대한수학교육학회지 <학교수학> 제 11 권 제 1 호
Journal of Korea Society of Educational Studies in Mathematics
School Mathematics Vol.11, No.1, 1-15. Mar. 2009

A Framework for Assessing Probability Knowledge and Skills for Middle School Students: A Case of U.S.

Park, Ji Yoon* · Lee, Kyung Hwa**

Some researchers (Jones et al., 1997; Tarr & Jones, 1997; Tarr & Lannin, 2005) have worked on students' probabilistic thinking framework. These studies contributed to an understanding of students' thinking in probability by depicting levels. However, understanding middle school students' probabilistic thinking is limited to the concepts in conditional probability and independence. In this study, the framework to understand middle school students' thinking in probability is integrated on the works of Jones et al. (1997), Polaki (2005) and Tarr and Jones (1997). As in their works, depicting levels of probabilistic thinking is focused on the concepts and skills for students in middle school. The concepts and skills considered as being necessary for middle school students were integrated from NCTM documents and NAEP frameworks.

I. Introduction

The need for coping with quantitative data led the school curriculum towards emphasizing the statistics and probability (National Council of Teachers of Mathematics [NCTM], 1989, 2000). The Curriculum and Evaluation Standards for School Mathematics [Standards] put statistics and probability as a main content strand which students should learn at every grade level K-12 (NCTM, 1989). Probability as a main content in the mathematics curriculum at all grade levels results from the need to prepare for out rapidly changing society where probabilistic thinking is necessary.

As uncertainty is increasing within the change of society, probabilistic thinking plays an important role, in that probability is a tool for quantifying uncertainty in decision-making (Amir & Williams, 1999). Uncertainty and the need to quantify it pervades our daily life in business, weather forecasting, medicine, or markets. In recent years mathematics curricula around the world has been recognizing the importance of chance and probability. In addition to the documents of NCTM in 1989 and 2000, Chance and Data is one of five mathematics content areas in the National Statement on Mathematics for Australian Schools (Austrailian Education Council, 1991). The National Curriculum: Mathematics (DfEE, 1999) of England also includes Probability in four attainment areas (as cited in Pratt, 2005).

* University of Minnesota(jiyoon212@gmail.com)
** Seoul National University(khmath@snu.ac.kr)

Studies on mathematics education also reflect the importance of learning probability. Gal (2004: 39) suggested two reasons for learning probability: (a) probability is part of mathematics and statistics, fields of knowledge that are important to learn in their own right, as part of modern education; and (b) the learning of probability is essential to help prepare students for life, since random events and chance phenomena permeate our lives and environments. The emphasis of probabilistic thinking in school mathematics is based on the second reason as an external consideration.

However, recent studies on students' probabilistic thinking report that students do not fully understand the concepts of chance and probability (Polaki, 2005; Tarr, 2002; Watson & Kelly, 2005). Most of instruction in probability deals with the formulae to get the measure of probability without building deep an understanding of the probabilistic concepts. Indeed, the instruction in many mathematics classrooms does not provide students with the appropriate experience of probabilistic thinking. To understand the essence of random events and chance phenomena in our lives, improving students' probabilistic reasoning with interpretation of the various concepts and skills should be the focus when teaching probability

When it comes to the probability, however, there is little research on the perspective of teaching and learning. Given that our rapidly changing society requires people to understand and deal with uncertain situations, research about probabilistic thinking is needed to help educators have a clear view on teaching fundamental ideas in probability. This study aims to develop a way to understand and characterize middle school students' probabilistic thinking. This work will be primarily based on the previous research on assessment of students' probabilistic thinking by Jones et al. (1997), Polaki (2005), and Tarr & Jones (1997). We will reconsider this framework in terms of the concepts considered as being necessary for middle school students, and criteria to classify the levels. Based on this reconsideration on the existing framework, we will present a developed framework which can provide more clear explanation for middle school students' probabilistic thinking.

II. Studies on Probabilistic Concepts and Understanding Students' Probabilistic Thinking

1. Uncertainty and random events

There is inconsistency with the age at which children are capable of understanding uncertainty in literature. Piaget and Inhelder (1975) asserted that the ability to differentiate between certainty and uncertainty did not appear until around 7 years of age. On the other hand, there are many evidences indicating that young children can recognize the concepts of uncertainty in the sense of differentiating random situations from determinacy (Byrnes & Beilin, 1991; Horvath & Lehrer, 1998).

For this conflicting ages suggested in the research literature, Metz (1998: 158) argued that there are "different criteria corresponding with different conceptualizations" to understand students' understanding on uncertainty. For example, at the age around 4 or 5, children have "a relatively rudimentary form" of "the distinction between phenomena with certain and predictable outcomes versus phenomena with uncertain and unpredictable outcomes". Also, from a study by Kuzmak and Gelman (1986), Metz (1998: 158) suggested that children at age 5 can give "appropriate explanations" of why a certain outcome is unpredictable.

Although there are many levels of complexity involving the notion of uncertainty, there seems to be a consistency that children at age 5 grasp the idea of uncertainty "in the sense of being able to consider what situations produce deterministic versus nondeterministic results" (Metz, 1998: 160).

An explicit definition of randomness does not exist in literature although uncertainty has been considered for a long history. A spectrum of the perspective for randomness, which was analyzed by Liu and Thompson (2002: 1), shows the change in views for uncertainty over the ages. In the age of the European Enlightment, probability was regarded as reflecting "human ignorance of a true determinist course of events". In a perspective of deterministic perspective, "absolute randomness does not exist", and therefore, "all probabilities will be 0 or 1" (Liu & Thompson, 2002: 1).

A review of literature conducted Liu and Thompson shows that the other end of the spectrum in randomness perspective came from "the renunciation of determinism" followed by the development of science in the twentieth century.

In particular, von Mises (1928/1952) considered a sequence "to be random if we are convinced of the impossibility of finding a method that lets us win in a game of chance where winning depends on forecasting that sequence" (as cited in Batanero et al., 2005: 26).

The definition of randomness has been attempted through meanings of random phenomena and random sampling (Liu & Thompson, 2002). Yates, Moore and McCabe (2000) defined 'random phenomena' as ones "uncertain but there is a regular distribution of outcomes in a large number of repetitions". Bluman (2001) defined random sampling that "all possible samples of a certain size "must have an equal chance of being selected from the population"" (as cited in Liu & Thompson, 2002: 1). These definitions are in the same line, in that both are considering a kind of regularity by a large number of repeated trials. Indeed, it is noted that "perfect randomness" can be regarded only in infinite outcomes, and therefore, randomness is a theoretical concept (Batanero et al., 2005: 26).

2. Independence and conditional event

The idea of independence of an event is intuitive. For example, "a die or a coin does not have a memory of preceding throws". More explicitly, intuitive idea was considered as independent "if there was no reason to think that one of them could influence the other (Batanero et al., 2005: 28).

However, according to Batanero et al. (2005) with an axiomatic theory of Kolmogorov, intuitive idea of independence was emptied. The probabilistic expression of independence by the multiplication rule, $P(A \cap B) = P(A) \times P(B)$ is stochastically independent, but not intuitively independent.

In previous heuristic research of his paper, Konold et al. (1993) showed that difficulty of understanding independence of events is related with heuristics. A problem, which of the following sequences was most likely and which was least likely to occur when tossing a coin five times: (a) HHHTT, (b) THHTH, (c) THTTT, (d) HTHTH and (e) all four sequences are equally likely, was presented to high school students and undergraduate students of remedial-level in mathematics course. Overall 72 % students correctly chose the answer (e) for the most likely problem. This result suggests that students understand that trials of successive tossing are independent resulting in all sequences of tossing a coin being equally likely.

The study mentioned above about understanding independence of successive events show that students have an intuitive idea of independence, but it doesn't necessarily mean that they can justify correctly the idea come from intuition.

Some studies showed that middle school students experience a transition when they have to understand the concept of compound event as a combining event of simple events. In a study by Watson, Collis, and Moritz (1997) involving a probability comparison between two events of rolling a die such as, "which is more likely, a 1 or a 6, or are they equally likely?", just a few students gave adequate justification for their right answer indicating that students do not have correct understanding on the concept of compound event.

Polaki (2005) denoted that the understanding compound events requires to be able to (a) generate complete sets of outcomes for each experiments, and (b) use sample space symmetry, composition or experimentation as a basis for making probability predictions. In his study with elementary and middle school students, he found that students use the strategies for listing sets of outcomes for simple and compound events: (a) arbitrary lists and incomplete lists based on subjective reasoning for simple events, (b) trial-and-error strategies, (c) partially-generative strategies for compound events, and (d) generative strategies for compound events, as the degree of sophistication.

According to Shaughnessy and Bergman (1993), students encounter extreme difficulty when learning the concept of independence and the laws of conditional probability. Theoretically, the relation of independence and the conditional probability is drawn by Bayesian Rule, 'if event A and event B are independent, $P(A \cap B) = P(A)P(B)$. And the conditional probability of event A given event B is $P(A \mid B) = P(A \cap B)/P(B)$. So, if two events A and B are independent, $P(A \mid B) = P(A)$.

However, as Tomlinson and Quinn (1997: 4-6) pointed out, this formula may be useful, but "it clearly does not provide the student with an intuition of the reasoning process necessary to solve such embedded problems". He also noted that the counter-intuitive laws of probability composed of abstract terms and complex equations should be taught with focus on "challenging the personal biases and cognitive heuristics identified by psychologists".

In a study of Watson and Moritz (2002: 82), students tended to reason more correctly in an item with frequency type than probability type. For a question, "which one has larger frequent between (a) people who are left-handed out of 100 men, and (b) people who are men out of 100 left-handed", success rate was relatively higher than a question with probability type. For this difference between frequency item and probability item involving conditional event, Watson and Moritz (2002) suggested that students experience transfer their probabilistic understanding "from countable situations to social settings in estimation, reasoning, and appropriate intuition". In this process of transfer, questions involving social contexts ask students to use their contextual knowledge of the environment as well as the numbers provided. To help students be familiar with the conditional event within social context, Watson and Moritz (2002) suggested the educational programs to include exposure to social context.

3. Students' Probabilistic Thinking

The attempts to characterize and assess students'probabilistic thinking have been conducted by some researchers (Tarr & Jones, 1997; Jones et al., 1997; Polaki, 2005; Tarr & Lannin, 2005). The researchers presented the frameworks which contribute to capturing the multi-facets of students' thinking in probability. The frameworks have been studied for different probability constructs, and validated for students at different grade levels. The idea within the frameworks provided us with a coherent view of students' understanding on various probability concepts.

A framework of Jones et al. (1997) describes young children's probabilistic thinking across four levels for each of the four constructs: sample space, probability of an event, probability comparisons, and conditional probability. The four key constructs were formulated adding the fourth construct to the former three key constructs which have been investigated by several researchers. The levels of thinking within specific knowledge domains are in concert with cognitive development research (as cited in Jones et al., 1997)

This work includes Piaget and Inhelder (1975), and "neo-Piagetian theories that postulate the existence of sub stages or levels that recycle during developmental stages" (as cited in Jones et al., 1997: 102). According to Biggs and Collis (1991), the levels reflect shifts in the structural complexity of student's thinking, and each level subsumes the preceding one. Further, they maintain that "this learning cycle is consistent across stages and is applicable to school-based tasks" (as cited in Jones et al., 1997: 104).

Ⅲ. Analysis of the documents of mathematics curriculum and a nationalized assessment

1. 'Standards(NCTM, 1989)' and 'Principles and Standards(NCTM, 2000)'

Comparing to the previous curriculum in

mathematics, the NCTM (1989) increased attention to "creating experimental and theoretical models of situations involving probabilities", instead of "memorizing formulas" (NCTM, 1989: 70). Traditional teaching emphasized measuring of probability using formulas such as the 'addition rule' or 'multiplication rule.' The NCTM (1989) shows the movement from teaching formulas in measuring probability to teaching probability within situated problem. This aspect is consistent with the overall trend of NCTM (1989) which aims that mathematics curricula help students "recognize the need to apply a particular concept or procedure and have a conceptual basis for reconstructing their knowledge at a later time" (NCTM, 1989: 10).

The Standard's expectations for probability can be summarized that they emphasize: modeling probabilistic situation through experiments or simulation understanding experimental and theoretical probability; and applying the probabilistic knowledge into the real-world situations.

In the Principles and Standards, the strand of Data Analysis and Probability has the expectations including only a few probabilistic concepts that middle school students need to know: Understand and use appropriate terminology to describe complementary and mutually exclusive events; Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; Compute probabilities for simple compound events, using such methods are organized lists, tree diagrams, and area models.

As in the Standards, middle school students' probabilistic thinking is developed from simple notions of chance they learned through K-4 mathematics. Middle school students need to learn formal knowledge in probability to develop their probabilistic thinking to higher level statistics or probability courses. Students' primary knowledge or primary intuition in probability needs to be formalized with well-organized conceptions and skills, and sometimes, it needs to be modified when it is not correct.

When students are acquiring abstract mathematical concepts, the transitional characteristic of the process sometimes makes middle grade students feel difficult to understand them. The difficulties students have at the stage of transition from probabilistic intuition to formal probabilistic thinking can be resolved by helping students experience with real activity or simulation involving probability concepts. In regards to this, as in the Standards, the Principles and Standards also considers 'modeling probabilistic situation through experiments or simulations' as an important role which provides students with understanding theoretical probability from experimental probability.

2. Analysis of the NAEP Mathematics Assessment

In the 1990 NAEP mathematics assessment for 8th grade students, two probability questions are asking the probabilities of simple events. In 1992 NAEP test, there are four probability questions among nine released questions in the content of Data Analysis, Statistics, and Probability. It is noticed that the questions cover various concepts in probability - likelihood of a simple event, representativeness of a sample, and sample space.

But some questions of them are still limited on the simple questions with typical format of probability item that can be answered even without appropriate understanding of the concepts. For example, in the following questions, 38 % and 73% students responded correctly, respectively.

There are 15 girls and 11 boys in a mathematics class. If a student is selected at random to run an errand, what is the probability that a boy will be selected?

A) 4/26 B) 11/26 C) 15/26 D) 11/15 E) 15/11

(NAEP Mathematics Assessment, 1990)

In a bag of marbles, 1/2 are red, 1/4 are blue, 1/6 are green, and 1/12 are yellow. If a marble is taken from the bag without looking, it is most likely to be

A) red B) blue C) green D) yellow

(NAEP Mathematics Assessment, 1992)

Considering that these questions were suggested to the 8th grade students, and that the formats of questions are multiple-choice, they would answer right if they have just a little knowledge in fraction regardless of understanding of probability. We need focus on that they were not able "to justify these [their answers] in an appropriate fashion"although students can achieve correct numerical answers (Watson & Callingham, 2005: 151). To assess students' understanding of probability on uncertain simple event, a question for evaluating their justifications should be suggested.

The NAEP assessment in 1996 has two probability related items - representative of a sample and reasoning about sample space. As emphasized in the objectives of probability in the framework for the NAEP assessments between 1996 and 2003, the questions are focusing on assessing students' understanding of sampling and sample space.

Two questions in 2003 NAEP assessment are about sampling procedure and estimating the sample size which are on the framework of 1996 NAEP assessment. Following question shows a question which is assessing students'understanding of bias in a sampling:

A survey is to be taken in a city to determine the most popular sport. Would sampling opinions at a baseball game be a good way to collect this data? Explain your answer. (NAEP Mathematics Assessment, 2003)

To answer correctly to this question with short-constructed-response format, students need to understand the biases that can happen in sampling procedure. 45% of students produced correct justifications, and 55% of students judged incorrectly (50%) or made omitted answer (5%). One incorrect answer, "No, that would only be referring to baseball, and no other information can be collected", shows that this student can reason that the sampling is not appropriate to determine the most popular sport in a city, but she/he does not understand the source of bias or sampling error.

The NAEP framework for 2005 and 2007 mathematics assessments has the objectives involving various probabilistic concepts for middle grade students such as sample space, randomness, independent/dependent events, or simple/compound events. The items for probability in 2005 and 2007 tests are trying to assess students'conceiving probabilistic thinking with various problem formats, not only the problems asking definitions or applying formulas. A question followed is to ask about the idea of conditional probability:

A package of candies contained only 10 red candies, 10 blue candies, and 10 green candies. Bill shook up the package, opened it, and started taking out one candy at a time and eating it. The first 2 candies he took out and are were blue. Bill thinks the probability of getting a blue candy on this third try is 10/30 or 1/3. Is Bill correct or incorrect? Explain your answer. (NAEP Mathematics Assessment, 2005)

This problem approaches to students' intuitive understanding on conditional probability. In the framework of NAEP assessment for 2005 and 2007, although the conditional probability is restricted for 12th grade students, 8th grade students are expected to be able to understand the situation that includes independence of an event. As Watson & Callingham (2005: 155) noted, "conditional events are considered at the same time as independent events". As considering the importance of the concept of independence, this question assesses intuitive idea about conditional event which will be taught high middle grade levels.

VII. Developing a Framework

The chronological analyses of the NCTM documents and NAEP mathematics assessments show a trend of mathematics curriculum in regard with the change of society. In particular, the analyses on probability content of each document focusing on middle grade level suggest probabilistic concepts and skills emphasized in middle grade levels, as well as some aspects of assessment in probability during each period.

Literature review of this paper showed the attempts of some researchers - a study of young children's probabilistic thinking by Jones et al. (1997); a study of middle school students'thinking in conditional probability and independence by Tarr and Jones (1997); and a study of elementary and middle school students' probability thinking framework by Tarr and Lannin (2005).

These studies contributed to an understanding of students'thinking in probability by depicting four levels which represent a continuum from level 1 (subjective) to level 4 (numerical). However, understanding middle school students' probabilistic thinking is limited to the concepts in conditional probability and independence.

In this study, the framework to understand middle school students' thinking in probability is integrated on the works of Jones et al. (1997), Polaki (2005) and Tarr and Jones (1997). As in their works, depicting levels of probabilistic thinking is focused on the concepts and skills for students in middle school. The concepts and skills considered as being necessary for middle grade students were integrated from NCTM documents and NAEP frameworks.

As seen in the result of the NCTM and frameworks for NAEP assessments, some probabilistic concepts are considered as necessary for middle grade students - samples and sampling, determining the probability of a simple event, compound event, independence, and conditional probability. For these five constructs in probability, four levels are identified as students' thinking and skills - subjective (Level 1), transitional (Level 2), informal quantitative (Level 3), and numerical (Level 4).

Level 1 (subjective)

Students at level 1 make their judgments for an uncertain situation based on "subjective beliefs". In random sampling, students at this level tend to provide their reasoning relying on their subjective judgments focusing on "what is more likely to happen rather than what is possible" (Jones et al., 1997: 114). When determining the probability of a simple event and compound event, they do not consider the quantitative information given and rely on their subjective judgments. They typically provide subjective probability using idiosyncratic and deterministic reasoning (Jones et al., 1997; Polaki et al., 2000). In situations involving conditional probability, students construct conditional probability with "their own reality" because of their "lack of quantitative referents" (Tarr & Lannin, 2005: 221). Therefore, students at level 1 do not think about the situations involving various probabilistic concepts with a meaningful reasoning.

Level 2 (Transitional)

Students who exhibit level 2 experience transition between subjective and informal quantitative judgments (Jones et al., 1997). Whereas students at level 1 do not approach to the "mental counting line", Level 2 students construct this counting line when generating sets of outcomes for compound experiments. According to Polaki (2005: 199), the mental counting line enables students "to coordinate the notions of number and ordering needed for comparing probabilities of simple events". But it does not enable them to generate sets of outcomes for compound events.

Level 2 students demonstrate their predictions on probability for simple events with "informal but valid quantitative judgments to the most-likely or least likely event, albeit inconsistently" (Polaki, 2005: 199). They can recognize the changes occurring in an event involving conditional probability - e.g., the probability in a "without-replacement" situation. However, their reasoning is still incomplete, sometimes confining to events that have previously occurred. Indeed, Level 2 students frequently use a "representativeness" strategy in consecutive events with inconsistent reasoning on independence.

Level 3 (Informal Quantitative)

Students in Level 3 have no difficulty in listing complete sets of outcomes for simple random experiments. Unlike students in Level 1 and Level 2, Level 3 students are also able to provide complete sets of outcomes for compound events using a 'partially generative strategy' (Polaki et al., 2000; Jones et al., 1997). Level 3 students generally use 'quantitative judgment' when determining probabilities. They recognize conditional probability both of 'with- and without-replacement'situations. To compare probabilities they use numerical information, "although such students do not usually assign precise numerical probabilities" (Tarr & Lannin, 2005: 223). They use appropriate strategies such as relative frequencies, ratios, or some form of odds to get the conditional probabilities. Students recognize independence of an event in 'with-replacement'situations but "they sometimes revert to a representativeness strategy after observing a run on one outcome in a sequence of independent trials (as cited in Tarr & Lannin, Shaughnessy, 1992: 224).

Table 1: framework for assessing students' probabilistic thinking

	Basic notion of probability	Additivity of probability	Sample space for compound event	Conditional event and independence
Level 1	Shows incomplete understanding on a chance Incompletely understands the axioms of probability, especially, $0 \leq P(E) \leq 1$ and $P(\Omega) = 1$ for a certain event E and any entire sample space Ω Shows difficulty in language expression about chance and probability of an event	Provides incomplete understanding that, for two disjoint sets A and B, the probability that A or B will happen is the sum of the probabilities that A will happen and B will happen experiments Exhibits inconsistent understanding of the probability of events which are involving more than two situations	Provides incomplete sets of outcomes for compound experiments Uses rial-and-error strategy to figure the sets of outcome, but incomplete Is unsuccessful in expecting sets of outcomes for compound experiments	Ignores the change of sample space in with-replacement and without-replacement random sampling Exhibits unwarranted confidence in predicting successive outcomes Is unable to differentiate independent and dependent events
Level 2	Demonstrates some awareness of chance and probability Understands informally the axioms of probability, $0 \leq P(E) \leq 1$ and $P(\Omega) = 1$ Uses appropriate language to explain a probability of an event Incompletely applies their understanding of a probability to real contexts	Is able to recognize the principle that $P(A \cup B) = P(A) + P(B)$ for two disjoint events A and B Is able to understand informally that the probability that A or B will happen becomes smaller when the events A and B are not disjoint Incompletely justifies the probability that A or B will happen	Is able to consider the sets of outcomes in compound events but sometimes it is incomplete Uses unsystematic strategy in reasoning of compound events Justifies the set of outcomes for compound events with trial-and-error strategy	Informally recognizes the change of probabilities in some events with- and without-replacement random sampling Incompletely justifies the independence of some events in terms of conditional events Sometimes depends their reasoning about probability of an independent event on inappropriate intuition
Level 3	Appropriately demonstrates awareness of chance and probability Completely understands and justifies $0 \leq P(E) \leq 1$ and $P(\Omega) = 1$ Is able to explain a probability informally as well as formally Inconsistently applies understanding of a probability to various contexts	Is able to justify that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Is able to deduce the inclusion and exclusion principle that $P(\Omega - A) = P(\Omega) - P(A)$ from the addition rule of probability Inconsistently applies the addition rule to various contexts	Is able to provide complete sets of outcomes for compound events using partially generative strategy Informally justifies the set of outcomes for compound events using multiplication rule	Recognizes the change of probabilities in all events in with- and without-replacement random sampling Is able to deduce informally meaning of independence of some events from conditional probability Exhibits appropriate understanding of the relationship between conditional events and independence of events Incompletely applies part-part or part-whole reasoning to the situations involving conditional events
Level 4	Completely recognizes the meaning of chance and probability Is able to exhibit a probability of a certain event with various expressions Consistently applies understanding of probability to various contexts Is able to explain an uncertain situation by using notion of probability	Recognizes and appropriately justifies the addition rule of probability and inclusion and exclusion principle Consistently applies the principles of inclusion-exclusion principles to various contexts	Is able to produce a complete set of outcomes for compound events using generative strategy Is able to justify the set of outcomes of compound event appropriately using the multiplication rule Is able to apply understanding of compound event to various situations	Completely deduce meaning of independence of some events from conditional probability Appropriately justifies their reasoning on independent and dependent events in terms of conditional probability Is able to apply appropriate proportional reasoning to various situations of conditional events

Level 4 (Numerical)

Whereas students in Level 3 use a partially generative approach in producing set of compound outcomes, students exhibiting Level 4 use a generative approach to list complete sets of outcomes (Polaki et al., 2000; Jones et al., 1997). In compound experiments, according to Polaki et al. (2000), Level 4 students consistently produce a complete set of outcomes using the generative strategy. They are also able to use and systematically coordinate arithmetical thinking using multiple counting lines. Polaki et al. (2005) argued that this multiple counting lines distinguish Level 4 from Level 3 students. They use sample space as a basis for finding and comparing numerical probabilities (Jones, Langrall, Thornton, & Mogill, 1997). Using numerical probability Level 4 students can recognize conditional probability in relation to all events of an experiment (Jones et al., 1997). In the situations 'with-replacement', Level 4 students are less likely to use the representativeness strategy (Tarr & Lannin, 2005).

VIII. Conclusion

The research on probabilistic thinking which has been studied so far more focuses on the psychological aspects of probabilistic thinking. The researchers try to identify what intuitions people have and what misconceptions people tend to show in some specific probabilistic situations. Rather than psychological and diagnostic approach to probabilistic thinking, we attempted to have an understanding of students' probabilistic thinking with a pedagogic perspective. With this approach, we more focused on how students think and what differences they show when they give a justification in some question or situation involving a specific probabilistic concept. The literature review of this paper showed that, even though students seem to understand a probabilistic concept in a test, their justification on the concept was not correct indicating they have misunderstanding on the concept.

In order to characterize students' probabilistic thinking, we first indicated four probabilistic concepts which are considered that middle school students should understand in middle school grades. For those concepts, we supposed that primary intuitions emerging in children's thinking can be a base for students to understand fundamental ideas of probability. In terms of curriculum in mathematics, traditional probability instruction, which mainly includes definitions and formulas in probability, cannot link with students' intuitive ideas. Therefore, our framework was developed with a consideration that understanding of probability develops from the primary intuition into meaningful secondary intuition as their thinking process develops.

The framework which has been studied by some researchers characterizes students' thinking in probability regardless of grade level. In this study, however, we considered that middle school students experience a transition in their thinking when they learn formal concepts in mathematics. Given that there are different probabilistic concepts supposed to be taught in mathematics curriculum, and that there is a big gap in students' thinking process among different grade levels, we more focused on the middle school level in probability which needs more systematic understanding for students' thinking.

In addition to more focused framework on middle school grade levels, we have different criteria to identify students' level of thinking from the existing framework. The previous framework classifies the levels according to how well students apply the strategy to get a right answer in a quantitative perspective. For example, they identify students who see the meaning of probability in a deterministic view as level 1, and students who can assign a numerical probability to an event as level 4 (Polaki, 2005). Rather than understanding students' probabilistic thinking with diagnostic perspective of whether or not they got correct, we approached to a developed framework which could explain 'how much students those concepts understand well', and 'how they justify their answers with what strategy'.

This framework does not involve all probabilistic concepts and skills, nor does it include valid evidence for the effectiveness of its application. Nevertheless, it is still worth when we are to see how students reason in a specific probabilistic concept. Moreover, the framework gives an insight for probability instruction which can help students develop their primary intuition as they learn formal probabilistic concepts.

References

Amir, G. S., & Williams, J.S. (1999). Cultural influences on children's probabilistic thinking. *Journal of Mathematical Behavior, 18,* 85 - 107.

Austrailian Education Council (AEC). (1991). *A National Statement on Mathematics for Australian Schools.* Carlton, VIC: Curriculum Corporation.

Batanero, C., Henry, M., and Parzysz, B. (2005). The nature of chance and probability. In G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning* (pp. 15-37). New York: Springer.

Biggs, J. B., and Collis, K. F. (1991). Multimodal learning and the quality of intelligent behavior. In H.A.H. Rowe (Ed.), *Intelligence: Reconceptualization and Measurement.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Byrnes, J. P., and Beilin, H. (1991). The cognitive basis of uncertainty. *Human Development, 34*(4), 189-203.

Fischbein, E., and Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education, 28*(1), 96-105.

Gal, I. (2004). Statistical literacy: Meanings, components, responsibilities. In D. Ben-Zvi and J. Garfield (Eds.), *The Challenge of Developing Statistical Literacy, Reasoning, and Thinking* (pp. 47-78). *Dordrecht: Kluwer Academic Publishers.*

Garfield, J., and Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education, 19*(1), 44-63.

Horvath, J. K., and Lehrer, R. (1998). A model-based perspective on the development of children's understanding of chance and uncertainty. In S. P. Lajoie (Ed.), *Reflections in Statistics: Learning, Teaching, and Assessment in Grades K-12* (pp. 121-148). New Jersey: Lawrence Erlbaum Associates.

Jones, G. A, Langrall, C. W., Thornton, C. A., & Mogill, T. A. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics 32*, 101-125.

Jones, G. A., and Thornton, C. A. (2005). An overview of research into the teaching and learning of probability. In G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning.* New York: Springer.

Konold, C., Pollatsek, A., Well, A., Lohmeier, J., & Lipson, A. (1993). Inconsistencies in students' reasoning about probability. *Journal for research in mathematics education, 24*(5), 392-414.

Langrall, C. W., & Mooney, E. S. (2005). Characteristics of elementary school students' probabilistic thinking. In G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning.* New York: Springer.

Liu, Y., and Thompson, P. (2002). Randomness: Rethinking the foundation of probability. In D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proceedings of the Twenty-fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 3,* 1331-1334.

Metz, K. E. (1998). Emergent understanding and attribution of randomness. *Cognition and Instruction, 16*(3), 285 - 365.

National Assessment Governing Board. (1995). *Mathematics framework for the 1996 National Assessment of Educational Progress:* U.S. Department of Education.

National Assessment Governing Board. (2004). *Mathematics framework for the 2005 National Assessment of Educational Progress:* U.S. Department of Education.

National Assessment of Educational Progress. (1988). Mathematics objectives. In The *Nation's Report Card.* ETS.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

NCTM. (2000). Principles and *standards for school mathematics.* Reston, VA: Author.

Piaget, J., and Inhelder, B. (1975). *The origin of the idea of chance in children.* New York: Norton.

Polaki, M. V. (2005). Dealing with compound event. In G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning.* New York: Springer.

Pratt, D. (2000). Making sense of the total of two dice. *Journal for research in mathematics education, 31*(5), 602-625.

Pratt, D. (2005). How do teachers foster students' understanding of probability? In

G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning.* New York: Springer.

Tarr, J. E. (2002). The confounding effects of the phrase "50-50 Chance" in making conditional probability judgments. *Focus on Learning Problems in Mathematics, 24*(4), 35-53.

Tarr, J. E., and Jones, G. A. (1997). A framework for assessing middle school students' thinking in conditional probability and independence. Mathematics Education *Research Journal, 9,* 39-59.

Tarr, J. E., & Lannin, J. K. (2005). How can teachers build notions of conditional probability and independence? In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning.* New York: Springer.

Shaughnessy, J. M. (1992). Researches in probability and statistics: Reflections and Directions. In Grouws, D.A. (Ed.). *Handbook of Research on Mathematics Teaching and Learning (pp.465-494).* New York: Michigan Publishing Company.

Shaughnessy, J. M., & Bergman, B. (1993). Thinking about uncertainty: Probability and statistics. In P. S. Wilson (Ed.), *Research Ideas for the Classroom: High School Mathematics.* New York: MacMillan.

Tomlinson, S., & Quinn, R. (1997). Understanding conditional probability. *Teaching Statistics, 19*(1), 2-7.

Watson, J. M., & Callingham, R.A. (2005). Statistical literacy: From idiosyncratic to critical thinking. In G. Burrill & M. Camden (Eds.), *Curricular Development in Statistics Education. International Association for Statistical Education (IASE) Roundtable,* Lund, Sweden, 2004. Voorburg, The Netherlands: International Statistical Institute.

Watson, J. M., & Kelly, B. (2005). Cognition and instruction: Reasoning about bias in sampling. *Mathematics Education Research Journal, 17*(1), 24-57.

Watson, J. M., Collis, K. F., & Moritz, J. B. (1997). The development of chance measurement. *Mathematics Education Research Journal, 9,* 60-82.

Watson, J. M., & Moritz, J. B. (2002). School students reasoning about conjunction and conditional events. *International Journal of Mathematical Education in Science and Technology, 33*(1), 59-84.

Yates, Moore, & McCabe. (2000) *The Practice of Statistics.* W. H. Freeman.

중학교 학생들의 확률적 사고 수준 평가 기준 개발 : 미국의 사례

박 지 윤 (미네소타대학교)
이 경 화 (서울대학교)

일부 연구자들 (Jones et al., 1997; Tarr & Jones, 1997; Tarr & Lannin, 2005)은 학생들의 확률적 사고틀에 대해 연구해왔다. 이들 연구는 학생들의 확률적 사고 수준을 이해하는 도구를 제공하였다. 그러나 중학교 학생들의 확률적 사고 수준 관련 연구는 조건부 확률과 독립성 개념에만 머물러 있었다. 이 연구에서는 Jones et al. (1997), Polaki (2005), and Tarr and Jones (1997)의 연구를 분석하고, 미국의 교육과정과 국가 수준의 평가 자료를 분석하여 중학교 학생들의 확률적 사고 수준을 평가할 수 있는 틀을 개발한다.

* key words : Probabilistic thinking(확률적 사고), Assessment(평가), Theoretical framework (이론적 틀)

논문접수 : 2009. 1. 30
논문수정 : 2009. 3. 5
심사완료 : 2009. 3. 12