Distributed Power and Rate Control for Cognitive Radio Networks

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Abstract: **In this paper, a distributed power and end-to-end rate control algorithm is proposed in the presence of licensed users. By Lagrangian duality theory, the optimal power and rate control solution is given for the unlicensed users while satisfying the interference temperature limits to licensed users. It is obtained that transmitting with either 0 or the maximum node power is the optimal scheme. The synchronous and asynchronous distributed algorithms are proposed to be implemented at the nodes and links. The convergence of the proposed algorithms are proved. Finally, further discussion on the utility-based fairness is provided for the proposed algorithms. Numerical results show that the proposed algorithm can limit the interference to licensed user under a predefined threshold.**

Index Terms: **Cognitive radio, fairness, Lagrangian duality, power control, rate control, wireless communication.**

I. INTRODUCTION

Cognitive radio is one of the emerging communication technologies recently. Because the unlicensed users are not allowed to utilize licensed spectrum even though the spectrum is used insufficiently, the utilization is always low in some licensed spectrum. For the short-to-come lack of spectrum, cognitive radio [1]–[4] is proposed to improve the spectrum utilization. The equipments with cognitive radio can be aware of its surrounding environment and adapt themselves by changing some operating parameters. Cognitive radio makes it possible that unlicensed users can access the unoccupied licensed spectrum, which can improve spectrum utilization a lot. The premise condition to use this kind of spectrum is guaranteeing the communication quality of licensed users.

One of the challenges in cognitive radio networks is how to handle the co-existence of licensed users and unlicensed users. There have been two types of models proposed so far for the licensed users' spectrum usage activities: dynamic spectrum model and interference temperature model. In the dynamic spectrum model, the primary users may not always use the spectrum, and the secondary users can utilize the spectrum only when it is not being used by the primary users [5]. In the interference temperature model, both primary and secondary users can coexist on the same spectrum. The secondary users' interference at the primary receivers is not allowed to exceed the interference temperature limit [6]. In order to evaluate the interference to licensed user, Federal Communications Commission (FCC) introduced the concept of interference temperature [7], which is intended to quantify and manage the interference in radio environment. In this paper, the interference temperature model is considered.

Power control is an efficient method to restrict the interference for handling the co-existence problem. It is mainly used for improving the system throughput and decreasing the interference. Some exist publications [8]–[10] have investigated the power control problem in wireless networks, but they do not take the interference caused by unlicensed users to licensed users into account. In [11], joint power and rate control is investigated by Lagrangian duality. Our previous works [12]–[14] give some preliminary results on power control. However, there is few publications on joint power and rate control for cognitive radio networks.

When the data of a session go through multiple links, all the related links in the path need to provide enough capacity for the session. On the other hand, to increase link capacity, the transmitter should increase the power, which causes the interference to other links enlarged. Therefore, power control and rate control affect each other and need to be considered jointly. In [15], we provide some preliminary results on joint power and rate control.

Fairness is also an important goal for QoS provision. In [16], a link algorithm is presented to achieve utility-based max-min fairness for bandwidth allocation. In [17], the authors consider different QoS requirements among users and propose a distributed flow control algorithm which can achieve utility-based fairness.

In this paper, we investigate the joint power and rate control algorithm in presence of licensed users for cognitive radio networks. The contribution of this paper includes:

- Based on [11], the optimal solution of the joint power and rate problem is obtained for cognitive radio networks by applying Lagrangian duality theory and sub-gradient method. Transmitting with either 0 or the maximum node power is the optimal scheme.
- For both synchronous and asynchronous scenarios, the algorithm is divided into two sub-algorithms, which are deployed at nodes and links as a distributed manner. The convergence of the proposed distributed algorithms are proved by strict theoretic deduction. The improvement of the performance is shown by numerical results.
- The proposed algorithms can achieve proportional fairness.

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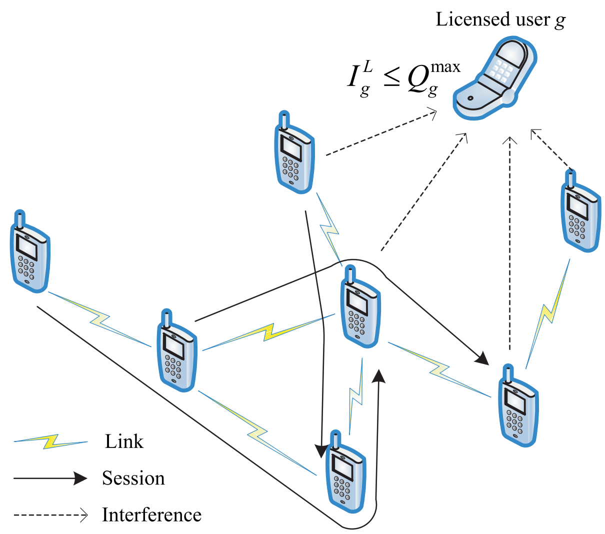

Fig. 1. Illustration of cognitive radio network model.

Utility-based fairness can be achieved to extend the applications of the proposed algorithms.

The rest of this paper is organized as follows. In Section II, the system model is presented. Section III gives the optimal solution of this problem. In Section IV, the synchronous distributed algorithm is proposed and its convergence is proved. In Section V, the proposed algorithm is extended for asynchronous scenarios. The utility-based fairness issue is discussed in Section VI. Following this, the numerical results are provided in Section VII. Finally, this paper is concluded in Section VIII.

II. SYSTEM MODEL

A. Cognitive Radio Network Model

Consider a cognitive radio network consisting of a set of unlicensed nodes denoted by $\mathbf{N}$, and a set of links denoted by $\mathbf{L}$. The signal-to-interference-plus-noise-ratio (SINR) η_l for link l can be expressed as

$$\eta_l = \frac{p_l g_{ll}}{\sum\limits_{i \in \mathbf{N}, i \neq l} p_i g_{il} + \sigma_l^2} \tag{1}$$

where p_l is the transmission power for the link l, g_{ll} is the path gain for link l, g_{il} is the path gain from the transmitter of link i to the receiver of link l, σ_l^2 denotes the thermal noise and the interference from primary users at the receiver of unlicensed link l. For simplicity of presentation, define I_l to represent the total interference of link l, (i.e., $I_l = \sum_{i \neq l} p_i g_{il} + \sigma_l^2$).

There is a set of sessions $\mathbf{M}$. Assume each session has a fixed or determined transmission path between its source node and destination node. In that case, the data of each session go through multiple links, as shown in Fig. 1. On the other hand, each link may transmit data for multiple sessions. Let $\mathbf{M}_l$ be the set of sessions which go through link l.

The capacities of links are time-varying parameters because of the complex wireless propagation environment. According to the Shannon formula, define c_l as the capacity of link l,

$$c_l = W \log(1 + \eta_l) \tag{2}$$

where W is the bandwidth.

In cognitive radio networks, the transmission powers of unlicensed users are restricted for protecting licensed users. Meanwhile, the unlicensed users suffer great interference from both licensed users and other interference sources from surrounding environment which result in low SINR for unlicensed users. Thus, we can use linear function of the SINR to represent their capacity. When the SINR is low, c_l can be simplified as a linear function of link l's SINR η_l

$$c_l = W \eta_l. \tag{3}$$

Even if the SINR is not low sometimes, the linear function of SINR can be also used to estimate the capacity approximately [11], [24].

To investigate joint power and rate control scheme for cognitive radio networks in presence of licensed users, a set of licensed receivers is denoted by $\mathbf{G}$. In order to protect licensed users, the interference temperature limit is adopted to restrict the interference to licensed receivers (either licensed base station or licensed user terminal) caused by unlicensed users. It is assumed that the communication between licensed users would not be interrupted if the interferences at the licensed receivers are below the interference temperature limit. Let Q_j^{max} denote the interference threshold, which is the maximum interference tolerance of primary receiver j. It is calculated by the multiplex of interference temperature limit and Boltzman's constant.

It is assumed that the unlicensed user can obtain the total interference and the interference threshold of Q_j^{max} for all the surrounding licensed receivers, as shown in Fig. 1. If the licensed users can cooperate with the unlicensed users and broadcast the related interference information, the assumption is reasonable. If not, extra sensors are needed to measure the interference. Once the interference to licensed users exceeds the interference temperature limit, these sensors will inform the interference information to the unlicensed users to decrease their transmit power for decreasing the interference to licensed users. The path gain information can be known by the unlicensed users using the estimation of the received signal or the feedback from other users. The path gain and interference information obtained by unlicensed users are not always perfect, so the performance results of this paper can be considered as an upper bound of the practice cognitive radio networks.

B. Problem Formulation

In this paper, our objective is to achieve high throughput with low power consumption. To achieve a balance of the tradeoff between throughput and power consumption, we define the system performance metric as

$$F(\mathbf{x}, \mathbf{p}) = \sum_{m \in \mathbf{M}} a_m U_m(x_m) - \sum_{i \in \mathbf{N}} b_i p_i \tag{4}$$

where x_m is the data rate of session m, p_i is the transmission power for node i, $\mathbf{x} = \{x_m\}_{m \in \mathbf{M}}$ is users' transmission rate vector, $\mathbf{p} = \{p_i\}_{i\in\mathbf{N}}$ is nodes' transmission power vector. a_m and b_i are the weighted parameters for the tradeoff, which can be adjusted for different networks according to the energy supplement of equipments. $U_m(x_m)$ is the session m's utility, which is an increasing function of the session's data rate, such as the linear function and the logarithmic function of the date rate x_m. It is assumed that there is no throughput requirement for the services of unlicensed users, which is suitable for best-effort services.

The limitations of node power, link capacity and the interference to licensed users impose several constraints on the optimization problem, and our problem can be formulated as

$$\max \quad F(\mathbf{x},\mathbf{p}) = \sum_{m\in\mathbf{M}} a_m U_m(x_m) - \sum_{i\in\mathbf{N}} b_i p_i \tag{5}$$

$$\text{s.t. } p_i \le p_i^{max}, \qquad i \in \mathbf{N} \tag{6}$$

$$\sum_{m\in\mathbf{M}_l} x_m \le c_l, \qquad l \in \mathbf{L} \tag{7}$$

$$\sum_{i\in\mathbf{N}} p_i h_{ij} \le Q_j^{max}, \quad j \in \mathbf{G} \tag{8}$$

where p_i^{max} is the maximum transmit power of node i and h_{ij} is the path gain from the unlicensed node i to the licensed receiver j. Constraint (6) indicates the limitation of the maximum transmit power because of the equipment capability, Constraint (7) denotes that the total data rate of link l should not exceed its capacity c_l, and Constraint (8) is used to protect licensed users, i.e., the total interference at licensed receiver j caused by unlicensed users cannot exceed the interference temperature limit Q_j^{max}.

III. OPTIMAL POWER AND RATE CONTROL

In this section, the optimal solution for the power and rate control is derived analytically. In last section, the problem was formulated as an optimization problem with constraints. To convert this problem to an optimization problem without any constraint, the Lagrangian multiplier method is adopted. The Lagrangian function can be written as

$$\begin{aligned} L(\mathbf{x},\mathbf{p},\boldsymbol{\mu},\boldsymbol{\lambda}) = &\sum_{m\in\mathbf{M}} a_m U_m(x_m) - \sum_{i\in\mathbf{N}} b_i p_i \\ &+ \sum_{l\in\mathbf{L}} \mu_l \left(c_l - \sum_{k\in\mathbf{M}_l} x_k \right) \\ &+ \sum_{j\in\mathbf{G}} \lambda_j \left(Q_j^{max} - \sum_{i\in\mathbf{N}} p_i h_{ij} \right) \end{aligned} \tag{9}$$

where $\boldsymbol{\mu} = \{\mu_l\}_{l\in\mathbf{L}}$ and $\boldsymbol{\lambda} = \{\lambda_j\}_{j\in\mathbf{G}}$ are Lagrangian multipliers satisfying $\boldsymbol{\mu} \ge 0$ and $\boldsymbol{\lambda} \ge 0$. μ_l can be considered as the link price for link l, denotes the link congestion condition. When μ_l is large, it means that high link price is needed to decrease the throughputs of the sessions on the link l because of heavy congestion. λ_j can be considered as the power price for licensed receiver j, which denotes the negative effect of the transmission power of unlicensed users to the licensed user j.

According to Lagrangian duality theory, the following theorem can be obtained to give the optimal scheme to control transmit power and data rate for cognitive radio networks.

Theorem 1: The optimal solution of the joint power and rate control problem can be achieved if the data rate of session m and the transmit power of node i satisfy

$$x_m = U_m'^{(-1)} \left(\sum_{l\in\mathbf{L}_m} \mu_l \Big/ a_m \right), \tag{10}$$

$$p_i = \begin{cases} 0, & W \sum\limits_{l\in\mathbf{L}(i)} \mu_l g_l / I_l \le b_i + \sum\limits_{j\in\mathbf{G}} \lambda_j h_{ij} \\ p_i^{max}, & W \sum\limits_{l\in\mathbf{L}(i)} \mu_l g_l / I_l > b_i + \sum\limits_{j\in\mathbf{G}} \lambda_j h_{ij} \end{cases} \tag{11}$$

where $\mathbf{L}_m$ is the set of links which the data of session m go through, and $\mathbf{L}(i)$ is the set of links which are from node i.

Proof: According to Lagrangian duality theory, the dual problem of the power and rate control problem can be described as

$$\min_{\boldsymbol{\mu},\boldsymbol{\lambda}} \; D(\boldsymbol{\mu},\boldsymbol{\lambda}) \tag{12}$$

where

$$D(\boldsymbol{\mu},\boldsymbol{\lambda}) = \max_{\mathbf{x},\mathbf{p}} \; L(\mathbf{x},\mathbf{p}). \tag{13}$$

Given $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$, the optimal power and rate control solution can be calculated as

$$\begin{aligned} L(\mathbf{x},\mathbf{p}) = &\sum_{m\in\mathbf{M}} a_m U_m(x_m) - \sum_{l\in\mathbf{L}} \mu_l \sum_{k\in\mathbf{M}_l} x_k \\ &+ \sum_{l\in\mathbf{L}} \mu_l c_l - \sum_{i\in\mathbf{N}} b_i p_i - \sum_{j\in\mathbf{G}} \lambda_j \sum_{i\in\mathbf{N}} p_i h_{ij} \\ &+ \sum_{j\in\mathbf{G}} \lambda_j Q_j^{max} \\ = &\sum_{m\in\mathbf{M}} (a_m U_m(x_m) - x_m \sum_{l\in\mathbf{L}_m} \mu_l) \\ &+ \sum_{i\in\mathbf{N}} p_i (W \sum_{l\in\mathbf{L}(i)} \mu_l \frac{g_l}{I_l} - b_i - \sum_{j\in\mathbf{G}} \lambda_j h_{ij}) \\ &+ \sum_{j\in\mathbf{G}} \lambda_j Q_j^{max}. \end{aligned} \tag{14}$$

From (14), the power control and rate control can be decomposed in the optimization process.

The first line of (14) is to calculate the data rate. Let $\partial a_m U_m(x_m)/\partial x_m = \sum_{l\in\mathbf{L}_m} \mu_l$, then the optimal rate control solution can be obtained as (10).

The second line of (14) is used to calculate the power control solution. Because (14) is a linear function of p_i, (11) is obtained considering the limitation of the maximum node power. g_l/I_l indicates the current channel condition for link l, $\sum_{j\in\mathbf{G}} \lambda_j h_{ij}$ denotes the situation of the interference to licensed user. When the channel condition is good (i.e., g_l/I_l is large), the node transmits data with its maximum power. On the contrary, when node i may cause large interference to licensed receivers (i.e., $\sum_{j\in\mathbf{G}} \lambda_j h_{ij}$ is large), the node should stop transmitting to avoid causing too large interference to licensed user. Transmitting with either 0 or the maximum node power is the optimal scheme.

The third line $\sum_{j\in\mathbf{G}} \lambda_j Q_j^{max}$ is an irrelevant parameter to the power and rate control solution, so it can be ignored. □

The optimal power and rate control solution is relevant to $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$. For given $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$, the optimal solution can be obtained according to Theorem 1.

To get the optimal $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$, the traditional sub-gradient method is adopted to adjust the values of $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$.

$$\begin{aligned}\mu_l^{n+1} =& \mu_l^n - \gamma^n(\partial D(\boldsymbol{\mu},\boldsymbol{\lambda})/\partial\mu_l)\\ =& \mu_l^n - \gamma^n\left(c_l^n(\mathbf{p}^n(\boldsymbol{\mu}^n,\boldsymbol{\lambda}^n)) - \sum_{m\in\mathbf{M}_l} x_m(\boldsymbol{\mu}^n)\right), \quad (15)\\ \lambda_j^{n+1} =& \lambda_j^n - \gamma^n\left(\partial D(\boldsymbol{\mu},\boldsymbol{\lambda})/\partial\lambda_j\right)\\ =& \lambda_j^n - \gamma^n\left(Q_j^{max} - \sum_{i\in\mathbf{N}} p_i^n h_{ij}^n\right) \quad (16)\end{aligned}$$

where γ^n is the step size of adaptation, which has great effect on the convergence rate. It has been proved in [18] that the algorithm can eventually converge to an unique point when the step size satisfies

$$\begin{aligned}&\gamma^n \geq 0,\\ &\lim_{n\to\infty}\gamma^n = 0,\\ &\sum_{n=0}^{\infty}\gamma^n = \infty, \quad \sum_{n=0}^{\infty}(\gamma^n)^2 < \infty. \end{aligned} \quad (17)$$

From (15), it can be seen that the Lagrangian multiplier μ_l will increase in $(n+1)$th iteration when the total data rate exceeds the link capacity in nth iteration. Because the data rate x_m in (10) is a decreasing function of μ_l, the source node of the related sessions will decrease its transmission rate. So the source nodes can adjust their transmission rate based on the link congestion condition. Similarly, λ_j can be used to adjust the transmit power based on the interference to licensed users.

It is noted that the sub-gradient method is only one of the methods to adjust the Lagrangian multipliers. Other methods for parameter adjustment, such as ellipsoid method and Frank-Wolfe method [20], [21], can provide difference convergence performance, which is beyond the scope of this paper.

This solution provided by Theorem 1 can achieve proportional fairness between the sessions which use the same link. More details of the fairness performance of the proposed algorithms are discussed in Section VI.

IV. DISTRIBUTED ALGORITHM FOR SYNCHRONOUS SCENARIOS

In this section, the theoretic results obtained in last section are used as a basis for the derivation of the computational algorithms that can be implemented in two system configurations. We first consider the synchronous scenarios, which means all the nodes and links execute the algorithm at the same time.

A. Algorithm Description

The optimal power and rate control scheme is decomposed based on the above analysis. For implementation, the system algorithm is divided into two parts: The node's algorithm and the link's algorithm.

Node i's algorithm:

- calculates the transmission power $p_i(n+1)$ for the next iteration based on (11)
- collects the sum value $\sum_{l\in\mathbf{L}_m}\mu_l$ from the links $l\in\mathbf{L}_m$ if node i is the source of session m.
- calculates the new data rate $x(n+1)$ for the next iteration based on (10) and communicates the result to the links $l\in\mathbf{L}_m$ if node i is the source of session m.

Link l's algorithm:

- receives the rate information from the source nodes whose data go through link l.
- calculates the Lagrangian multiplier μ_l^{n+1} based on (15) and communicate the result to the nodes.
- calculates the Lagrangian multiplier $\boldsymbol{\lambda}^{n+1}$ using (16) based on the interference information from nearby licensed users, and communicates the result to the source nodes of the sessions $m\in\mathbf{M}_l$.

The proposed algorithm needs the cooperation between the licensed users and the unlicensed users, or extra measurement sensors to report the interference information. When computing the Lagrangian multiplier $\boldsymbol{\lambda}^{n+1}$, the unlicensed users need to know the information about the interference at the licensed receivers.

B. Convergence

In this subsection, we prove the convergence of the proposed distributed power and rate control algorithm. Extending the result in [21] to multiple multiplier cases, the following lemma can be obtained for the convergence of the Lagrangian duality based algorithms.

Lemma 1: Let $\boldsymbol{\mu}^*$ and $\boldsymbol{\lambda}^*$ denote the optimal Lagrangian multipliers of the problem respectively, then the algorithm (15) and (16) is said to converge to $\boldsymbol{\mu}^*$ and $\boldsymbol{\lambda}^*$, if for any given δ, the following conditions are satisfied

$$\lim_{n\to\infty}\frac{1}{n}\sum_{\tau=1}^{n}\left(L(\boldsymbol{\mu}^\tau) - L(\boldsymbol{\mu}^*)\right) \leq \delta, \quad (18)$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{\tau=1}^{n}\left(L(\boldsymbol{\lambda}^\tau) - L(\boldsymbol{\lambda}^*)\right) \leq \delta. \quad (19)$$

The following theorem guarantees the convergence of the proposed distributed algorithm.

Theorem 2: Let $\boldsymbol{\mu}^*$ and $\boldsymbol{\lambda}^*$ be the optimal Lagrangian multipliers. If the norm of the subgradients is bounded, i.e., there exists G such that $\|\partial L(\mu_l^n,\lambda_j^n)/\partial\mu_l^n\|_2 \leq G$ for $\boldsymbol{\mu}$, the algorithm (15) and (16) converge to the points within $\gamma G^2/2$ of the optimal points $\boldsymbol{\mu}^*$ and $\boldsymbol{\lambda}^*$.

Proof: If the convergence of $\boldsymbol{\mu}$ is proved, the convergence of $\boldsymbol{\lambda}$ is proved similarly. So, we only prove the convergence of $\boldsymbol{\mu}$ for simplicity.

The norm of the distance between $\boldsymbol{\mu}^{n+1}$ at the $(n+1)$th iteration and the optimal point $\boldsymbol{\mu}^*$ can be described as

$$\begin{aligned}\|\boldsymbol{\mu}^{n+1}-\boldsymbol{\mu}^*\|_2^2 =& \|\boldsymbol{\mu}^n - \gamma\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}} - \boldsymbol{\mu}^*\|_2^2\\ =&\|\boldsymbol{\mu}^n-\boldsymbol{\mu}^*\|_2^2 + \gamma^2\left\|\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}}\right\|_2^2\\ &-2\gamma\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}}(\boldsymbol{\mu}^n-\boldsymbol{\mu}^*)\\ \le& |\boldsymbol{\mu}^n-\boldsymbol{\mu}^*\|_2^2 + \gamma^2\left\|\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}}\right\|_2^2\\ &-2\gamma(L(\boldsymbol{\mu}^n,\boldsymbol{\lambda})-L(\boldsymbol{\mu}^*,\boldsymbol{\lambda})). \end{aligned} \tag{20}$$

Applying the inequalities recursively,

$$\begin{aligned}\|\boldsymbol{\mu}^{n+1}-\boldsymbol{\mu}^*\|_2^2 \le& \|\boldsymbol{\mu}^1-\boldsymbol{\mu}^*\|_2^2\\ &+\gamma^2\sum_{\tau=1}^{n}\left\|\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}}\right\|_2^2\\ &-2\gamma\sum_{\tau=1}^{n}(L(\boldsymbol{\mu}^\tau,\boldsymbol{\lambda})-L(\boldsymbol{\mu}^*,\boldsymbol{\lambda})).\end{aligned} \tag{21}$$

Since $\|\boldsymbol{\mu}^{n+1}-\boldsymbol{\mu}^*\|_2^2 \ge 0$,

$$\begin{aligned}&2\gamma\sum_{\tau=1}^{n}(L(\boldsymbol{\mu}^\tau,\boldsymbol{\lambda})-L(\boldsymbol{\mu}^*,\boldsymbol{\lambda}))\\ &\le\|\boldsymbol{\mu}^1-\boldsymbol{\mu}^*\|_2^2+\gamma^2\sum_{\tau=1}^{n}\left\|\left\{c_l - \sum_{k\in\mathbf{M}_l} x_k\right\}_{l\in\mathbf{L}}\right\|_2^2\\ &\le\|\boldsymbol{\mu}^1-\boldsymbol{\mu}^*\|_2^2+n\gamma^2G^2.\end{aligned} \tag{22}$$

So it can be obtained that

$$\begin{aligned}&\lim_{n\to\infty}\frac{1}{n}\sum_{\tau=1}^{n}(L(\boldsymbol{\mu}^\tau,\boldsymbol{\lambda})-L(\boldsymbol{\mu}^*,\boldsymbol{\lambda}))\\ &\le\lim_{n\to\infty}\frac{\|\boldsymbol{\mu}^1-\boldsymbol{\mu}^*\|_2^2+n\gamma^2G^2}{2n\gamma}\\ &=\frac{\gamma G^2}{2}.\end{aligned} \tag{23}$$

Therefore, with the sub-gradient method, our proposed algorithm (15) can converge to a point within $\gamma G^2/2$ of the optimal value. In the similar way, $\boldsymbol{\lambda}$ converges to such a point $\boldsymbol{\lambda}^*$ too. □

V. DISTRIBUTED ALGORITHM FOR ASYNCHRONOUS SCENARIOS

In the above analysis, it is assumed that the distributed algorithm at all the nodes and links are executed at the same time which is a strict condition for some asynchronous systems. In this section, we propose the iterative algorithm deployed at nodes and links for asynchronous scenarios. This algorithm is also divided into two parts: The node's algorithm and the link's algorithm. The node's algorithm is used to calculate the data rate $\mathbf{x}$ and transmit power $\mathbf{p}$. The link algorithm is used to adjust the lagrangian multipliers $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$.

Let $T_l^1,\ l=1,2,\cdots$ denotes the set of time at which the links update their parameters, and $T_i^2\ i=1,2,\cdots$ denotes the set of time at which the nodes update their parameters.

At time $n\in T_l^1$, based on (15), link l updates the Lagrangian multiplier μ_l^n as

$$\mu_l^{n+1}=\mu_l^n-\gamma^n\left(c_l^n(\mathbf{p}^n(\boldsymbol{\mu}^n,\boldsymbol{\lambda}^n))-\sum_{m\in\mathbf{M}_l}x_m(\boldsymbol{\mu}^n)\right) \tag{24}$$

where

$$x_m(\boldsymbol{\mu}^n)=\sum_{n'=n-n_0}^{n}A(n',n)x_m(\boldsymbol{\mu}^{n'})$$

and $A(n',n)$ is the weight of the data rate at n'th iteration. n_0 latest iterations are considered for accumulated value

$$\sum_{n'=n-n_0}^{n}A(n',n)=1.$$

Then, the Lagrangian multiplier $\boldsymbol{\mu}^n$ for asynchronous scenarios can be updated as

$$\begin{aligned}\mu_l^{n+1}=&\mu_l^n-\gamma^n\Bigg[c_l^n(\mathbf{p}^n(\boldsymbol{\mu}^n,\boldsymbol{\lambda}^n))\\ &-\sum_{m\in\mathbf{M}_l}\sum_{n'=n-n_0}^{n}A(n',n)x_m(\boldsymbol{\mu}^{n'})\Bigg].\end{aligned} \tag{25}$$

At time $n\in T_i^2$, node i update the data rate and transmission power based on (10) and (11)

$$x_m=U_m'^{(-1)}\left(\sum_{l\in\mathbf{L}_m}\mu_l\Big/a_m\right) \tag{26}$$

where

$$\mu_l^n=\sum_{n'=n-n_0}^{n}B(n',n)\mu_l^{n'}.$$

$B(n',n)$ is the weight for $\mu_l^{n'}$ at n'th iteration, which satisfies

$$\sum_{n'=n-n_0}^{n}B(n',n)=1.$$

Then, the data rate for asynchronous algorithm can be updated as

$$x_m^{n+1}=U_m'^{(-1)}\left(\sum_{l\in\mathbf{L}_m}\sum_{n'=n-n_0}^{n}B(n',n)\mu_l^{n'}\Big/a_m\right). \tag{27}$$

When calculating $\mathbf{x}$, the data rates before and after adjusting $\mathbf{p}$ are totally different. Therefore, we need to consider more latest iterations to calculate the accumulated $\mathbf{x}$ and its corresponding Lagrangian multiplier $\boldsymbol{\mu}$. On the contrary, $\mathbf{x}$ is relatively stable. The update of the transmit power $\mathbf{p}$ and the Lagrangian multiplier $\boldsymbol{\lambda}$ can be calculated by (11) and (16), which are the same as the synchronous case. The latest information is utilized to estimate the current condition, so the estimation is relatively correct.

Node i's algorithm:

- calculates the transmission power $p_i(n+1)$ for the next iteration based on (11)
- collects the sum value $\sum_{l \in \mathbf{L}_m} \mu_l$ continually from the links $l \in \mathbf{L}_m$ if node i is the source of session m.
- calculates the new data rate $x(n+1)$ for the next iteration based on (27) and communicates the result to the links $l \in \mathbf{L}_m$ if node i is the source of session m.

Link l's algorithm:

- receives the rate information continually from the source nodes whose data go through link l.
- calculates the Lagrangian multiplier μ_l^{n+1} based on (25) and communicate the result to the nodes.
- calculates the Lagrangian multiplier $\boldsymbol{\lambda}^{n+1}$ using (16) based on the interference information from nearby licensed users, and communicates the result to the source nodes of the sessions $m \in \mathbf{M}_l$.

Compared with the proposed algorithm for synchronous scenarios, the asynchronous distributed algorithm has several differences.

- The links and nodes need to collect the information all the time, because other links and nodes send information at the time when they execute the proposed distributed algorithm.
- The data rate $\mathbf{x}$ and the Lagrangian multiplier $\boldsymbol{\mu}$ are updated considering the accumulated information, using (25) and (27), respectively, instead of (10) and (15).

From the process described above, all the necessary parameters can be obtained based on the local information. The node's algorithm and link's algorithm can be executed in a distributed manner. Although exchanging information between two algorithms is needed in every iteration, the cost is very low that only the calculated results need to be exchanged. Meanwhile, proved by [19], the proposed asynchronous algorithm can achieve the convergence as the algorithm for synchronous scenarios. As the synchronous algorithm, the algorithm for asynchronous scenarios also needs to know the information about the interference at the licensed receivers.

VI. Utility-Based Fairness

In this section, the fairness issues are considered for the proposed algorithms. The proposed algorithms for power and rate control can provide proportional fairness to the sessions. Utility-based fairness can extend the application of the proposed algorithms by defining different utilities to provide the required kinds of fairness to the sessions.

The concept of proportional fairness was first introduced in [22]. A proportional fair rate vector $(x_m, m \in \mathbf{M})$ is defined such that for any other feasible rate vector $(y_m, m \in \mathbf{M})$, it is satisfied that

$$\sum_{m \in M} \frac{y_m - x_m}{x_m} \leq 0. \tag{28}$$

This definition is motivated by the assumption that all users have the same logarithmic bandwidth utility function $U_m(x_m) = \log(x_m)$.

In this sense, the algorithm we proposed can achieve rate-based proportional fairness as in the convergence point x^*,

$$x_m^* = U_m^{'(-1)} \left(\sum_{l \in \mathbf{L}_m} \mu_l^* \Big/ a_m \right). \tag{29}$$

If the utility function is set as $U_m(x_m) = a_m \log(x_m)$, then

$$x_m^* \sum_{l \in \mathbf{L}_m} \mu_l^* = a_m. \tag{30}$$

If the link resource is valued as the product of bandwidth and link price, (30) implies that each session can occupy a_m units of the system resource and achieve rate-based proportional fairness.

From the user's viewpoint, as long as its utility requirement is satisfied, it is not important how much bandwidth is allocated to the user. In a more general sense, achieving utility fair allocation is more useful in practice. In [23], the conception of utility fairness is proposed. A transformation function $f_m(\mu_m)$ is used to describe the available utility for the path used of session m, which includes the links that the data of session m go through. $\mu_m = \sum_{l \in \mathbf{L}_m} \mu_l$ denotes the aggregate link congestion measure.

Theorem 3: If $f_m(\mu_m)$ is a continuous, differentiable, strictly decreasing function of μ_m and can be valued as the available utility the system can offer to node m, the proposed algorithm with the following objective function can achieve utility proportional fairness.

$$F(x_m, p_i) = \sum_{m \in \mathbf{M}} \overline{U(x_m)} - \sum_{i \in \mathbf{N}} b_i p_i \tag{31}$$

where

$$\overline{U_m(x_m)} = \int f_m^{-1}(a_m U_m(x_m)) dx_m.$$

Proof: In equilibrium, the equation $U_m(x_m^*) = f(\mu_m^*)$ holds, so $x_m^* = U_m^{-1}(f(\mu_m^*))$.

Define $x_m = G_m(\mu_m)$, we construct a second order utility function $F_m(x_m)$ as the integral of $G_m^{-1}(x_m)$,

$$F_m(x_m) = \int G_m^{-1}(x_m) dx_m. \tag{32}$$

The equilibrium rate is the unique solution of the optimization problem

$$\max\ F_m(x_m) - \mu_m x_m \tag{33}$$

because the first order necessary optimality condition is

$$G_m^{-1}(x_m) - \mu_m = 0. \tag{34}$$

In the equilibrium, this equation holds.

Considering all the sessions, the objective function can be redefined as (31). Let $f_m(\mu_m) = 1/\mu_m$,

$$a_m U_m(x_m^*) = f_m(\mu_m^*) = \frac{1}{\mu_m^*}. \tag{35}$$

In equilibrium point, it is obtained that

$$U_m(x_m^*) \sum_{l \in \mathbf{L}_m} \mu_l^* = \frac{1}{a_m}. \tag{36}$$

The rate allocation vector in equilibrium points $\mathbf{x}^*$ is a utility proportional fair rate vector, as for any other rate vector $\mathbf{y}$, there exists

$$\sum_{m \in \mathbf{M}} \frac{\partial \overline{U_m(x_m^*)}}{\partial x_m}(y_m - x_m^*) = \sum_{m \in \mathbf{M}} \frac{y_m - x_m^*}{U_m(x_m^*)} \leq 0. \tag{37}$$

Therefore, it is proved that the proposed algorithm can achieve proportional fairness between sessions. □

As discussed previously, for achieving proportional fairness, the utility function $U_m(\cdot)$ needs to be a strict concave function. In real time systems, user's utility function can be different for different classes of applications, such as stepwise utility function in some applications(i.e., streaming services). In the new proposed utility function $\overline{U_m(x_m)}$, we only need the utility $U_m(x_m)$ to be a non-negative, continuous, increasing function with data rate x_m. Then, we can have

$$\frac{\partial^2 \overline{U_m(x_m)}}{\partial^2 x_m} = a_m f_m'^{-1}(a_m U_m(x_m)) U_m'(x_m). \tag{38}$$

Because $f^{-1}(\cdot)$ is a strict decreasing function, $\overline{U_m(x_m)}$ is a strict concave function and ensures the utility proportional fairness.

VII. NUMERICAL RESULTS

In order to investigate the performance, the simulation in a simple scenario is deployed to give a physical insight for the proposed algorithms.

A. Simulation Configuration

A cognitive radio network with two licensed receivers and six unlicensed nodes is employed for system level simulation. Both the licensed users and unlicensed users are distributed randomly. The area covers 2 km×2 km. Each node has a session, which has a random destination and determines the corresponding min-hop path. The path gains are calculated by Free Space Propagation model, and COST207-BU Rayleigh fading model [25] is considered. The average performance is investigated considering different fading situations. The power region of the unlicensed users is set from -100 dBm to 20 dBm. The maximum interference temperature limit for the licensed users is assumed to be -100 dBm. All a_m and b_i are set to 1. Different values can be configured according to the specific networks.

The simulation results on the performance in the synchronous scenarios are provided in this section. The results for asynchronous scenarios are similar with the synchronous case, just like our theoretic analysis. In the simulation, the rate-based fairness scheme is applied in the algorithms, $U_m(x_m) = a_m \log(x_m)$.

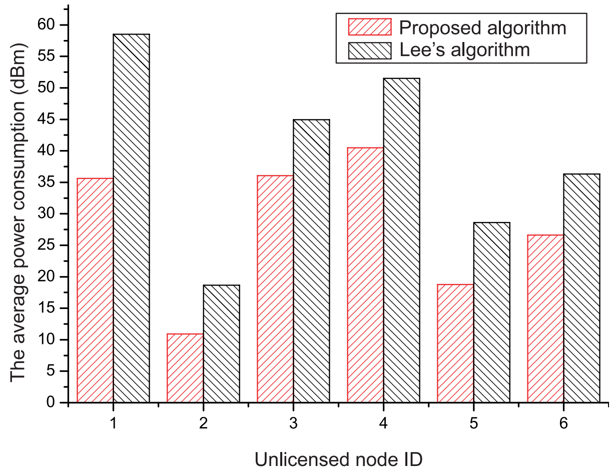

Fig. 2. The average power consumption of unlicensed nodes.

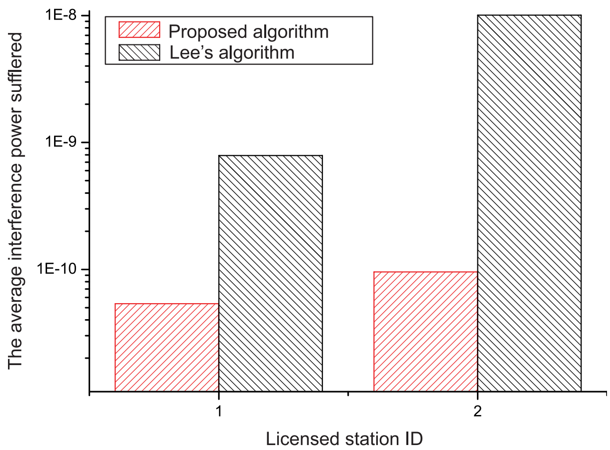

Fig. 3. The average interference power to licensed receivers.

B. Performance Comparison

The performance of the proposed joint power control and rate control algorithm is investigated and compared with the one proposed in [11] (Lee's algorithm). Even though the proposed algorithm developed in this paper is specialized to cognitive radio systems and the algorithm in [11] is developed for ad hoc networks, the results obtained are providing an insight into the operation of these algorithms in different scenarios.

Fig. 2 shows that the power consumption can be decreased dramatically by the proposed algorithm. Due to the presence of the licensed users, the unlicensed users who transmit in the same spectrum band have to decrease their transmission power to avoid causing too large interference to licensed users.

Fig. 3 denotes that the interference to licensed users with the proposed algorithm can be restricted below the maximum interference temperature limit. That is because that the interference constraint is a hard constraint in the optimization problem. In Lee's algorithm, the authors do not consider the existence of the

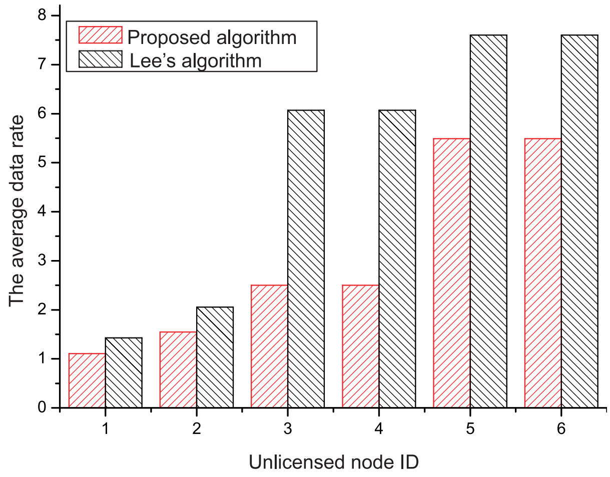

Fig. 4. The average data rate of unlicensed users.

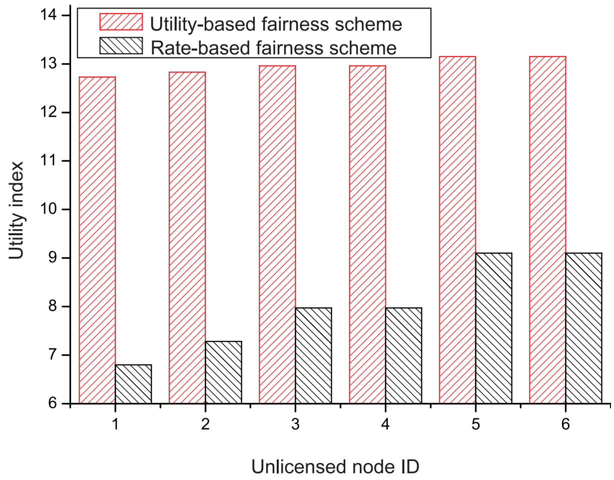

Fig. 6. Unlicensed users' utility.

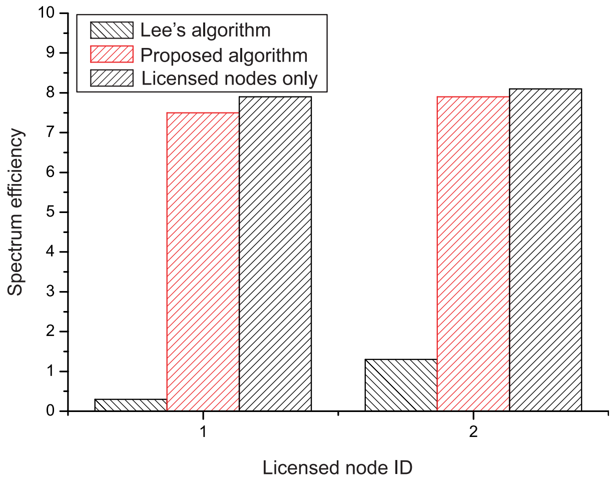

Fig. 5. Licensed users' spectrum efficiency.

licensed users. When the licensed nodes are closed to the unlicensed nodes, the interference to licensed users is large. As shown in Fig. 3, the interference caused by Lee's algorithm is much larger compared with the proposed algorithm.

Fig. 4 shows that the average data rate achieved in the proposed algorithm is relatively lower because it is assumed in the simulation that the licensed users always exist. The unlicensed users have to restrict their transmission power for protecting licensed users, which results in a lower data rate.

Fig. 5 illustrates the effect to licensed users' performance caused by unlicensed users. From the results, licensed users' utilities in the proposed algorithm are almost the same as the situation without any unlicensed users. In contrast, the licensed users' utilities are much lower in Lee's algorithm. That is because the licensed users have target SINRs. When their SINRs are below the target, their utilities decrease dramatically. In the proposed algorithm, the interference to licensed users is restricted under the thresholds, so the licensed users can always reach their target SINRs and achieve high utilities. Compared with Lee's algorithm, the proposed algorithm can restrict the interference to licensed users very well, with the cost that the performance of unlicensed system degrades gracefully. Thus, the proposed algorithms is suitable to be applied to cognitive radio networks.

C. Utility-based Fairness

By changing the utility function, the proposed algorithms can provide different kind of fairness to the sessions. In the algorithm for utility-based fairness, the utility function is defined as $U_m(x_m) = a_m \log(1 + x_m)$.

Fig. 6 illustrates the fairness performance comparison between the algorithm for rate-based fairness and the one for utility-based fairness. In the utility-based fairness scheme, the sessions can obtain almost equal utilities. In this sense, the utility-based fairness scheme can achieve more flexibility for different kinds of networks.

VIII. CONCLUSION

In this paper, we proposed a distribted power and rate control algorithm considering the co-existence of the licensed users and unlicensed users. The problem is formulated as an optimization problem with multiple constraints. Applying Lagrangian duality theory, the optimal solution is obtained. Then, the distributed algorithms for both synchronous and asynchronous scenarios are proposed. The convergence of the proposed algorithms are proved. Finally, we give a discussion to the fairness performance for utility-based fairness. Numerical results show the performance comparison between the proposed algorithm and the existed algorithm in [11]. In our proposed algorithm, the unlicensed users can use the licensed band without causing large negative effect on the licensed users' utilities.

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