JPE 9-3-19

ICPE'07 Selected Paper

Mathematical Analysis and Simulation Based Survey on Initial Pole Position Estimation of Surface Permanent Magnet Synchronous Motor

Tae-Woong Kim[†], Patrick Wheeler[*], and Jaeho Choi[**]

[†]Dept. of Control and Instrumentation Eng., Gyeongsang National University, Jinju, Korea
[*]Dept. Of Electrical and Electronic Engineering, The University of Nottingham, Nottingham, United Kingdom
[**]School of Electrical and Computer Engineering, Chungbuk National University, Cheongju, Korea

ABSTRACT

In this paper, the initial pole-position estimation of a surface (non-salient) permanent magnet synchronous motor is mathematically analyzed and surveyed on the basis of simulation analysis, and developed for accurate servo motor drive. This algorithm is well carried out under the full closed-loop position control without any pole sensors and is completely insensitive to any motor parameters. This estimation is based on the principle that the initial pole-position is simply calculated by the reverse trigonometric function using the two feedback currents in the full closed-loop position control. The proposed algorithm consists of the predefined reference position profile, the information of feedback currents, speed, and relative position, and the reverse trigonometric function for the initial-pole position estimation. Comparing with the existing researches, the mathematical analysis is introduced to get a more accurate initial pole-position of the surface permanent magnet motor under the closed-loop position control. It is found that the proposed algorithm can be easily applied in servo drive applications because it satisfies the following user's specifications; accuracy and moving distance.

Keywords: Surface permanent magnet synchronous motor, Initial pole-position estimation, Sensorless control, Servo drive

1. Introduction

The permanent magnet synchronous motor (PMSM) has been widely applied to the industrial-servo drive fields such as machine tools and semiconductor manufacturing [1-8]. With the well-defined appropriate control strategies, it can provide significant energy savings and high performance. However, it requires the precise initial pole-position information (typically obtained by a pole sensor) for a smooth start-up and a precise servo-motion control.

It may be prohibited to install a pole sensor on the motor shaft due to compactness, low-cost requirements, and mechanical mis-installation which often yields the initial pole position (IPP) error. Depending on the type of SPMSM, some servo motor drives would require the

---

Manuscript received Jan. 31, 2009; revised April 15, 2009

[†]Corresponding Author: kimtw@gnu.ac.kr
Tel: +82-55-751-5373, Fax: +82-55-757-3974, Gyeongsang Univ.
Dept. of Control and Instrumentation Eng., Gyeongsang Univ. Korea
[*]Dept. Of Electrical and Electronic Engineering, University of Nottingham, U.K.
[**]School of Electrical Computer Eng., Chungbuk National Univ., Korea

expensive pole-sensors and often be exposed to heat, dust, electric noise, mechanical vibration, etc. such that the position sensor signals get distorted. On the other hand, if the initial pole-position cannot be accurately known, the performances of a motor itself can not be obtained. The motor may produce less torque or it may become unstable. Moreover, during start-up it may rotate in the wrong direction and lose control.

Recently, several IPP estimation algorithms for SPMSM have been reported [1-5]. The principle of these algorithms is based on the agreement of two reference frames in the control axes and the motor axes which are carried under the current control or the speed control. They show good estimation results, however problems such as long stroke and weakness against mechanical disturbances and complex implementation still remain [1-2]. The accuracy of the IPP estimation algorithm of surface PMSM proposed by the authors is dependant on the integral period of torque-component currents. The estimation accuracy is maximally increased and constantly obtained by the simplified implementation for SPMSM sensorless servo drives.

The authors will introduce the absolute accumulation of q-axis current integral into the previous IPP estimation [1] to clear the above problems. Using the absolute accumulation of q-axis currents makes the accuracy of the IPP estimation not dependant on the integral accumulation period within the same position interval mode and higher accuracy to keep constant. The effectiveness of the proposed IPP estimation algorithm will be verified through both mathematical analysis and simulation based analysis.

2. Mathematical Analysis of IPP Estimation

2.1 Principle of IPP estimation based on agreement of two coordinate frames

The principle of the initial pole-position estimation shown in Fig. 1 was developed by the author [2-3]. Suppose that there are two dq reference frames; they are a control side dq reference frame (virtual reference frame) and a motor dq reference frame (actual reference frame). And the d-coordinate axis of the control side dq reference frame is fixed to 0 degree. For example, the actual IPP of a motor may be placed on a temporary control side dq reference frame and the actual IPP coincides with the d-coordinate axis of a motor side dq reference frame. The actual IPP shown in this figure is deviated from a temporary control side dq reference frame, the deviated angle of which is defined as the deviated IPP (θ_{err}). To detect the actual initial-pole position without any pole sensor, the deviated angle of the IPP should be estimated using the information of some reference or feedback signals.

Under the constant flux, some thrust force is necessary for a motor to be moved at the same reference speed, or to the same reference position, regardless of any deviated IPP, which is the principle of the initial pole-position estimation.

Supposing the reference thrust force T_{e1}^* is given at the control side q axis ($\theta_{\text{est}} + \theta_{\text{shift1}}$) shifted from the original control side q axis when the motor is controlled in agreement with its reference value; the actual thrust force T_{e1} at the motor side q axis can be expressed as (1) with the reference thrust force and the deviated position.

Supposing the reference thrust force T_{e2}^* is given at the control side q axis ($\theta_{\text{est}} + \theta_{\text{shift2}}$) shifted from the original control side q axis when the position of the motor is controlled in agreement with its reference value; the actual thrust force T_{e2} at the motor side q axis can be expressed as (2) with the reference thrust force and the deviated position. And the shift position functions (θ_{shift1} and θ_{shift2}) can be set to any value if the position difference between the 1st shift position θ_{shift1} and the 2nd shift position θ_{shift2} is kept at 90 degrees.

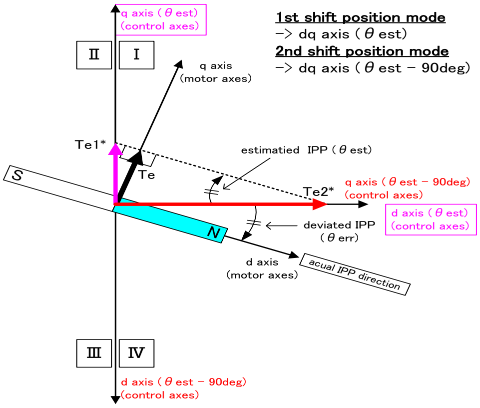


Fig. 1. Principle of initial-pole position estimation.

$$\begin{aligned} T_{e1} = T_{e1}^{\text{shift1}} &= T_{e1}^{*}\cos(\theta_{fb}^{\text{shift1}} - \theta_{\text{act}}) \\ &= T_{e1}^{*}\cos(\theta_{fb} - \theta_{\text{act}} + \theta_{\text{shift1}}) \\ &= T_{e1}^{*}\cos(\theta_{\text{err}} + \theta_{\text{shift1}}) \\ &= T_{e1}^{*}\cos(\theta_{\text{err}} + 0) \end{aligned} \tag{1}$$

$$\begin{aligned} T_{e2} = T_{e2}^{\text{shift2}} &= T_{e2}^{*}\cos(\theta_{fb}^{\text{shift2}} - \theta_{\text{act}}) \\ &= T_{e2}^{*}\cos(\theta_{fb} - \theta_{\text{act}} + \theta_{\text{shift2}}) \\ &= T_{e2}^{*}\cos(\theta_{\text{err}} + \theta_{\text{shift2}}) \\ &= T_{e2}^{*} = \cos(\theta_{\text{err}} - 90^{\circ}) \end{aligned} \tag{2}$$

where $\theta_{fb}^{\text{shift}} = \theta_{fb} + \theta_{\text{shift}}$, $\theta_{\text{shift1}} = 0^{\circ}$, $\theta_{\text{shift2}} = -90^{\circ}$, θ_{act} is the absolute actual position, and θ_{fb} is the incremental feedback position.

The position information in the current controller is adjusted by (3), which includes the shift function for selecting the reference thrust force direction in any control side q axis reference frame.

$$\theta_{\text{ref}} = \theta_{\text{fb}} + \theta_{\text{shift}} + \theta_{\text{cmp}} + \theta_{\text{default}} \tag{3}$$

where θ_{cmp} is the compensation position that increases the estimation accuracy and is not mentioned in detail in this paper, and θ_{default} is the default IPP(=0).

According to the principle of IPP estimation, each necessary thrust force of both the 1st shift position mode and the 2nd shift position mode is equal and then (1) is equal to (2).

$$T_{e1}^{*}\cos(\theta_{\text{err}} + 0^{\circ}) = T_{e2}^{*}\cos(\theta_{\text{err}} - 90^{\circ}) \tag{4}$$

Using the relation between torque and q axis current and solving (4), the deviated IPP can be calculated by the following equation and replaced as the estimated IPP;

$$\theta_{\text{est}} = \tan^{-1}\left(\frac{T_{e1}^{*}}{T_{e2}^{*}}\right) = \tan^{-1}\left(\frac{I_{q1}^{*}}{I_{q2}^{*}}\right) \tag{5}$$

2.2 Novel IPP estimation algorithm

If the actual speed or the actual position is equal at each shift position mode, respectively, (5) can be used for initial pole-position estimation. But in the estimation, the actual speed or the actual position is not always equal at each shift position mode. Therefore, to improve the estimation accuracy of the IPP, the actual speed or actual position should be taken into consideration at each shift position mode. Assuming that the load is constant at any position in a short stroke, the load term in the motion equation can be neglected for the IPP estimation.

$$\begin{aligned} J\frac{d}{dt}\omega &= T_e - T_L - D\omega \\ &= T_e - D\omega \end{aligned} \tag{6}$$

where D is the damping factor, T_L is the load, and J is the inertia.

Integrating both sides of (6), (7) can be produced and rearranged as (8).

$$J\omega = \int T_e dt - D\int \omega dt = \int T_e dt - DP \tag{7}$$

$$\omega = \frac{1}{J}\left(\int T_e dt - DP\right) \tag{8}$$

The shift position is set to ±45[deg] at each shift position mode. As (1) or (2) is put into (8) according to each shift position mode, (1) and (2) can be rewritten as (9) and (10), respectively.

$$\omega_1 = \frac{1}{J}\left(\int T_{e1}^{*}\cos(\theta_{\text{err}} + 45°)dt - DP_1\right) \tag{9}$$

$$\omega_2 = \frac{1}{J}\left(\int T_{e2}^{*}\cos(\theta_{\text{err}} - 45°)dt - DP_2\right) \tag{10}$$

where P is the feedback relative position, and 1 and 2 are the subscripts of the 1st and 2nd shift position modes.

If $\omega_1 = \omega_2 = 0$ and $P_1 \neq P_2 \neq 0$, the following relations of (11) and (12) can be obtained from (9) and (10).

$$P_1 = \frac{1}{D}\int T_{e1}^{*}dt \cdot \cos(\theta_{err} + 45°) \tag{11}$$

$$\begin{aligned} P_2 &= \frac{1}{D}\int T_{e2}^{*}dt \cdot \cos(\theta_{err} - 45°) \\ &= \frac{1}{D}\int T_{e2}^{*}dt \cdot \sin(\theta_{err} + 45°) \end{aligned} \tag{12}$$

From (11) and (12), the proportional relation can be obtained as follows;

$$P_1 : P_2 = \int T_{e1}^{*} dt \cdot \cos(\theta_{\text{err}} + 45^{\circ}) : \int T_{e2}^{*} dt \cdot \sin(\theta_{\text{err}} + 45^{\circ}) \tag{13}$$

Solving (13) for the deviated position θ_{err} produces

$$\theta_{\text{est}} = \theta_{\text{err}} = -45^{\circ} + \tan^{-1}\left(\frac{P_2}{P_1} \cdot \frac{\int i_{q1}^{*} dt}{\int i_{q2}^{*} dt} \right) \tag{14}$$

As the deviated IPP is equal to the estimated IPP, the estimated IPP is calculated by (14).

Putting torque equation (15) into (14), the estimated IPP of (16) can be calculated through the q axis currents instead of torque information.

$$T_e = \frac{3}{2} p \lambda_f i_q \tag{15}$$

$$\theta_{\text{est}} = -45^{\circ} + \tan^{-1}\left(\frac{P_2}{P_1} \cdot \frac{\int i_{q1}^{*} dt}{\int i_{q2}^{*} dt} \right) \tag{16}$$

where p is pole pair of motor, λ_f is flux, and i_q is the q-axis current in which 1 and 2 are the subscripts of the 1st and 2nd shift position modes.

2.3 IPP estimation algorithm with absolute integration of torque-component currents

In general, (16) can be used for calculating the initial pole position of surface PMSM. However depending on the integral period of the torque component current (q-axis current) within the same position interval, the accuracy of the IPP estimation is not constant because the accumulated amount regarding q-axis current is different. If the integral period of q-axis current is wrongly set, the integral amount may be smaller and neglected even if the position profile is the same. To clear these problems and to increase estimation accuracy, the accumulated amount of the q-axis should be maximized. This means that the larger the accumulated amount is, the higher the resolution for the IPP estimation accuracy is. According to the above conditions, the absolute integration of q-axis currents is introduced into (16) and the following equation of (17) is used to calculate the IPP estimation instead of (16).

$$\theta_{est} = -45^{\circ} + \tan^{-1}\left(\frac{P_2}{P_1} \cdot \frac{\int \left| i_{q1}^{*} \right| dt}{\int \left| i_{q2}^{*} \right| dt} \right) \tag{17}$$

2.4 Profiles of position, speed, and torque

Fig. 2 shows the predefined reference position profile, which is used to estimate the IPP. The calculation of the IPP is performed after obtaining the available two feedback q-axis currents and positions, which should be obtained at the 1st and 2nd shift position modes (reference position intervals). If there is an initial pole-position error between the actual dq axes and the control dq axes, the produced thrust force will be decreased compared to the reference thrust force, as shown in Fig. 3. These relations are well expressed by (18)~(19) and shown in Fig. 4. The speed and the position will be reduced according to the cosine function with a position error. Furthermore, if the IPP error is in the ranges of 90[deg]~270[deg], the generated thrust force is negative against the reference thrust force and the motor cannot be controlled.

$$T_{e\,\text{loss}} = T_e^{*} \times (1 - \cos\theta_{\text{err}}) \tag{18}$$

$$T_e = T_e^{*} \times \theta_{\text{err}} \tag{19}$$

where $T_{e\,\text{loss}}$ is a loss torque (or thrust force), T_e is a generated torque (or thrust force), T_e^{*} is a reference torque (or thrust force), and θ_{err} is a deviated IPP.

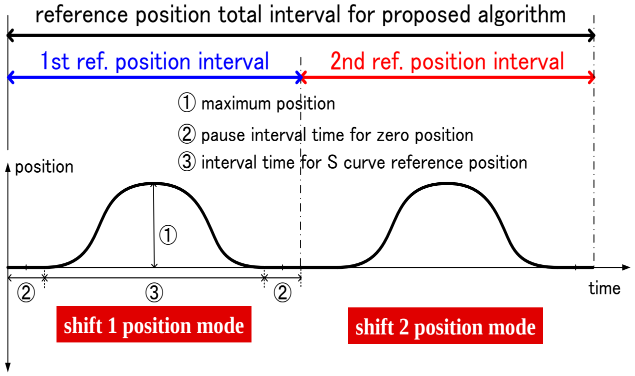


Fig. 2. Pre-defined reference position profile with S curve.

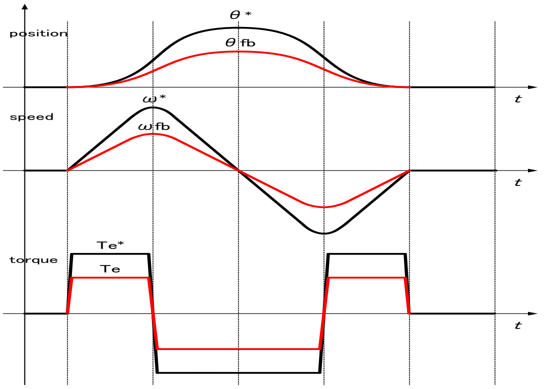


Fig. 3. Mechanical behaviors at non-zero deviated IPP; (upper) position, (middle) speed, and (bottom) torque.

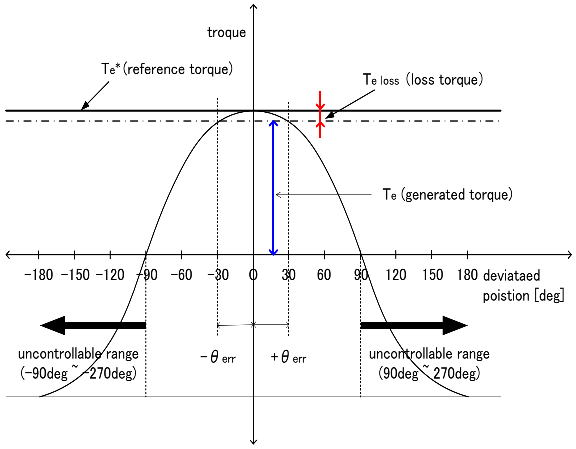


Fig. 4. Relationships between generated torque and loss torque according to the deviated IPP.

3. Simulation Based Analysis

3.1 System configuration

The overall system configuration of simulation is shown in Fig. 5 to confirm the feasibility of the improved estimation algorithm of surface PMSM. The block of proposed IPP estimation algorithm shown in this figure has the following functions; 1) to switch the position mode into the 1st shift position mode or the 2nd shift position mode, 2) to generate the reference position profile with S curve, 3) to adjust the compensated position to increase estimation accuracy, 4) finally to calculate the IPP with feedbacked incremental position and absolute currents.

3.2 Simulation analysis

To verify the feasibility of the proposed algorithm, the simulation based analysis is performed by PSIM simulator. Fig. 6 shows the estimation result when the IPP error is set to any given deviated IPP(±15[[deg], ±30[deg], ±45[deg]) under no load. As shown in Fig. 6, the moving distance of a motor is suppressed within the reference position peak and the estimation is performed in a very short time. Additionally, higher estimation accuracy is well obtained because of the absolute integrals of q-axis currents accumulated during each position interval. The non-absolute integral for (16) and the absolute integral for (17) are individually used to estimate the IPP. Their results are shown in Table 1. This table shows that the estimation variance at any given deviated IPP is smaller as

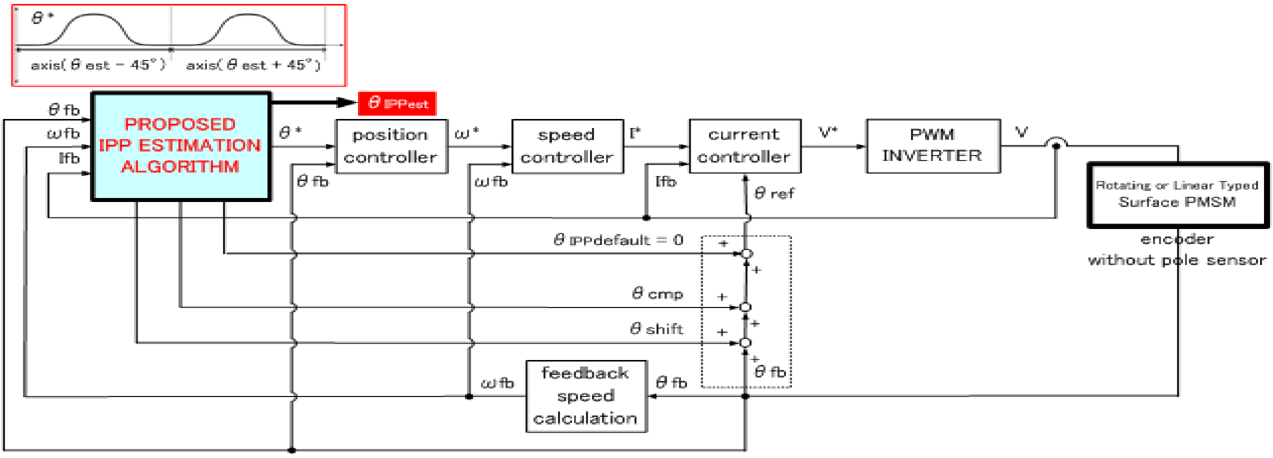
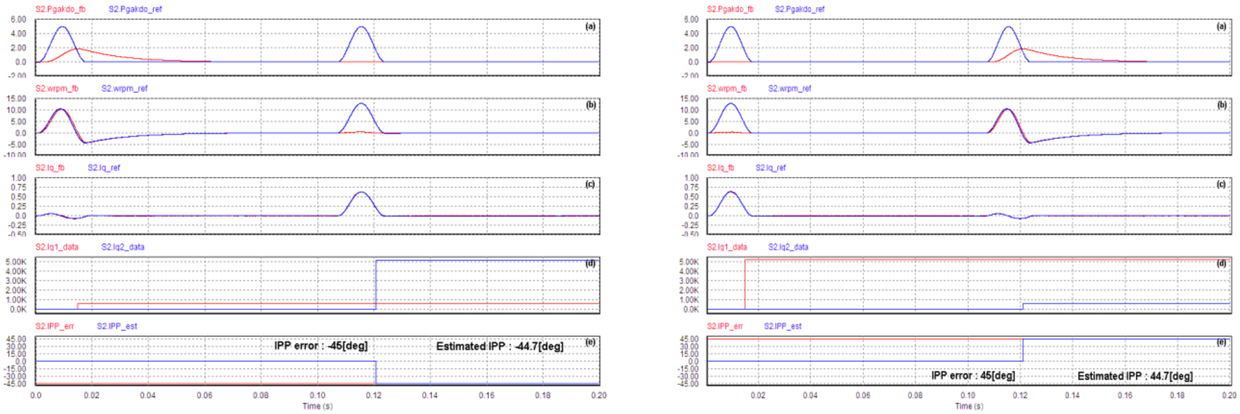
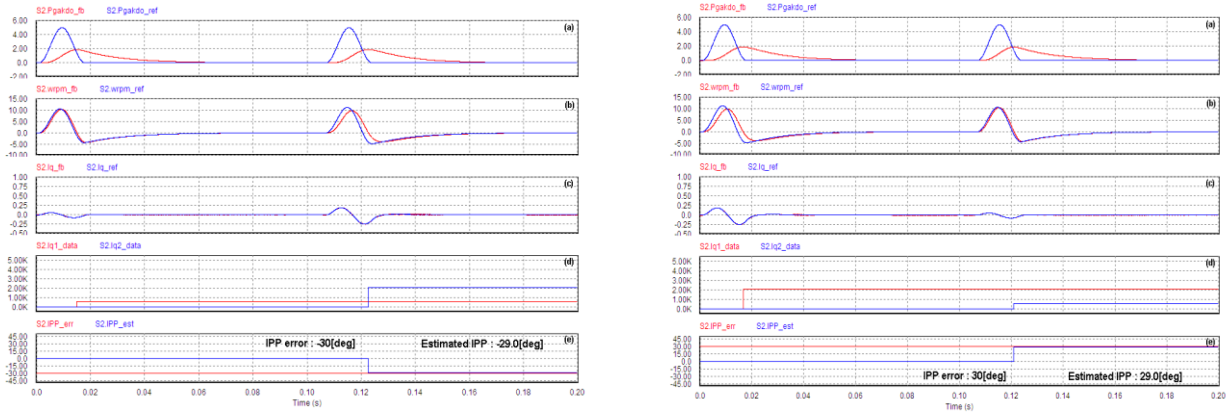


Fig. 5. Overall system configuration for calculating IPP estimation of surface PMSM.



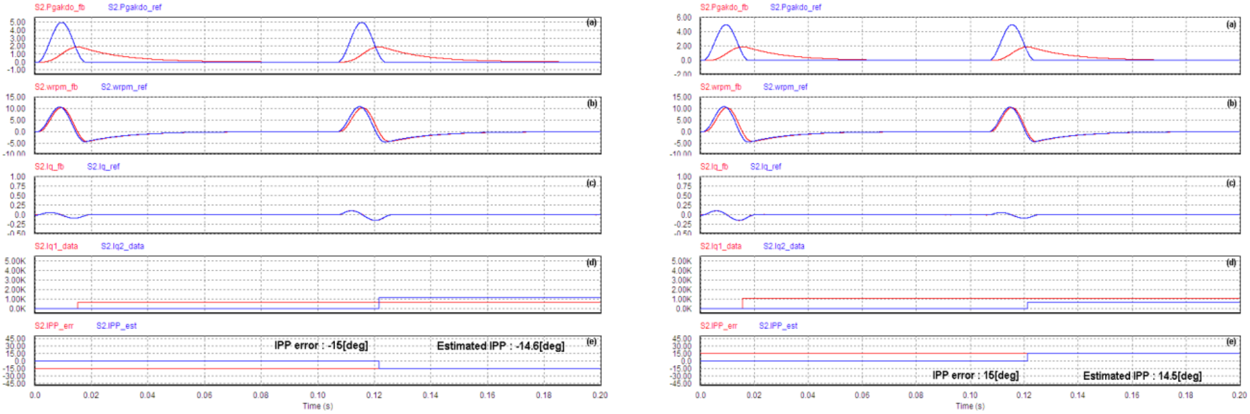
(f1) deviated IPP : -45[deg]

(f2) deviated IPP : +45[deg]



(f3) deviated IPP : -30[deg]

(f4) deviated IPP : +30[deg]



(f5) deviated IPP : -15[deg]

(f6) deviated IPP : +15[deg]

Fig. 6. Simulation results at any given deviated IPP (±45[[deg], ±30[deg], ±15[deg]); (upper) reference position[deg] and feedback position[deg], (secondly upper) reference speed[rpm] and feedback speed[rpm], (middle) q-axis reference current[A] and q-axis feedback current[A], (secondly bottom) absolute integral based q-axis current[A], (bottom) deviated IPP[deg] and estimated IPP[deg].

in the case of (17). Therefore, this algorithm with the absolute integrals of torque-component currents can be more effective and economic in applications such as in precise servo drives without any pole sensor. Overall, it has been found that the proposed algorithm can provide excellent IPP estimation with reduced estimation time while also suppressing the moving distance.

Table 1 Evaluation of proposed IPP estimation according to deviated IPP.

evaluation item	deviated IPP						
	-45	-30	-15	0	15	30	45
[1]estimated IPP [deg]	-44.7	-29.0	-14.6	0.1	14.5	29.0	44.7
	578	597	666	816	1141	2102	5237
	5178	2097	1146	816	667	597	575
moving distance [deg]	1.87	1.89	1.89	1.88	1.87	1.88	1.87
[2]estimated IPP [deg]	-45.0	**-32.7**	-15.7	**-1.7**	**17.1**	31.1	45.0
	8	8	10	12	20	36	5237
	5178	30	18	13	10	9	8

Note: 1) used by absolute integral, 2) is used by no-absolute integral.

4. Conclusions

In this paper, the novel initial pole-position estimation of surface PMSM is proposed. The proposed algorithm is mathematically analyzed and surveyed through simulation based analysis, and more accurately improved for SPMSM sensorless servo drive by the absolute integration of torque-component currents.

This IPP estimation is based on the principle that the initial pole-position is simply calculated by the reverse trigonometric function using the two feedback absolute currents in the full closed-loop position control. The proposed algorithm was simple in implementation and was highly accurate in estimation even close to standstill. This estimation can be widely applied to both rotating typed and linear typed surface PMSM without any limitations of their motor structures.

Acknowledgment

The authors would like to thank for the project supported by R2005-B-109 of KESRI (Korea Electrical Engineering and Science Research Institute), which is funded by MOCIE (Ministry of Commerce Industry and Energy) in Korea.

References

[1] T.W. Kim, J. P. Wheeler, C.I Jeong, J.H. Choi and A. Kawamura, “Accurate Initial Pole-Position Estimation of Surface PM-LSM in the Position Control,” *in Proc. on EPE 2007*, pp. 1-8, 2007.

[2] T.W. Kim, J. Watanabe, S. Sonoda, and J. Hirai, “New initial pole position estimation of surface PM-LSM using reference currents,” *in IEEE Trans. on IAS*, Vol. 41, No. 3, pp. 817-824, 2005.

[3] T.W. Kim, J. Watanabe, S. Sonoda, J. Hirai, “Initial Pole Position Estimation of Surface PM-LSM,” *Journal of Power Electronics*, Vol.1, No.1, pp.1-8, 2001.

[4] J.W. Choi, W.E. Yun, and H.-G. Kim, “Initial pole-position estimation of linear motor,” *in IEE Proc. on Electric Power Applications*, Vol. 152, No. 4, pp. 997-1002, 2005.

[5] K. Ide, H.S. Song, M. Takaki, S. Morimoto, and S.K. Sul, “Fast initial pole-position estimation for non-salient PM-LSM based on agreement of two reference frames,” *in IEEE Proc. on PESC06*, pp. 1-7, 2006.

[6] K.Y. Cho, “Sensorless Control for a PM Synchronous Motor in a Single Piston Rotary Compressor,” *Journal of Power Electronics*, Vol.6, No.1, pp.29-37, 2006.

[7] J.S Ko, S.H. Ko, Y.-C. Kim, “Robust Adaptive Precision Position Control of PMSM,” *Journal of Power Electronics*, Vol.6, No.4, pp.347-355, 2006.

[8] S.C. Yoon, J.M. Kim, “Sensorless Control of a PMSM at Low Speeds using High Frequency Voltage Injection,” *Journal of Power Electronics*, Vol.5, No.1, pp.110-19, 2005.

Tae-Woong Kim received his B.S. and M.S. degrees in Electrical Engineering from Chungbuk National University, Chungbuk, Korea, in 1990 and 1993, respectively. He received his Ph.D. from Yokohama National University, Japan, in 1996. He joined the

Yaskawa Electric Corporation, Japan, from April 1996 to September 2001. He was a research associate professor for one year at Aalborg University, Denmark. Later, he joined the faculty of Gyeongsang National University, Jinju, Korea, in October 2002, where he is presently an Associate Professor in the Department of Control and Instrumentation Engineering. His interests include power electronics, motion control, electric vehicles, and applied embedded control systems. Dr. Kim is an active member of KIPE, IEEE, IEEJ, RICIC, and GSERI. He received the IEEE IE Transaction Best Paper Award in 2000 and the outstanding academic co-work project award from LG Electronics DAC Lab in 2008.

Patrick Wheeler received his Ph.D. degree in Electrical Engineering from the University of Bristol, Bristol, U.K., in 1993. Since 1993, he has been with The University of Nottingham, Nottingham, U.K., where he has been a Research Assistant with the School of Electrical and Electronic Engineering. In 1996, he was a Lecturer (subsequently a Senior Lecturer in 2002 and a Professor in Power Electronic Systems since 2007) with the Power Electronics, Machines and Control Group, School of Electrical and Electronic Engineering, The University of Nottingham. His research interests include variable-speed ac motor drives, particularly different circuit topologies, power converters for power systems, and semiconductor switch use. Dr. Wheeler is a member of the Institution of Electrical Engineers, U.K.

Jaeho Choi received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from the Seoul National University, Seoul, Korea in 1979, 1981, and 1989, respectively. From 1981 to 1983, he was with the Jungkyoung Technical College, Daejeon, Korea, as a Full-time Lecturer. Since 1983, he has been with the School of Electrical and Computer Engineering, Chungbuk National University in Cheongju, Korea, where he is currently a professor. In 1993, 1998 and 2003, he was a visiting professor at the University of Toronto in Canada each for one year and he was a Danfoss Visiting Professor at the Aalborg University in Denmark in 2000. His research areas include power electronics, power quality problems and solutions, energy storage systems, and renewable energy and microgrid systems. He is an active member of KIEE, KIPE, and IEEE, and currently the Editor-in-Chief of Journal of Power Electronics.