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Theoretical Investigation of First-order and Second-order Polarization-mode Dispersion Tolerance on Various Modulation Formats in 40 Gb/s Transmission Systems with FEC Coding

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We investigated the polarization-mode dispersion (PMD) tolerance for 40Gb/s non-return to zero (NRZ), duobinary NRZ, return to zero (RZ), carrier-suppressed RZ (CS-RZ), and duobinary-carrier-suppressed RZ (DCS-RZ) modulation formats with a forward error correction (FEC) coding. The power penalty has been calculated as a measure of the system performance due to PMD. After comparison of the PMD tolerance of various modulation formats, our results suggest that RZ signals have the best tolerance against the effect of first-order PMD only. The duobinary NRZ modulation format is most resilient to PMD when both first- and second-order PMD are considered. However, the duobinary NRZ modulation format is the most sensitive to the incident angle of the input signal to a fiber axis in the presence of first- and second-order PMD, leading to incident angle-dependent power penalty. The coding gain by FEC can cope with the power penalties induced by first- and second-order PMD up to a DGD value of 16ps.

Keywords: Polarization mode dispersion, Tolerance, Forward error correction, Coding gain

OCIS codes : (060.0060) Fiber optics and optical communications; (060.2360) Fiber optics links and subsystems; (060.4080) Modulation; (060.4510) Optical communications

I. INTRODUCTION

Polarization mode dispersion (PMD) is a serious impairment for high-bit-rates and long-haul optical transmission systems. PMD is caused by asymmetry and stress in the fiber core, which locally lead to birefringence [1]. In the first-order approximation for PMD, due to the fiber birefringence, single mode fiber supports two orthogonal polarization modes traveling at different velocities along a fiber. The difference in traveling time between the two principal states of polarization (PSP) is called as differential group delay (DGD). It is the dominant source of pulse distortion in dispersion managed high bit rate transmission systems [2], [3].

Many methods to overcome a PMD limitation in high capacity and long-haul transmission systems have been developed in optical and electrical domains. Among them, the PSP method and post compensation scheme are a popular optical PMD compensators [4], [5]. The PMD compensations in the electrical domain are achieved by equalizers such as transversal filters (TF) and decision feedback equalizers (DFE) [6]. Although the PMD compensation techniques are promising, they have an issue of dynamic features in implementation. Therefore, it may be interesting to consider modulation formats and FEC coding to mitigate how large the effect of PMD is in a fiber.

In this paper, we investigated first- and second-order PMD tolerance of modulation formats such as NRZ, duobinary NRZ, RZ, CS-RZ, and DCS-RZ in a 40Gb/s transmission system. Some modulation formats may play an important role in the transmission performance of high speed systems with the presence of PMD. The use of a modulation format more resilient to PMD also helps to design a marginal system impaired by PMD. Therefore, we analyzed the PMD tolerance on modulation

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formats, which was decided by the amount of degraded receiver sensitivity at 10^{-9} BER by a given DGD for optically preamplified receivers, to find an optimum modulation format in transmission systems impaired by PMD. Also, we investigated the coding gain by FEC coding according to DGD values in 40 Gb/s transmission systems to validate restoration of the system performance impaired by first- and second-order PMD.

The paper is organized as follows. In section Ⅱ, we present our modeling method of the PMD effect and the configuration of a schematic diagram used in simulation. Section III is dedicated to an analysis on the PMD tolerance of the modulation formats. In detail, it consists of power penalty due to first-order PMD only with and without the SPM effect, power penalty due to first- and second-order PMD, the PMD tolerance according to incident angle, and coding gain by FEC coding using the Reed-Solomon (RS) (255, 239) code in 40 Gb/s systems with first- and second-order PMD. Finally, in section IV we summarize our results and discuss modulation formats resilient to PMD.

II. CONFIGURATION AND MODELING FOR PMD TOLERANCE

1. Wave Propagation in Optical Fibers in the Presence of PMD

PMD in a fiber can be modeled as a concatenation of several birefringent segments. The input field of birefringent segments is represented by $E_{in}(t) = E_{in}(t)j_{in}$, where jin is the input Jones polarization vector which depends on the azimuth and the ellipticity of the input SOP. The output field is given by [7], [8]

$$E_{out}(\omega) = T(\omega)E_{in}(\omega)j_{in} = E_{out}(\omega)j_{out}(\omega) \tag{1}$$

where $T(\omega)$ represents the transfer matrix of the fiber and $j_{out}(\omega)$ is given by matrix multiplication of the unitary matrix $U(\omega)$ and the input Jones polarization vector j_{in}. Here, $E_{out}(\omega)$ is obtained by the Fourier transformation of output signals of a lossy and dispersive fiber with the nonlinear effects.

In Eq. (1), the unitary matrix $U(\omega)$ can be written as $U(\omega)=R^{-1}(\omega)D(\omega)R(\omega)$, where $R(\omega)$ takes into account the rotation of PSPs, and the dispersive matrix $D(\omega)$ takes into account the different propagation speeds on the two PSPs [7], [8].

$$R(\omega) = \begin{bmatrix} \cos k\omega & \sin k\omega \\ -\sin k\omega & \cos k\omega \end{bmatrix} \tag{2}$$

$$D(\omega) = \begin{bmatrix} \exp(j\Delta\tau\omega/2) & 0 \\ 0 & \exp(-j\Delta\tau\omega/2) \end{bmatrix} \tag{3}$$

where $\Delta\tau = \Delta\tau_{0+}\Delta\tau_{\omega}\omega$ under the second-order approximation. Thus, the PMD dispersion vector represented by the second-order approximation is given by $\vec{\Omega}(\omega) = (\Delta\tau_0 + \omega\cdot\Delta\tau_\omega)\cdot\vec{q_0} + \Delta\tau_0\cdot\ \omega\cdot\ 2\vec{k}$, where $\Delta\tau_\omega = \partial\Delta\tau/\partial\omega$ and $2\vec{k} = \partial\vec{q}/\partial\omega$ [7], [8]. The second-order PMD effects are represented by frequency dependent terms, the linear DGD frequency dependence ($\Delta\tau_\omega$) and the PSP rotation with a constant angular rate ($|2\vec{k}|$). Here, $2\vec{k}$ perpendicular to $\vec{q_0}$ causes depolarization. This depolarization is a result of the rotation of PSPs with frequency. The linear DGD frequency dependence induces the polarization dependent chromatic dispersion (PCD). In the second-order PMD approximation, the time domain output field obtained by inverting Eq. (1) can be represented as [7], [8]

$$\begin{aligned}\overrightarrow{E_{out}} = \frac{1}{2\sqrt{2}}\{&(a\vec{u}+b\vec{u})(E^{+}_{out}(t+\Delta\tau_0/2)+E^{-}_{out}(t-\Delta\tau_0/2)) \\ &+a\vec{u}(E^{+}_{out}(t-2k+\Delta\tau_0/2)-E^{-}_{out}(t-2k-\Delta\tau_0/2)) \\ &+b\vec{u}^{*}(E^{+}_{out}(t+2k+\Delta\tau_0/2)-E^{-}_{out}(t+2k-\Delta\tau_0/2))\end{aligned} \tag{4}$$

where $E^{\pm}_{out}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_{in}(\omega)e^{-\alpha z}e^{j\{-\bar{\beta}(\omega)z \pm \Delta\tau_\omega\omega^2/2\}}e^{j\omega t}d\omega$ is the time domain output field which loss, chromatic dispersion, and polarization-dependent chromatic dispersion are considered, $a = e^{j\theta}\cos(\varepsilon+\pi/4)$ and $b = e^{j\theta}\cos(\varepsilon-\pi/4)$, where θ is an azimuth, and ε is an ellipticity of the input SOP, $\vec{u} = [1\ j]$, $\vec{u} = [1 - j]$, and $\beta(\omega)$ is the mode propagation constant.

2. Transmission System

Fig. 1 shows the schematic diagram for a transmission system. Transmitters of various modulation formats (NRZ, RZ, duobinary NRZ, CS-RZ, and DCS-RZ) generate 40Gb/s signals with 2^7 bit. One span consists of a standard single-mode fiber (SSMF), two erbium-doped fiber amplifiers (EDFAs), and a dispersion compensating fiber (DCF). The chromatic dispersion in the

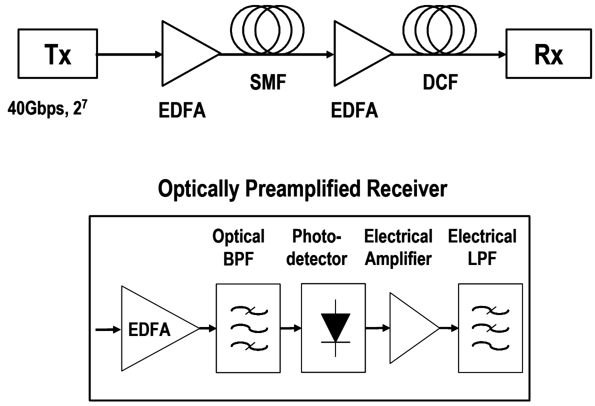

FIG. 1. Schematic diagram of a 40 Gb/s transmission system with various modulation formats.

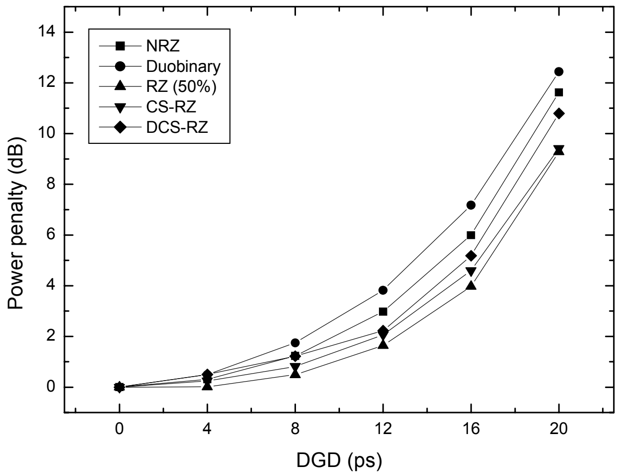

FIG. 2. Power penalties due to first-order PMD at 10^{-9} BER for different modulation formats with only considering first-order PMD and incident angle of 45°.

SSMF with the fiber length of 80km was compensated by the DCF with a fiber length of 15.72 km. The first EDFA was used to adjust power of the input signal launched into the SSMF and the second EDFA was used to compensate a loss in the DCF [9], [10]. The receiver model consists of an optical amplifier, followed by a Gaussian band-pass optical filter, a square law detector, and a fifth-order Bessel-Thomson electrical filter. The shot noise, beat noises generated by signal and amplified spontaneous emission (ASE) noise, and the receiver circuit noise are considered in the calculation of bit error rates (BER), which was optimized with both threshold level and sampling time [9]. The receiver sensitivity at 10^{-9} BER was used to measure the PMD-induced penalty.

III. SIMULATION RESULTS AND DISCUSSIONS

1. Power Penalty due to First-Order PMD Only

To analyze the first-order PMD tolerance of NRZ, RZ, duobinary NRZ, CS-RZ, and DCS-RZ modulation formats, we neglected $\Delta\tau_\omega$ and $|2\vec{k}|$ representing second-order PMD. Fig. 2 shows the first-order PMD-induced power penalties at 10-9 BER for different modulation formats when the incident angle is 45˚. The incident angle is an angle between the SOP of input signals and the PSPs of the fiber. When the incident angle is 45˚,

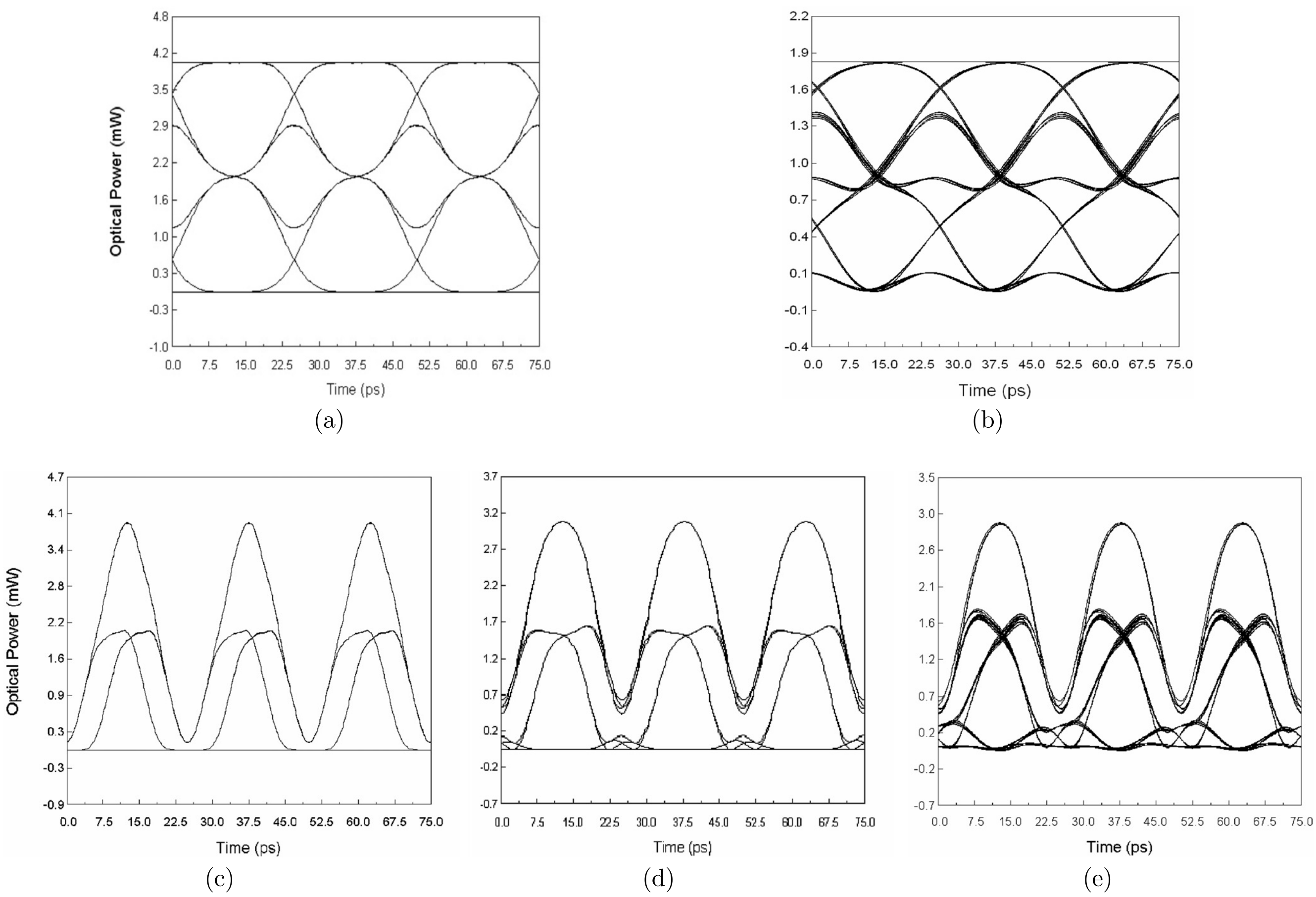

FIG. 3. Optical eye diagrams of various modulation formats for a DGD value of 20ps with only considering first-order PMD and incident angle of 45° for (a) NRZ, (b) Duobinary, (c) RZ, (d) CS-RZ, and (e) DCS-RZ signals.

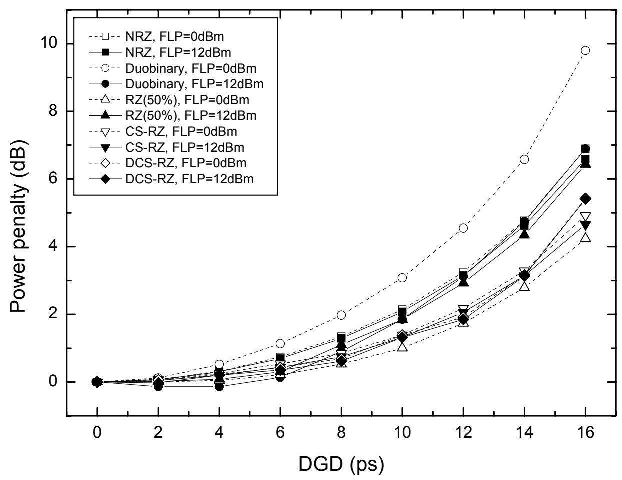

FIG. 4. Power penalties due to first-order PMD at 10-9 BER for 40 Gbps transmission with including SPM (incident angle =45°).

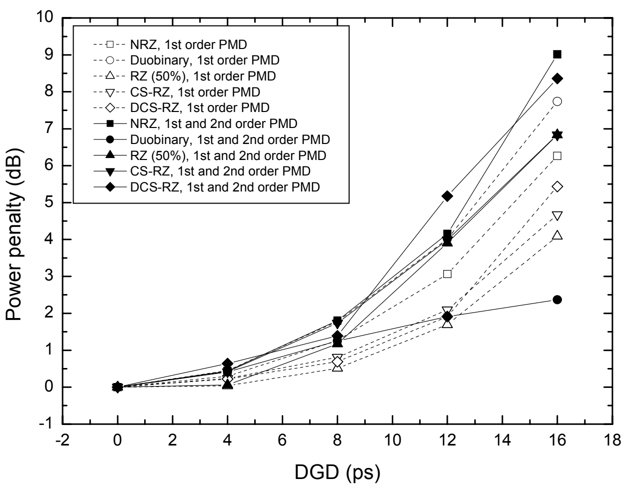

FIG. 5. Power penalties due to first- and second-order PMD at 10-9 BER for different modulation formats with the incident angle of 45°.

TABLE 1. rms values of $\Delta\tau_\omega$ and $|2\vec{k}|$ for a given DGD.

$\Delta\tau$(PS)	$\Delta\tau_\omega$(PS/GHz)	$\lvert 2\vec{k}\rvert$(degree/GHz)
0	0	0
4	3.6272×10^{-3}	0.1659
8	14.5088×10^{-3}	0.3318
12	32.6448×10^{-3}	0.4976
16	58.0352×10^{-3}	0.6635

the deformation of signal by DGD is maximized since inter-symbol interference (ISI) between the PSPs of the fiber is maximized. As shown in Fig. 2, the first-order PMD-induced power penalty for the duobinary modulation format is the largest among the modulation formats since the signal with larger duty cycle tends to have larger ISI [11]. However, the RZ modulation format has the best tolerance against first-order PMD due to its small duty cycle.

Fig. 3 shows the eye diagrams of the modulation formats for a DGD value of 20ps. The effect of ISI due to first-order PMD deforms signals after transmission. However, the RZ modulation format has clearly opened eye diagrams compared with the NRZ and duobinary NRZ modulation formats for large DGD value up to 20ps.

2. Power Penalty due to First-Order PMD in Optical Link with SPM

Fig. 4 shows the first-order PMD-induced power penalties at 10-9 BER for a dispersion-managed link with SPM included. The effect of SPM was analyzed for a fiber launching power from 0dBm to 12dBm. As shown in Fig. 4, the power penalty of the RZ modulation format increases with fiber launching power due to its characteristics easily affected by the fiber nonlinearity. However, the first-order PMD-induced power penalty of the duobinary NRZ modulation format decreases fiber launching power although it is easily impaired by fiber nonlinearity. This phenomenon is due to the interaction of first-order PMD and SPM. The RZ modulation format is deformed by the interaction between first-order PMD and SPM, leading to increase the penalty for higher fiber launching power. On the other hand, the duobinary NRZ modulation format has relatively better receiver sensitivity at high fiber launching power due to the interaction between first-order PMD and SPM. Meanwhile, NRZ, CS-RZ, and DCS-RZ modulation formats have similar results regardless of fiber launching power since they are hardly affected by SPM.

3. Power Penalty due to First-Order and Second-Order PMD

If second-order PMD is considered in analyzing the PMD tolerance, the PMD tolerance of various modulation formats becomes closer to that of an actual optical link with PMD. To analyze the tolerance of first- and second-order PMD for various modulation formats, we set $\Delta\tau_\omega$ and $|2\vec{k}|$ representing second-order PMD to their rms values for a given DGD. The rms values of $\Delta\tau_\omega$ and $|2\vec{k}|$ are decided by the theoretical scaling rule based on the measurement of several fibers with different mean DGD and a statistically significant amount of data for each fiber. Table 1 shows the rms value of $\Delta\tau_\omega$ and $|2\vec{k}|$ for a given DGD.

Fig. 5 shows the power penalties due to first- and second-order PMD at 10^{-9} BER for different modulation formats with 45° incident angle. The chromatic dispersion in the SSMF is perfectly compensated by the DCF and fiber nonlinearity is not included in considering only the effect of first- and second-order PMD. Power penalties due to first- and second-order PMD are larger than the

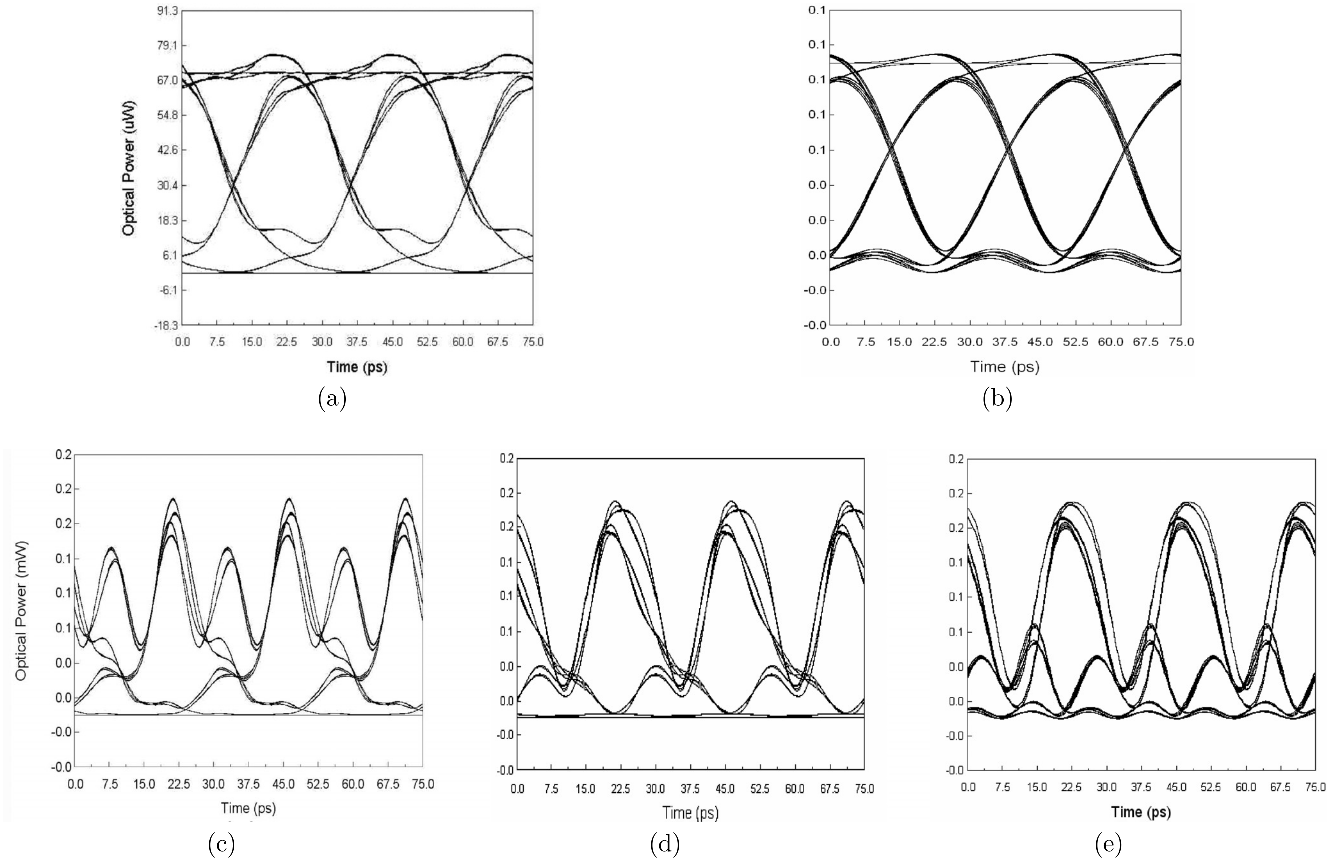

FIG. 6. Optical eye diagrams of various modulation formats at DGD =12ps, $\Delta\tau_{\omega} = 32.6448\times10^{-3}$ ps/GHz, and $|2\vec{k}| = 0.4976°/\text{GHz}$ with considering first- and second-order PMD, and incident angle=45° for (a) NRZ, (b) Duobinary, (c) RZ, (d) CS-RZ, and (e) DCS-RZ signals.

first-order PMD-induced power penalties except for the duobinary NRZ modulation format. RZ-shaped signals are more affected by second-order PMD. While the NRZ modulation format is more sensitive to first-order PMD due to their high susceptibility to ISI, RZ-shaped signals are usually less tolerant to second-order PMD than NRZ signals due to their wide optical spectra. The duobinary modulation format outperforms the NRZ modulation format in the tolerance to first- and second-order PMD due to its narrower power spectrum, whereas NRZ signals outperform the duobinary modulation format in tolerance to first-order PMD [12]. Therefore, power penalties due to first- and second-order PMD of the duobinary NRZ modulation format are smaller than that of the NRZ modulation format at DGD above 5ps that second-order PMD becomes dominant. Fig. 6 shows the eye diagrams of the modulation formats at DGD = 12ps, $\Delta\tau_{\omega} = 32.6448 \times10^{-3}$ ps/GHz, and $|2\vec{k}| = 0.4976°/\text{GHz}$. In this condition, the duobinary NRZ modulation format has a relatively small power penalty of 1.9 dB. On the other hand, the NRZ and RZ modulation formats have a relatively large power penalties of 4.2 dB and 3.9 dB. As shown in Fig. 6, the duobinary NRZ modulation format has clearly opened eye diagrams compared with the NRZ and RZ modulation formats.

4. Analysis of PMD Tolerance According to Incident Angle

The incident angle plays an important role in the deformation of signals by PMD. As previously mentioned, the first-order PMD-induced power penalties become a maximum at 45° incident angle for a given DGD. Thus, to analyze the effect of the incident angle on power penalties induced by PMD, we investigated power penalties according to the incident angle for a given DGD. Fig. 7 shows the power penalties due to first- and second-order PMD at 10^{-9} BER according to incident angles of input signals for the NRZ and duobinary NRZ modulation formats. The results show that the NRZ modulation format has very small variance compared with an average value in power penalties according to the incident angle. However, the duobinary NRZ modulation format has slightly smaller variances than the average value of power penalties. The duobinary NRZ modulation format is more sensitive to the incident angle than the NRZ modulation format in the presence of first- and second-order PMD, leading to incident angle-dependent power penalties.

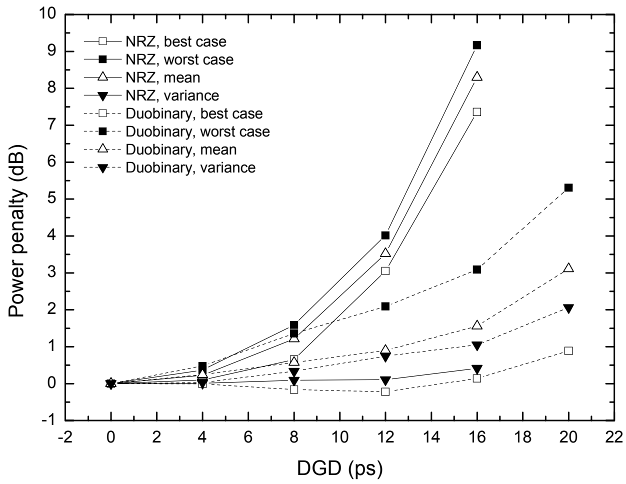

FIG. 7. Power penalties due to first- and second-order PMD at 10^{-9} BER according to the incident angle of input signals for NRZ and duobinary NRZ modulation formats.

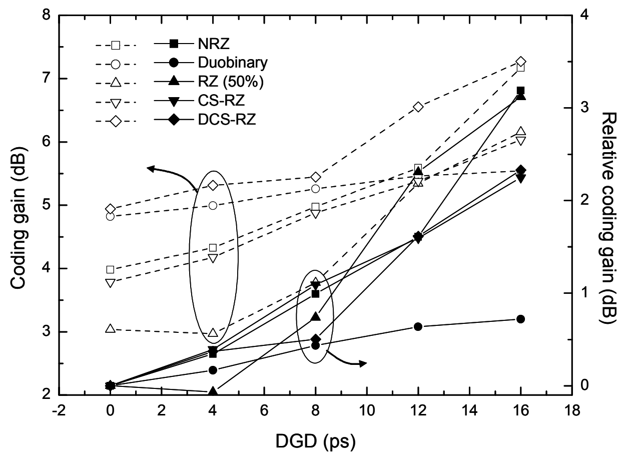

FIG. 8. Coding gain of modulation formats obtained by the ITU-T G.975 FEC coding, RS(255, 239) code with considering first- and second-order PMD and SPM.

5. Coding Gain by FEC Coding using RS(255,239) Code in 40Gb/s Systems with First-Order and Second-Order PMD

It's interesting to see the PMD tolerance on FEC coding for different modulation formats. The FEC coding has been employed to improve a system margin in high-speed long-distance lightwave systems. Therefore, we estimated coding gain as a measure of FEC performance for each modulation format. Coding gain was investigated in 40 Gb/s systems with first-order and second-order PMD, and SPM. 39.81312 Gb/s 211-bit pseudorandom binary sequence (PRBS) signals are encoded with the RS(255, 239) code to produce 42.47843 Gb/s signals with 6.7% overhead. The out-of-band FEC scheme based on the RS(255, 239) code is the standard FEC for undersea cable systems determined by ITU-T G.975.

Fig. 8 shows the coding gains by the ITU-T G.975 FEC code, RS(255, 239) code. All used modulation formats except for the duobinary NRZ modulation format have a tendency that the coding gain increases according to increased DGD by the RS code's characteristics strong to burst error since the pulse shape of signals is degraded due to increased DGD, leading to more burst error [13]. However, as shown in Fig. 5, the duobinary NRZ modulation format is very resilient to first- and second-order PMD. It has a small power penalty of 2.37dB at DGD = 16ps, $\Delta\tau = 58.0352\times10^{-3}$ ps/GHz, and $|2\vec{k}| = 0.6635°/$GHz. Therefore, it has relatively small coding gain, compared with other modulation formats. This result is well shown in relative coding gain (RCG) by first- and second-order PMD. The RCG was calculated by the coding gain difference between at DGD=0 and a given DGD. The duobinary NRZ modulation format has a RCG of 0.72dB at DGD = 16 ps, $\Delta\tau = 58.0352\times10^{-3}$ ps/GHz, and $|2\vec{k}| = 0.6635°/$GHz. The NRZ and RZ modulation formats have relatively large RCG of 3.19 dB and 3.12 dB, respectively, compared with that of the duobinary NRZ modulation format.

IV. CONCLUSION

We investigated the PMD tolerance limit of a 40 Gb/s transmission system using different modulation formats; NRZ, duobinary NRZ, RZ, CS-RZ, and DCS-RZ. We showed that the NRZ modulation format with larger pulse-width has larger power penalties in the presence of first-order PMD only, while the modulation formats with wider optical spectra have larger power penalties in the presence of second-order PMD. The simulation results suggest that the duobinary NRZ modulation format is the most tolerant to first- and second-order PMD among the modulation formats in the transmission system. However, we validated that the duobinary NRZ modulation format is more sensitive to incident angle than the NRZ modulation format. Also, we investigated a coding gain obtained by the standard FEC code, the RS(255, 239) code recommended by ITU-T G.975. Our results show that the coding gain by the FEC coding copes with power penalties induced by first- and second-order PMD up to 16ps DGD. The modulation formats except for the duobinary NRZ modulation format have a tendency that the coding gain increases according to increased DGD by the RS code's characteristics strong to burst errors. The duobinary NRZ modulation format has a RCG of 0.72dB at DGD =16ps, $\Delta\tau = 58.0352\times10^{-3}$ ps/GHz, and $|2\vec{k}| = 0.6635°/$GHz. The NRZ and RZ modulation formats have a relatively large RCG of 3.19 dB and 3.12 dB, respectively, compared with that of the duobinary NRZ modulation format.

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REFERENCES

1. J. Sakai and T. Kimura, "Birefringence and polarization characteristics of single-mode optical fibers under elastic deformations," IEEE J. Quantum Electron. **17**, 1041-1051 (1981).
2. H. Jang, K. Kim, J. Lee, and J. Jeong, "PMD tolerance of 10 Gbps modulated signals due to SOA-induced chirp in SOA booster amplifiers," J. Opt. Soc. Korea **12**, 232-239 (2008).
3. S.-M. Kim, "PMD effect on the clock-based optimum dispersion compensation monitoring technique," J. Opt. Soc. Korea **13**, 112-115 (2009).
4. H. Sunnerud, C. Xie, M. Karlsson, R. Samuelsson, and P. A. Andrekson, "A comparison between different PMD compensation techniques," IEEE J. Lightwave Tech. **20**, 368-378 (2002).
5. R. Noe, D. Sandel, and V. Mirvoda, "PMD in high-bit-rate transmission and means for its mitigation," IEEE J. Sel. Topics Quantum Electron. **10**, 341-355 (2004).
6. J. Wang and J. M. Kahn, "Performance of electrical equalizers in optically amplified OOK and DPSK systems," IEEE Photonics Technology Letters **16**, 1397-1399 (2004).
7. F. Bruyere, "Impact of first and second order PMD in optical digital transmission systems," Optical Fiber Technol. **2**, 269-280 (1996).
8. C. Francia, F. Bruyere, D. Penninckx, and M. Chbat, "PMD second-order effects on pulse propagation in single-mode optical fibers," IEEE Photonics Technology Letters **10**, 1739-1741 (1998).
9. R. A. A. Lima, M. C. R. Carvalho, and L. F. M. Conrado, "On the simulation of digital optical links with EDFA's: an accurate method for estimating BER through Gaussian approximation," IEEE J. Select. Topics Quantum Electron. **3**, 1037-1044 (1997).
10. Y. Kim, J. Lee, Y. Kim, and J. Jeong, "Evaluation of transmission performance in cost-effective optical duobinary transmission utilizing modulator's bandwidth or low-pass filter implemented by a single capacitor," Optical Fiber Technol. **10**, 312-324 (2004).
11. C. Xie, L. Möller, H. Haunstein, and S. Hunsche, "Comparison of system tolerance to polarization-mode dispersion between different modulation formats," IEEE Photonics Technology Letters **15**, 1168-1170 (2003).
12. N. Kaneda, X. Liu, Z. Zheng, X. Wei, M. Tayahi, M. Movassaghi, and D. Levy, "Improved polarization-mode-dispersion tolerance in duobinary transmission," IEEE Photonics Technology Letters **15**, 1005-1007 (2003).
13. J. Youn, H. Jang, K. Kim, and J. Jeong "BER performance due to irregularity of row-weight distribution of the parity-check matrix in irregular LDPC codes for 10-Gb/s optical signals," IEEE J. Lightwave Tech. **23**, 2673-2680 (2005).