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Shape Recognition and Classification Based on Poisson Equation- Fourier-Mellin Moment Descriptor

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Abstract – In this paper, we present a new shape descriptor, which is named Poisson equation-Fourier-Mellin moment Descriptor. We solve the Poisson equation in the shape area, and use the solution to get feature function, which are then integrated using Fourier-Mellin moment to represent the shape. This method develops the Poisson equation-geometric moment Descriptor proposed by Lena Gorelick, and keeps both advantages of Poisson equation-geometric moment and Fourier-Mellin moment. It is proved better than Poisson equation-geometric moment Descriptor in shape recognition and classification experiments.

***Keywords*:** **Poisson equation, Fourier-Mellin moment, shape recognition,shape classification**

1. Introduction

Shape is one of the most important features of objects, and how to recognize and classify shapes is an important topic not only in pattern recognition, but also in computer vision. At present, shape variant is the mainstream of the shape recognition methods, and the Moment method is widely used. Fourier-Mellin Moment is one of complex Moments, and it can be transformed to Rotation-and- Translation Invariant. It attains good result in shape recognition. However, moment can only describe the global feature of the shape, and can not include the detail ones.

In 2006Lena Gorelick with her group proposed a novel approach using the solution to the Poisson equation to represent a shape[1]. They used the solution to the Poisson equation in the shape area, to extract useful properties of a shape, which are then integrated using geometric moment. We improve this method by integrating the properties with Fourier-Mellin moment instead of the geometric moment, and obtain Poisson equation-Fourier-Mellin moment Descriptor. This new method combines the advantages of Poisson equation-geometric moment descriptor and Fourier-Mellin moment. It can be transformed to Rotation-and-Translation Invariant, and also describe a shape more accurately. We use both methods in shape recognition and classification experiments, and find our method obtain better results. Here, we introduce Poisson equation-geometric moment Descriptor proposed by Lena Gorelick first.

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2. The Poisson Equation-geometric moment Descriptor

Consider a shape S (Fig. 1) surrounded by a simple, closed contour ∂S. Based on the thought of random walk, we calculate the mean time required for every point in the shape to hit the boundaries.

Let $U(x, y)$ denote this measure, we can get

$$\Delta U(x,y) = -1 \quad (1)$$

$(x,y) \in \partial S$, subject to Dirichlet boundary conditions $U(x, y) = 0$, at the bounding contour ∂S.

Solve the equation, we can get this measure. Then use this measure to construct feature function:

$$\phi(x,y) = U(x,y) + \|\nabla U(x,y)\|^2 \quad (2)$$

$\phi(x,y)$ is used in hierarchical representation. By simply thresholding ϕ, we can divide a shape into parts.

Another feature function is the leading eigenvector of the Hessian matrix of U, which describes the local orientation of the shape.

Using these two features, construct two measures:

$$H_\theta(x,y) = e^{-\gamma|\theta - |\alpha(x,y')||} \quad (3)$$

Fig. 1. A collection of shapes.

$-\pi/2\le\alpha(x,y)\le\pi/2$, and γ is a constant (we used $\gamma=3$). This measure identifies vertical and horizontal regions of a shape by detecting points for which the orientation computed with the Hessian matrix is close to either zero or $\pi/2$.

Denote by $\hat{\phi}(x,y)$, the function $\phi(x,y)$ centered about its saddle point value (the value at the point where U is maximal) and normalized so that its maximal absolute value is 1.

$$K_\phi(x,y)=1/1+e^{-\delta\hat{\phi}(x,y)} \tag{4}$$

(We used $\delta=4$). The second measure $K_\phi(x,y)$identifies concave regions as well as elongated convex sections by emphasizing points with high values of ϕ.

Then the two measures are integrated by geometric moment in [1], instead of which, we will use Fourier-Mellin moment. Before that, we introduce Fourier-Mellin moment first.

3. Fourier-Mellin Moment

Definition:

$$F_{kl}(g)=\int_{r=0}^{¥}\int_{q=0}^{2p} r^{k-1}e^{-ilq}g(r,q) \tag{5}$$

is called Fourier-Mellin moment of $g(r,\theta)$, where (r,θ) is the polar coordinate of image, $g(r,\theta)$ is the weight function in polar coordinate, l is a integer, k is a positive integer which is called the order of the moment.

Fourier-Mellin moment is a common complex moment. To make sure the descriptor will be a Rotation-Translation- and Scale-Invariant, we transform it into Fourier-Mellin moment invariant.

First, we calculate the moment center (x_c,y_c), and then move the original point of polar coordinate to the moment centre. This makes sure it is Translation invariable. Second, we will consider the relationship between the original moment and the moment after rotation and scale transformation.

When a shape rotates an angle α, the scale factor of the scale transformation is s, the relationship bet-
ween the Fourier-Mellin moment after rotation-scale transformation and the original shape moment is:

$$F_{kl}'(g)=s^k e^{il\theta}F_{ki}(g) \tag{6}$$

To ensure it is Translation-and Scale-Invariant, here we make $l=0$, then definite

$$\Psi_{k0}(g)=F_{k0}(g)/F_{20}(g) \tag{7}$$

Easily proved:

$$\Psi_{k0}(g)=F_{k0}(g)/F_{20}(g) \tag{8}$$

$\Psi_{k0}(g)$ is the Fourier-Mellin moment invariant of g. Next, we use Ψ_{k0} to describe a shape, and construct Poisson equation-Fourier-Mellin moment Descriptor.

4. Poisson equation-Fourier-Mellin moment Descriptor

As we can see in part 2, for each shape, we get 3 measures: $H_0(x,y)$, $H_{\pi/2}(x,y)$ and $K_{\phi/(x,y)}$. We change the 3 functions into polar coordinate form, and we get $H_0'(r,\theta)$, $H_{\pi/2}'(r,\theta)$ and $K_\phi'(r,\theta)$. Then we calculate

$$(\Psi_{k0}(H_0'),\Psi_{k0}(H_{\pi/2}'),\Psi_{k0}(K_\phi'))k=0,1,2\ldots \tag{10}$$

The function we got is named Poisson equation-Fourier-Mellin moment Descriptor (*PFD*).

5. The Shape Simplicity

To calculate the simplicity between shape i and j, we calculate the distance between Poisson equation-Fourier-Mellin moment Descriptors:
PFE_i and PFD_j:

$$\text{distance}=\sqrt{\sum_l\|PFD_i(l)-PFD_j(l)\|^2} \tag{11}$$

The smaller the distance is, the more similar the two shapes will be.

6. Shape Recognition and Classification Experiments

In this part, we calculate Fourier-Mellin moment invariant when $k=0,1,2,3$. For each shape, we got *PFD* which is a 12 dimensions vector.

6.1. Shape Recognition Experiment

To the 12 shapes in Fig. 2.(from general shape database, the first 6 are in same class, and the rest from different ones), we use *PFD* and *PGD* in our shape recognition experiment. The results are in Table 1 and Table 2.

Table 1 and table 2 shows the simplicity of 12 shapes in Figure 2. The black data is those figures which are lower than threshold and the red data higher than

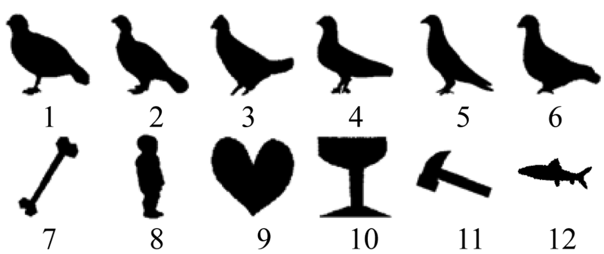

Fig. 2. 12 shapes from general shape database

Table 1. *PGD* Shape recognition experiment result (Threshold is 0.23)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.151	0.221	0.316	0.196	0.318	0.393	0.494	0.596	0.664	0.469	0.269
2	0.151	0.000	0.139	0.200	0.097	0.189	0.355	0.453	0.683	0.703	0.348	0.284
3	0.221	0.139	0.000	0.139	0.146	0.140	0.285	0.341	0.722	0.673	0.282	0.207
4	0.316	0.200	0.139	0.000	0.176	0.046	0.260	0.345	0.835	0.678	0.172	0.296
5	0.196	0.097	0.146	0.176	0.000	0.181	0.335	0.413	0.695	0.692	0.299	0.279
6	0.318	0.189	0.140	0.046	0.181	0.000	0.287	0.373	0.835	0.702	0.188	0.314

Table 2. *PFD* Shape recognition experiment result£ (Threshold is 0.23)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.048	0.079	0.205	0.082	0.125	0.605	0.588	0.403	0.566	0.568	0.456
2	0.048	0.000	0.082	0.226	0.071	0.139	0.634	0.607	0.356	0.611	0.598	0.495
3	0.079	0.082	0.000	0.146	0.025	0.067	0.558	0.525	0.395	0.561	0.521	0.428
4	0.205	0.226	0.146	0.000	0.167	0.113	0.413	0.383	0.519	0.464	0.377	0.301
5	0.082	0.071	0.025	0.167	0.000	0.079	0.581	0.542	0.370	0.582	0.542	0.451
6	0.125	0.139	0.067	0.113	0.079	0.000	0.522	0.481	0.428	0.513	0.474	0.381

threshold (The same below). We can see the simplicity of most shapes from different classes are above the threshold, while the simplicity of shapes from same class are below. Comparing the two methods, the accuracy of *PFD* is 100%, while the accuracy of *PGD* is 90.3%, which proves the former is better than the latter.

6.2. Shape Classification Experiment

We take apple, children, rat, cell phone, face etd. 5 classes from the general shape database, 5 shapes each class (Fig. 3.), then use them in shape classification experiment. The result is showed in attached list.2

In the attached list, G is the experiment result of *PGD* is of *PFD*. We can see that the two shape descriptors both can attain good result, and if we take a appropriate threshold (it is 0.1 for *PGD*, while 0.13 for *PFD*), they both can get 100% accuracy.

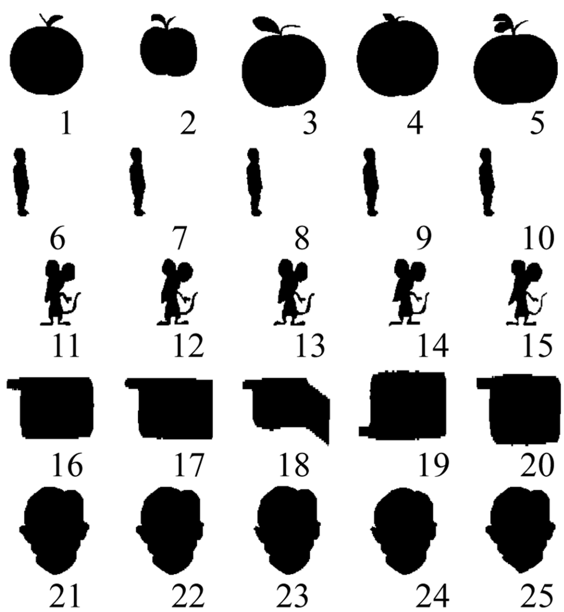

Fig. 3. Shapes used in shape classification experiment

7. Conclusion

Through the experiments we can see that compared with Poisson equation-geometric moment Descriptor, Poisson equation-Fourier-Mellin moment Descriptor is Rotation-Translation-and Scale -Invariable, so it is superior in classifying a large number of shapes. In the future, we hope to find a fast algorithm, and induce the time of shape classification experiment.

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Zou Jiancheng

Lu Yan

Ke nan-nan

G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	0.059	0.079	0.076	0.07	0.66	0.66	0.66	0.66	0.66	0.452	0.454	0.453	0.453	0.451	0.926	0.937	0.908	0.909	0.913	0.554	0.556	0.556	0.556	0.559
2	0.059	0	0.075	0.055	0.072	0.605	0.605	0.605	0.605	0.605	0.397	0.398	0.397	0.398	0.396	0.87	0.88	0.852	0.853	0.856	0.511	0.512	0.512	0.512	0.515
3	0.079	0.075	0	0.069	0.013	0.635	0.635	0.635	0.635	0.635	0.434	0.435	0.435	0.435	0.433	0.889	0.898	0.872	0.872	0.876	0.49	0.492	0.492	0.492	0.496
4	0.076	0.055	0.069	0	0.065	0.615	0.615	0.615	0.615	0.615	0.409	0.41	0.41	0.41	0.408	0.881	0.89	0.862	0.862	0.866	0.506	0.508	0.508	0.508	0.511
5	0.07	0.072	0.013	0.065	0	0.642	0.642	0.643	0.642	0.642	0.441	0.442	0.441	0.441	0.44	0.898	0.907	0.881	0.881	0.885	0.5	0.502	0.503	0.503	0.506
6	0.66	0.605	0.635	0.615	0.642	0	0	0	0	0	0.223	0.223	0.225	0.222	0.225	0.311	0.324	0.285	0.289	0.291	0.432	0.425	0.424	0.424	0.42
7	0.66	0.605	0.635	0.615	0.642	0	0	0	0	0	0.223	0.223	0.225	0.222	0.225	0.311	0.324	0.285	0.289	0.291	0.432	0.425	0.424	0.424	0.42
8	0.66	0.605	0.635	0.615	0.643	0	0	0	0	0	0.223	0.223	0.225	0.222	0.225	0.311	0.324	0.285	0.289	0.291	0.432	0.425	0.425	0.424	0.42
9	0.66	0.605	0.635	0.615	0.642	0	0	0	0	0	0.223	0.223	0.225	0.222	0.225	0.311	0.324	0.285	0.289	0.291	0.432	0.425	0.424	0.424	0.42
10	0.66	0.605	0.635	0.615	0.642	0	0	0	0	0	0.223	0.223	0.225	0.222	0.225	0.311	0.324	0.285	0.289	0.291	0.432	0.425	0.424	0.424	0.42
11	0.452	0.397	0.434	0.409	0.441	0.223	0.223	0.223	0.223	0.223	0	0.005	0.007	0.004	0.005	0.508	0.52	0.486	0.49	0.492	0.401	0.394	0.394	0.394	0.392
12	0.454	0.398	0.435	0.41	0.442	0.223	0.223	0.223	0.223	0.223	0.005	0	0.005	0.004	0.003	0.506	0.518	0.485	0.489	0.491	0.401	0.395	0.395	0.394	0.392
13	0.453	0.397	0.435	0.41	0.441	0.225	0.225	0.225	0.225	0.225	0.007	0.005	0	0.007	0.005	0.508	0.52	0.486	0.49	0.493	0.402	0.395	0.395	0.395	0.393
14	0.453	0.398	0.435	0.41	0.441	0.222	0.222	0.222	0.222	0.222	0.004	0.004	0.007	0	0.004	0.506	0.517	0.484	0.488	0.49	0.399	0.392	0.392	0.392	0.39
15	0.451	0.396	0.433	0.408	0.44	0.225	0.225	0.225	0.225	0.225	0.005	0.003	0.005	0.004	0	0.508	0.52	0.486	0.491	0.493	0.401	0.395	0.394	0.394	0.392
16	0.926	0.87	0.889	0.881	0.898	0.311	0.311	0.311	0.311	0.311	0.508	0.506	0.508	0.506	0.508	0	0.016	0.032	0.04	0.03	0.543	0.534	0.534	0.533	0.527
17	0.937	0.88	0.898	0.89	0.907	0.324	0.324	0.324	0.324	0.324	0.52	0.518	0.52	0.517	0.52	0.016	0	0.044	0.045	0.038	0.548	0.539	0.539	0.538	0.532
18	0.908	0.852	0.872	0.862	0.881	0.285	0.285	0.285	0.285	0.285	0.486	0.485	0.486	0.484	0.486	0.032	0.044	0	0.026	0.015	0.539	0.53	0.53	0.529	0.523
19	0.909	0.853	0.872	0.862	0.881	0.289	0.289	0.289	0.289	0.289	0.49	0.489	0.49	0.488	0.491	0.04	0.045	0.026	0	0.013	0.532	0.524	0.523	0.523	0.517
20	0.913	0.856	0.876	0.866	0.885	0.291	0.291	0.291	0.291	0.291	0.492	0.491	0.493	0.49	0.493	0.03	0.038	0.015	0.013	0	0.537	0.529	0.528	0.528	0.521
21	0.554	0.511	0.49	0.506	0.5	0.432	0.432	0.432	0.432	0.432	0.401	0.401	0.402	0.399	0.401	0.543	0.548	0.539	0.532	0.537	0	0.013	0.014	0.014	0.02
22	0.556	0.512	0.492	0.508	0.502	0.425	0.425	0.425	0.425	0.425	0.394	0.395	0.395	0.392	0.395	0.534	0.539	0.53	0.524	0.529	0.013	0	0.002	0.002	0.009
23	0.556	0.512	0.492	0.508	0.503	0.424	0.424	0.425	0.424	0.424	0.394	0.395	0.395	0.392	0.394	0.534	0.539	0.53	0.523	0.528	0.014	0.002	0	0	0.008
24	0.556	0.512	0.492	0.508	0.503	0.424	0.424	0.424	0.424	0.424	0.394	0.394	0.395	0.392	0.394	0.533	0.538	0.529	0.523	0.528	0.014	0.002	0	0	0.007
25	0.559	0.515	0.496	0.511	0.506	0.42	0.42	0.42	0.42	0.42	0.392	0.392	0.393	0.39	0.392	0.527	0.532	0.523	0.517	0.521	0.02	0.009	0.008	0.007	0
F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	0.041	0.125	0.055	0.094	0.607	0.607	0.607	0.607	0.607	0.384	0.384	0.386	0.386	0.384	0.913	0.928	0.895	0.887	0.894	0.457	0.46	0.46	0.461	0.463
2	0	0	0.107	0.08	0.072	0.574	0.574	0.574	0.574	0.574	0.357	0.356	0.358	0.358	0.357	0.877	0.892	0.858	0.851	0.858	0.42	0.423	0.424	0.424	0.425
3	0.041	0.107	0	0.13	0.037	0.503	0.503	0.504	0.503	0.503	0.278	0.277	0.278	0.279	0.277	0.81	0.826	0.793	0.786	0.792	0.385	0.386	0.386	0.386	0.389
4	0.125	0.08	0.13	0	0.104	0.601	0.601	0.601	0.601	0.601	0.372	0.372	0.373	0.374	0.372	0.915	0.931	0.897	0.89	0.896	0.458	0.461	0.462	0.462	0.465
5	0.055	0.072	0.037	0.104	0	0.52	0.52	0.52	0.52	0.52	0.297	0.296	0.298	0.298	0.296	0.827	0.842	0.809	0.802	0.808	0.387	0.389	0.389	0.389	0.392
6	0.094	0.574	0.503	0.601	0.52	0	0	0.001	0.001	0.001	0.237	0.237	0.237	0.235	0.237	0.328	0.346	0.311	0.306	0.312	0.224	0.218	0.217	0.216	0.221
7	0.607	0.574	0.503	0.601	0.52	0	0	0.001	0.001	0.001	0.237	0.237	0.237	0.236	0.237	0.327	0.346	0.311	0.306	0.311	0.224	0.218	0.217	0.217	0.221
8	0.607	0.574	0.504	0.601	0.52	0.001	0.001	0	0	0	0.238	0.237	0.237	0.236	0.238	0.327	0.346	0.311	0.306	0.311	0.224	0.218	0.217	0.216	0.22
9	0.607	0.574	0.503	0.601	0.52	0.001	0.001	0	0	0	0.238	0.237	0.237	0.236	0.238	0.327	0.346	0.311	0.306	0.311	0.223	0.217	0.217	0.216	0.22
10	0.607	0.574	0.503	0.601	0.52	0.001	0.001	0	0	0	0.238	0.237	0.237	0.236	0.238	0.327	0.346	0.311	0.306	0.311	0.223	0.217	0.217	0.216	0.22
11	0.607	0.357	0.278	0.372	0.297	0.237	0.237	0.238	0.238	0.238	0	0.004	0.004	0.003	0.003	0.561	0.579	0.544	0.538	0.544	0.206	0.204	0.204	0.204	0.211
12	0.384	0.356	0.277	0.372	0.296	0.237	0.237	0.237	0.237	0.237	0.004	0	0.003	0.002	0.001	0.56	0.578	0.543	0.537	0.543	0.207	0.204	0.204	0.204	0.211
13	0.384	0.358	0.278	0.373	0.298	0.237	0.237	0.237	0.237	0.237	0.004	0.003	0	0.003	0.002	0.56	0.578	0.543	0.538	0.543	0.209	0.206	0.206	0.206	0.214
14	0.386	0.358	0.279	0.374	0.298	0.235	0.236	0.236	0.236	0.236	0.003	0.002	0.003	0	0.002	0.558	0.577	0.542	0.536	0.542	0.206	0.203	0.203	0.203	0.211
15	0.386	0.357	0.277	0.372	0.296	0.237	0.237	0.238	0.238	0.238	0.003	0.001	0.002	0.002	0	0.56	0.578	0.544	0.538	0.544	0.207	0.205	0.205	0.205	0.212
16	0.384	0.877	0.81	0.915	0.827	0.328	0.327	0.327	0.327	0.327	0.561	0.56	0.56	0.558	0.56	0	0.023	0.024	0.032	0.024	0.493	0.488	0.487	0.486	0.485
17	0.913	0.892	0.826	0.931	0.842	0.346	0.346	0.346	0.346	0.346	0.579	0.578	0.578	0.577	0.578	0.023	0	0.036	0.042	0.035	0.506	0.501	0.5	0.499	0.498
18	0.928	0.858	0.793	0.897	0.809	0.311	0.311	0.311	0.311	0.311	0.544	0.543	0.543	0.542	0.544	0.024	0.036	0	0.01	0.003	0.472	0.466	0.465	0.465	0.463
19	0.895	0.851	0.786	0.89	0.802	0.306	0.306	0.306	0.306	0.306	0.538	0.537	0.538	0.536	0.538	0.032	0.042	0.01	0	0.008	0.464	0.459	0.458	0.458	0.456
20	0.887	0.858	0.792	0.896	0.808	0.312	0.311	0.311	0.311	0.311	0.544	0.543	0.543	0.542	0.544	0.024	0.035	0.003	0.008	0	0.472	0.466	0.465	0.465	0.464
21	0.894	0.42	0.385	0.458	0.387	0.224	0.224	0.224	0.223	0.223	0.206	0.207	0.209	0.206	0.207	0.493	0.506	0.472	0.464	0.472	0	0.007	0.008	0.008	0.014
22	0.457	0.423	0.386	0.461	0.389	0.218	0.218	0.218	0.217	0.217	0.204	0.204	0.206	0.203	0.205	0.488	0.501	0.466	0.459	0.466	0.007	0	0.001	0.002	0.011
23	0.46	0.424	0.386	0.462	0.389	0.217	0.217	0.217	0.217	0.217	0.204	0.204	0.206	0.203	0.205	0.487	0.5	0.465	0.458	0.465	0.008	0.001	0	0.001	0.011
24	0.46	0.424	0.386	0.462	0.389	0.216	0.217	0.216	0.216	0.216	0.204	0.204	0.206	0.203	0.205	0.486	0.499	0.465	0.458	0.465	0.008	0.002	0.001	0	0.011
25	0.461	0.425	0.389	0.465	0.392	0.221	0.221	0.22	0.22	0.22	0.211	0.211	0.214	0.211	0.212	0.485	0.498	0.463	0.456	0.464	0.014	0.011	0.011	0.011	0