necessity measures.

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2. Definitions and Preliminaries

In this section we list the set-theoretical arithmetic operations on the set of subintervals of an unit interval $I=[0,1]$ in $\mathbb{R}$. We denote $[I]$ by

$$[I]=\{\overline{a}=[a^-,a^+]\mid a^-,a^+\in I \text{ and } a^-\leq a^+\}.$$

For any $a\in I$, we define $a=[a,a]$. Obviously, $a\in[I]$.

Definition 2.1 ([7-9]) If $\overline{a},\overline{b}\in[I], k\in I$, then we define

(1) $\overline{a}+\overline{b}=[a^-+b^-,a^++b^+]$,
(2) $k\overline{a}=[ka^-,ka^+]$,
(3) $\overline{a}\wedge\overline{b}=[a^-\wedge b^-,a^+\wedge b^+]$,
(4) $\overline{a}\vee\overline{b}=[a^-\vee b^-,a^+\vee b^+]$,
(5) $\overline{a}\leq\overline{b}$ if and only if $a^-\leq b^-$ and $a^+\leq b^+$,
(6) $\overline{a}<\overline{b}$ if and only if $\overline{a}\leq\overline{b}$ and $\overline{a}\neq\overline{b}$,
(7) $\overline{a}\subset\overline{b}$ if and only if $b^-\leq a^-$ and $a^+\leq b^+$.

Definition 2.2 ([7-9]) A set function $d_H: [I] \times [I] \to [0, \infty]$ is called the Hasdorff metric if

$$d_H(A,B) = \max\{\sup_{x \in A} \inf_{y \in B} |x-y|, \sup_{y \in B} \inf_{x \in A} |x-y|\},$$

for all $A, B \in [I]$.

Theorem 2.3 ([7-9]) If $d_H: [I] \times [I] \to [0, \infty]$ is the Hausdorff metric, then for $\bar{a} = [a^-, a^+], \bar{b} = [b^-, b^+] \in [I]$

$$d_H(\bar{a}, \bar{b}) = \max\{|a^- - b^-|, |a^+ - b^+|\}.$$

Let Ω be a non-empty set and $\mathfrak{I}(\Omega)$ a non-empty family of subsets of Ω. A function $X: \Omega \to I$ is said to be $\mathfrak{I}(\Omega)$-measurable if for every $\alpha \in (0,1)$,

$$\{w \in \Omega \mid X(w) \geq \alpha\} \in \mathfrak{I}(\Omega).$$

Let $B(\Omega, \mathfrak{I}(\Omega))$ be the set of $\mathfrak{I}(\Omega)$-measurable functions. We remark that $B(\Omega, \mathfrak{I}(\Omega))$ is not convex (see [11]). We also list non-additive measures, possibility measures, and necessity measures.

Definition 2.4 ([3, 7-9, 10-13]) A set function μ on $\mathfrak{I}(\Omega)$ is called a non-additive measure if $\mu(\varnothing) = 0$ and $\mu(A) \leq \mu(B)$ whenever $A, B \in \mathfrak{I}(\Omega)$ and $A \subset B$.

Definition 2.5 ([11, 14]) (1) A set function μ on $\mathfrak{I}(\Omega)$ is called a possibility measure if $\mu(\varnothing) = 0$ and $\mu(X) = 1$ and

$$\mu(\bigcup_i A_i) \leq \max_i \mu(A_i)$$

for all collections $\{A_i\} \subset \mathfrak{I}(\Omega)$.

(2) A set function ν on $\mathfrak{I}(\Omega)$ is called a necessity measure if $\nu(A) = 1 - \mu(A^c)$ for all $A \in \mathfrak{I}(\Omega)$ and $A^c = \{w \in \Omega | w \notin A\}$.

We note that every possibility measure and necessity measure is a non-additive measure. Let us discuss the following Choquet integral.

Definition 2.6 ([3, 7-9, 10-13]) Let μ be a non-additive measure on $\mathfrak{I}(\Omega)$ and $X \in B(\Omega, \mathfrak{I}(\Omega))$. The Choquet integral of X with respect to μ is defined by

$$(C)\int f d\mu = \int_0^1 \mu_X(\alpha)\, d\alpha$$

where $\mu_X(\alpha) = \mu(\{w \in \Omega \mid X(w) > \alpha\})$ and the integrals on the right hand side are Lebesgue integral.

Definition 2.7 ([3, 7-9, 10-13]) Let $X, Y \in B(\Omega, \mathfrak{I}(\Omega))$. We say that X and Y are comonotonic, in symbol $X \sim Y$ if

$$X(w) < X(w') \Rightarrow Y(w) \leq Y(w')$$

for all $w, w' \in \Omega$.

3. Main results

In this section, we will denote the set of necessity measures on $\mathfrak{I}(\Omega)$ by $\mathrm{Nec}(\mathfrak{I}(\Omega))$ and the set of interval-valued necessity measures on $\mathfrak{I}(\Omega)$ by $\mathrm{INec}(\mathfrak{I}(\Omega))$.

First, we list binary relations on $\mathrm{Nec}(\mathfrak{I}(\Omega))$ and $\mathrm{INec}(\mathfrak{I}(\Omega))$ and discuss weak Choquet integral representation of preferences with a monotone set function for interval-valued necessity measures.

Definition 3.1 ([11]) (1) A binary relation $\succcurlyeq$ on $\mathrm{Nec}(\mathfrak{I}(\Omega))$ is said to be complete if for all $(v, w) \in \mathrm{Nec}(\mathfrak{I}(\Omega))^2$ we have $v \succcurlyeq w$ or $w \succcurlyeq v$.

(2) A binary relation $\succcurlyeq$ on $\mathrm{Nec}(\mathfrak{I}(\Omega))$ is said to be transtive if for all $(u, v, w) \in \mathrm{Nec}(\mathfrak{I}(\Omega))^3$, whenever $u \succcurlyeq v$ and $v \succcurlyeq w$ we have $u \succcurlyeq w$.

(3) A weak order $\succcurlyeq$ on $\mathrm{Nec}(\mathfrak{I}(\Omega))$ is called a binary relation on $\mathrm{Nec}(\mathfrak{I}(\Omega))$ which is complete and transtive.

Note that we write $v \succ w$ for $v \succcurlyeq w$ and not $(w \succcurlyeq v)$ and $v \sim w$ for $v \succcurlyeq w$ and $w \succcurlyeq v$. A functional I: $\mathrm{Nec}(\mathfrak{I}(\Omega)) \to I$ represents the binary relation $\succcurlyeq$ if and only if for all v, w in $\mathrm{Nec}(\mathfrak{I}(\Omega))$ it holds
$v \succcurlyeq w \Leftrightarrow I(v) \geq I(w)$.

We also state some axioms that the binary relation $\succcurlyeq$ may fufill.

(WO) $\succcurlyeq$ is a weak order.

(MON) Monotonicity: $\forall v, w$ in $\mathrm{Nec}(\mathfrak{I}(\Omega))$,

$$[v \geq w] \Leftrightarrow [v \succcurlyeq w].$$

(AGR) Agreement: $\forall u, v, w$ in $\mathrm{Nec}(\mathfrak{I}(\Omega))$, $\forall \alpha \in (0,1)$,

$$[u \sim w, v \sim w, u \sim v] \Rightarrow [\alpha u + (1-\alpha)w \sim \alpha v + (1-\alpha)w].$$

(ARCH) $\succcurlyeq$ is Archimedean: $\forall v, w$ in $\mathrm{Nec}(\mathfrak{I}(\Omega))$,

$$[v \prec w] \Rightarrow [\exists \alpha \in (0,1) \text{ s.t. } v \prec \alpha w + (1-\alpha)u_\Omega]$$

and

$$[\exists \alpha \in (0,1) \text{ s.t. } \alpha w + (1-\alpha)u_\Omega \prec v \preccurlyeq w] \Rightarrow [\exists \alpha' \in (0,1) \text{ s.t. } \alpha' w + (1-\alpha')u_\Omega \preccurlyeq v].$$

where

$$\forall A \subset \Omega, u_\Omega(A) = \begin{cases} 1 & \text{if } A = \Omega, \\ 0 & otherwise \end{cases}.$$

(NDEG) $\succcurlyeq$ is not degenerate:
$\exists v, w$ in $\mathrm{Nec}(\mathfrak{I}(\Omega))$ s.t. $v \succ w$.

We recall that $A^u = \{B | A \subset B \subset \Omega\}$ stands for the upset generated by A.

Theorem 3.3 ([11] Theorem 3.1) Let $\succcurlyeq$ be a binary relation on $\mathrm{Nec}(\mathfrak{I}(\Omega))$. If $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), and (NDEG), then there exists a monotone set function $\beta: B(\Omega,\mathfrak{I}(\Omega))\rightarrow I$ such that for all $v, w \in \mathrm{Nec}(\mathfrak{I}(\Omega))$,

$$v \succcurlyeq w \Leftrightarrow (C)\int v\, d\beta \geq (C)\int w\, d\beta.$$

Conversely, if the binary relation is representation by a Choquet integral with respect to a monotone set function $\beta: B(\Omega,\mathfrak{I}(\Omega))\rightarrow I$ such that $\beta(\{\Omega\})=0$ and $\beta(\{w_1\}^u)=1$ for some $w_1 \in \Omega$ then $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), and (NDEG).

Secondly, we introduce further axiom in order to obtain strong Choquet integral representation of preferences with an interval-valued utility function for necessity measures.

(INCL) Inclusion: for all $A,B \in \mathfrak{I}(\Omega), \neq \varnothing$,
$[u_A \succcurlyeq u_B] \Rightarrow [u_{A\cup B} \sim u_B]$.

Theorem 3.4 ([11] Theorem 3.2) Let $\succcurlyeq$ be a binary relation on $\mathrm{Nec}(\mathfrak{I}(\Omega))$. If $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), (NDEG), and (INCL), then there exists an utility function (which means normalized measurable function) $X: \Omega\rightarrow I$ such that for all $v, w \in \mathrm{Nec}(\mathfrak{I}(\Omega))$,

$$v \succcurlyeq w \Leftrightarrow (C)\int X\, dv \geq (C)\int X\, dw.$$

Conversely, if the binary relation is representation by Choquet integral of an utility function $X: \Omega\rightarrow I$ then $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), (NDEG), and (INCL).

Definition 3.5 ([5-9]) An interval-valued set function $\overline{\mu}: \mathfrak{I}(\Omega)\rightarrow [I]$ is a non-additive interval-valued measure if $\overline{\mu}(\varnothing)=\overline{0}$ and $\overline{\mu}(A) \leq \overline{\mu}(B)$, whenever $A,B \in \mathfrak{I}(\Omega)$ and $A \subset B$.

It is easily to see that for each $\overline{\mu}$, there are uniquely two non-additive measures μ^- and μ^+ on $\mathfrak{I}(\Omega)$ such that $\overline{\mu}=[\mu^-,\mu^+]$.

Definition 3.6 ([5-9]) (1) The Choquet integral with respect to $\overline{\mu}=[\mu^-,\mu^+]$ of $X \in B(\Omega,\mathfrak{I}(\Omega))$ is defined by

$$(C)\int X d\overline{\mu} = [(C)\int X d\mu^-, (C)\int X d\mu^+].$$

(2) The Choquet integral with respect to a necessity measure $v \in \mathrm{Nec}(\mathfrak{I}(\Omega))$ of an interval-valued utility function $\overline{X}=[X^-,X^+]$ $(X^-, X^+ \in B(\Omega,\mathfrak{I}(\Omega)))$ is defined by

$$(C)\int \overline{X} dv = [(C)\int X^- dv, (C)\int X^+ dv].$$

Now, we consider a binary relation $\succcurlyeq_i$ on $\mathrm{INec}(\mathfrak{I}(\Omega))$ defined by

$$\overline{v} \succcurlyeq_i \overline{w} \Leftrightarrow v^- \succcurlyeq w^- \text{ and } v^+ \succcurlyeq w^+ \tag{3.1}$$

and list some axioms that the binary relation $\succcurlyeq_i$ may fufill.

Definition 3.7 (1) A binary relation $\succcurlyeq_i$ on $\mathrm{INec}(\mathfrak{I}(\Omega))$ is said to be complete if for all $(\overline{v},\overline{w}) \in \mathrm{INec}(\mathfrak{I}(\Omega))^2$ we have $\overline{v} \succcurlyeq_i \overline{w}$ or $\overline{w} \succcurlyeq_i \overline{v}$.

(2) A binary relation $\succcurlyeq_i$ on $\mathrm{INec}(\mathfrak{I}(\Omega))$ is said to be transtive if for all $(\overline{u},\overline{v},\overline{w}) \in \mathrm{INec}(\mathfrak{I}(\Omega))^3$, whenever $\overline{u} \succcurlyeq_i \overline{v}$ and $\overline{v} \succcurlyeq_i \overline{w}$ we have $\overline{u} \succcurlyeq \overline{w}$.

(3) A weak order $\succcurlyeq_i$ on $\mathrm{INec}(\mathfrak{I}(\Omega))$ is called a binary relation on $\mathrm{INec}(\mathfrak{I}(\Omega))$ which is complete and transtive.

Note that we write $\overline{v} \succ_i \overline{w}$ for $\overline{v} \succcurlyeq_i \overline{w}$ and not $(\overline{w} \succcurlyeq_i \overline{v})$ and $\overline{v} \sim \overline{w}$ for $\overline{v} \succcurlyeq_i \overline{w}$ and $\overline{w} \succcurlyeq_i \overline{v}$. A functional $\overline{I}$: $\mathrm{INec}(\mathfrak{I}(\Omega))$(or $\mathrm{Nec}(\mathfrak{I}(\Omega))$) $\rightarrow [I]$ represents the binary relation $\succcurlyeq_i$ if and only if for all $\overline{v}, \overline{w}$ in $\mathrm{INec}(\mathfrak{I}(\Omega))$ it holds

$$\overline{v} \succcurlyeq_i \overline{w} \text{ (or } v \succcurlyeq_i w) \Leftrightarrow \overline{I}(\overline{v}) \geq \overline{I}(\overline{w}) \text{(or } \overline{I}(v) \geq \overline{I}(w)).$$

Similary, we can state some axioms that the binary relation $\succcurlyeq$ may fufill.

$(\mathrm{WO})_i$ $\succcurlyeq_i$ is a weak order.

$(\mathrm{MON})_i$ Monotonicity: $\forall \overline{v}, \overline{w}$ in $\mathrm{INec}(\mathfrak{I}(\Omega))$, $[\overline{v} \geq \overline{w}] \Leftrightarrow [\overline{v} \succcurlyeq_i \overline{w}]$.

$(\mathrm{AGR})_i$ Agreement: $\forall \overline{u},\overline{v},\overline{w}$ in $\mathrm{INec}(\mathfrak{I}(\Omega))$,

$\forall \alpha \in (0,1)$,
$[\overline{u} \sim \overline{w}, \overline{v} \sim \overline{w}, \overline{u} \sim \overline{v}]$
$\Rightarrow [\alpha\overline{u}+(1-\alpha)\overline{w} \sim \alpha\overline{v}+(1-\alpha)\overline{w}]$.

$(\mathrm{ARCH})_i$ $\succcurlyeq_i$ is Archimedean: $\forall \overline{v}, \overline{w}$ in $\mathrm{INec}(\mathfrak{I}(\Omega))$,

$[\overline{v} \prec_i \overline{w}] \Rightarrow [\exists \alpha \in (0,1)$ s.t. $\overline{v} \prec_i \alpha\overline{w}+(1-\alpha)\overline{u_\Omega}]$
and $[\exists \alpha \in (0,1)$ s.t. $\alpha\overline{w}+(1-\alpha)\overline{u_\Omega} \prec_i \overline{v} \preccurlyeq_i \overline{w}]$
$\Rightarrow [\exists \alpha' \in (0,1)$ s.t. $\alpha'\overline{w}+(1-\alpha')\overline{u_\Omega} \preccurlyeq_i \overline{v}]$.

where $\forall A \subset \Omega, \overline{u_\Omega}(A)=[u_\Omega^-, u_\Omega^+]$.

$(\mathrm{NDEG})_i$ $\succcurlyeq_i$ is not degenerate:

$\exists \overline{v},\overline{w}$ in $\mathrm{INec}(\mathfrak{I}(\Omega))$ s.t. $\overline{v} \succ_i \overline{w}$.

$(\mathrm{INCL})_i$ Inclusion: for all $A,B \in \mathfrak{I}(\Omega), \neq \varnothing$,

$[\overline{u_A} \succcurlyeq_i \overline{u_B}] \Rightarrow [\overline{u_{A\cup B}} \sim_i \overline{u_B}]$.

From the definition of $\succcurlyeq_i$, we note that (A) $\succcurlyeq_i$ satisfies $(\mathrm{WO})_i$, $(\mathrm{MON})_i$, $(\mathrm{AGR})_i$, $(\mathrm{ARCH})_i$, and $(\mathrm{NDEG})_i$ if and only if $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), and (NDEG) and that (B) $\succcurlyeq_i$ satisfies $(\mathrm{WO})_i$, $(\mathrm{MON})_i$, $(\mathrm{AGR})_i$, $(\mathrm{ARCH})_i$, $(\mathrm{NDEG})_i$, and $(\mathrm{INCL})_i$ if and only if $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), (NDEG), and (INCL).

Finally, we obtain the following two theorems which are weak Choquet integral representation of preferences with a monotone set function for interval-valued necessity measures and strong Choquet integral representation of preferences with an interval-valued utility function for necessity measures.

Theorem 3.8 Let $\succcurlyeq_i$ be a binary relation on INec $(\Im(\Omega))$. If $\succcurlyeq_i$ satisfies $(WO)_i$, $(MON)_i$, $(AGR)_i$, $(ARCH)_i$, and $(NDEG)_i$, then there exists a monotone set function $\beta: B(\Omega,\Im(\Omega))\rightarrow I$ such that for all $\overline{v},\overline{w}\in \mathrm{INec}(\Im(\Omega))$,

$$\overline{v} \succcurlyeq_i \overline{w} \Leftrightarrow (C)\int \overline{v}\, d\beta \geq (C)\int \overline{w}\, d\beta.$$

Conversely, if the binary relation is representation by a Choquet integral with respect to a monotone set function $\beta: B(\Omega,\Im(\Omega))\rightarrow I$ such that $\beta(\{\Omega\})=0$ and $\beta(\{w_1\}^u)=1$ for some $w_1\in\Omega$ then $\succcurlyeq_i$ satisfies $(WO)_i$, $(MON)_i$, $(AGR)_i$, $(ARCH)_i$, and $(NDEG)_i$.

Proof. By (A), we have $\succcurlyeq$ satisfies (WO), (MON), (AGR), (ARCH), and (NDEG). By Theorem 3.3, there exists a monotone set function $\beta: B(\Omega,\Im(\Omega))\rightarrow I$ such that for all $v,w\in \mathrm{Nec}(\Im(\Omega))$,

$$\text{(C)} \qquad v\succcurlyeq w \Leftrightarrow (C)\int v\, d\beta \geq (C)\int w\, d\beta.$$

By (C) and the definition of a binary relation $\succcurlyeq_i$, we can obtain

$$\begin{aligned}\overline{v}\succcurlyeq_i\overline{w} \quad \Leftrightarrow \quad & (C)\int v^-\, d\beta \geq (C)\int w^-\, d\beta \text{ and}\\ & (C)\int v^-\, d\beta \geq (C)\int w^-\, d\beta\\ \Leftrightarrow \quad & (C)\int \overline{v}\, d\beta = [(C)\int v^- d\beta, (C)\int v^+ d\beta]\\ = & [(C)\int w^-\, d\beta, (C)\int w^+ d\beta] = (C)\int \overline{w}\, d\beta.\end{aligned}$$

Conversely, if we define $\succcurlyeq_i$ by

$$\overline{v} \succcurlyeq_i \overline{w} \Leftrightarrow (C)\int \overline{v}\, d\beta \geq (C)\int \overline{w}\, d\beta.$$

for some a monotone set function $\beta: B(\Omega,\Im(\Omega))\rightarrow I$. By the definition of interval-valued Choquet integral(see [7,8,9]), it is clearly to see that $\succcurlyeq_i$ satisfies $(WO)_i$, $(MON)_i$, $(AGR)_i$, $(ARCH)_i$, and $(NDEG)_i$.

We can consider a binary relation $\succcurlyeq_u$ with an interval-valued utility function like $\succcurlyeq_i$ as follows

$$v\succcurlyeq_u w \Leftrightarrow (C)\int \overline{X}\, dv \geq (C)\int \overline{X}\, dw$$

and hence, by the same method of the proof in Theorem 3.8, we can obtain the following theorem.

Theorem 3.9 (1) Let $\succcurlyeq_u$ be a binary relation on INec $(\Im(\Omega))$. If $\succcurlyeq_u$ satisfies $(WO)_u$, $(MON)_u$, $(AGR)_u$, $(ARCH)_u$, and $(NDEG)_u$, then there exists an interval-valued function $\overline{X}: \Omega\rightarrow[I]$ such that for all $v,w\in$ $\mathrm{Nec}(\Im(\Omega))$,

$$v\succcurlyeq_u w \Leftrightarrow (C)\int \overline{X}\, dv \geq (C)\int \overline{X}\, dw.$$

Conversely, if the binary relation $\succcurlyeq_u$ is representation by a Choquet integral with an interval-valued utility function $\overline{X}: \Omega\rightarrow[I]$ as follows

$$v\succcurlyeq_u w \Leftrightarrow (C)\int \overline{X}\, dv \geq (C)\int \overline{X}\, dw$$

then $\succcurlyeq_u$ satisfies $(WO)_u$, $(MON)_u$, $(AGR)_u$, $(ARCH)_u$, and $(NDEG)_u$.

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