

Numerical Calculation Study on the Generalized Electron Emission Phenomenon

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Abstract

There are two kinds of well-known electron emissions from metal: field and thermionic emission. For thermionic emission, electrons come out of a metal due to the thermal energy, whereas for field emission, electrons tunnel out of a metal through the strong electric field. In this study, the most general electron emission caused by the temperature and electric field with a free electron gas model was considered. The total current density of electron emission comes from the field emission effect, where the electron energy is lower than vacuum, and from the thermionic-emission effect, where the electron energy is higher than vacuum. The total current density of electron emission is shown as a function of the temperature for a constant electric field, and as a function of the electric field for a constant temperature.

Keywords: field emission, thermionic emission, electron emission, CNT backlight, and field emission display

1. Introduction

Thermionic emission is well known, and the electrons in a metal obtain enough thermal energy to come out of the metal [1]. The most well-known thermionic emission is the electron emission from a tungsten filament in an incandescent light bulb. Electrons begin to be emitted from the tungsten when its temperature goes beyond 1500 K. The electrons in the metal tunnel through the barrier, which becomes thin due to the high electric field. This is called “field emission” or “Fowler-Nordheim tunneling”. The application of a tungsten tip by a high voltage is an example of field emission. The electric field in the tip is proportional to the applied voltage and inversely proportional to the tip’s radius. Field emission has been well studied in many papers [2, 3], and in particular, the field emission from tungsten emitters into many types of liquids has been investigated [4]. The field emission of a diamond-coated tungsten tip showed stable electron emission without damaging the tip [5], and the electric-tunnel effect was investi-

gated in a thin insulating film [6]. The angular dispersion of the electron beam emitted from an ultra-sharp tungsten tip with a few atoms on it was measured [7]. The field ionization consists of electron tunneling from the atoms or molecules under the action of even higher electric fields ($2\text{-}5 \times 10^8 \text{ V/cm}$) than that required for field emission ($3\text{-}6 \times 10^7 \text{ V/cm}$) [2]. Helium ionization was investigated with a tungsten tip as a function of gas pressure [8]. CNTs (carbon nanotubes) have been used as electron sources due to their small radius, in the order of 10 \AA , and their consequently lower critical voltage for emitting electrons. CNTs have been developed for the manufacture of field emission display and of the backlight for liquid crystal display (LCD). The gas ionization sensor, in which the breakdown voltage depends on different types of gas, was developed using CNTs [9]. It turns out that the temperature of the tip of a CNT becomes about 1500 K when electrons come out of the surface [10]. It is expected that the CNTs will show field emission and some thermionic emission because of the high-temperature effect.

In this paper, the most general electron emission case, which involves the temperature and electric field, was considered. From a free electron gas model, the density of the electrons in a metal follows the Fermi-Dirac distribution. The electrons above vacuum cause thermionic emission, and those below vacuum cause field emission; as such, the total current density of electron emission comes from the thermionic-emission and field emission effects. In the zero

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temperature limit, these researchers' unified theory of electron emission is approximated to the Fowler-Nordheim equation. The total current density is shown as a function of the electric field for a constant temperature, and as a function of the temperature for a constant electric field.

2. Theory

As electrons obey the Pauli exclusion principle, each energy level can accommodate no more than two electrons. It is thus adequate to consider only one partially filled conduction band and to treat this as if it corresponds to the electrons contained in a rectangular three-dimensional potential well. The electrons near the top of the band behave in many respects like free electrons. A metal contains a number of free electrons that are detached from the metal atoms, act as a perfect gas, and can move through the whole metal colliding only with the atoms [11].

The free electrons in a metal obey the Fermi-Dirac distribution, and the electron density in three dimensions in terms of energy can be written as

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2} \sqrt{E}}{h^3 (1 + \exp(-\frac{E - E_F}{k_B T}))}, \quad (1)$$

where E_F is the Fermi energy, E the energy of the electron, m the mass of the electron, h the Plank constant, and k_B the Boltzmann constant. At the temperature of 0 K, the electrons fill up to the Fermi energy level, and the density of electrons increases with \sqrt{E} according to the increase in density of states in three dimensions. According to equation (1), the electron density appears above the Fermi energy level and decreases exponentially from a few $k_B T$ above the Fermi energy level and reaches above vacuum at a high temperature. The Fermi energy correction by the temperature is shown as $E_F(T) = E_F(0)(1 - \frac{\pi^2}{12}(\frac{k_B T}{E_F(0)})^2)$, and the Fermi energy correction can be negligible within a few thousand Kelvin because the thermal energy is much lower than the Fermi energy level at zero temperature.

The unified theory of field and thermionic emission is considered at an arbitrary temperature and electric field. Fig. 1 shows a schematic diagram of the unified electron emission theory. The electron density distribution follows

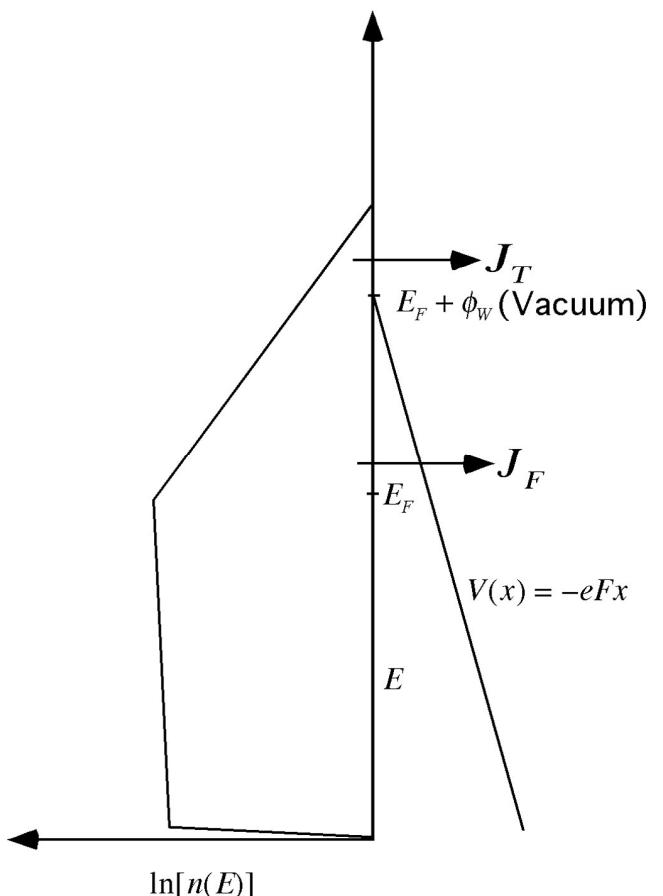


Fig. 1. A cartoon for the unified theory of field and thermionic emission. The electron density distribution is shown in terms of the energy level at a certain temperature. The line represents the potential energy $V(x)=-eFx$ according to the applied-electric-field F . The electrons with an energy level higher than vacuum have enough thermal energy to overcome the work function and to cause thermionic emission. The electrons with an energy level lower than vacuum, on the other hand, tunnel through the thin barrier caused by the electric field, and cause field emission. The total current density of electron emission consists of the current density of the thermionic and field emission.

equation (1) for a given temperature. At zero bias, the electrons below vacuum cannot tunnel due to the infinite-thickness barrier. The thickness of the barrier is changed by the potential energy $V(x)=-eFx$ due to the applied-electric-field F , as shown in Fig. 1. The electrons with an energy level lower than vacuum tunnel into the vacuum through the thin barrier caused by the electric field, and cause field emission. The electrons with an energy level higher than vacuum have enough thermal energy to overcome the work function and to cause thermionic emission. The total current density of the electron emission consists of the current density of the thermionic and field emission.

The total current density of electron emission can be expressed as

$$J = J_F + J_T, \quad (2)$$

where J_F is the current density of the field emission in which electrons come out of the metal surface through the electric field, and J_T the current density of the thermionic emission in which electrons come out of the metal surface via the thermal energy. *Field emission* is defined as the electron emission from the surface of a condensed phase into another phase, usually a vacuum, under the action of high electric fields ($3-6 \times 10^7 V/cm$) [2, 3]. Electrons go through the deformed potential barrier of the metal surface via a strong electric field.

The current density of field emission is

$$J_F = \int_0^{E_F + \phi_w} en(E)v_x(E)D(E)dE, \quad (3)$$

where ϕ_w is the work function, v_x the electron velocity in the x-direction, and D the electron tunneling coefficient. The energy range of the electrons in equation (3) is considered from zero to vacuum, where electron tunneling is possible. The tunneling coefficient D in equation (3) is given by [12]

$$D = \frac{4\sqrt{E}\sqrt{E_F + \phi_w - E}}{E_F + \phi_w} e^{-4k(E_F + \phi_w - E)^{3/2}/3F}, \quad (4)$$

where $k = \sqrt{\frac{8\pi^2 m}{h^2}}$ is the electron wave number and F the electric field. The tunneling coefficient increases along with the energy of the electron and the electric field because the barrier becomes thinner according to the applied electric field. Using equation (1), the current density of the field emission, equation (3) can be expressed as [13]

$$J_F = \frac{4\pi n e k_B T}{h^3} \int_0^{E_F + \phi_w} \int_0^\infty \frac{dy}{e^{\frac{(E-E_F)+y}{k_B T}} + 1} D(E)dE. \quad (5)$$

Equation (5) is then simplified to

$$J_F = \frac{4\pi n e k_B T}{h^3} \int_0^{E_F + \phi_w} D(E) \ln(1 + \exp(-\frac{E - E_F}{k_B T})) dE. \quad (6)$$

From equations (4) and (6), the following is obtained:

$$J_F = \frac{16\pi n e k_B T}{h^3 (E_F + \phi_w)} \int_0^{E_F + \phi_w} dE \sqrt{E} \sqrt{E_F + \phi_w - E} \cdot \ln(1 + \exp(-\frac{E - E_F}{k_B T})) e^{-4k(E_F + \phi_w - E)^{3/2}/3F} \quad (7)$$

The current density of the field emission, equation (7), can be solved numerically. The field emission strongly depends on the work function and electric field. It turns out that the field emission does not depend much on the temperature for the high value of the work function and Fermi energy. The field emission does not increase much even if the temperature increases. The reason for this is that the number of thermally excited electrons around the Fermi energy is small even if the excited electrons have a higher quantum tunneling probability according to equation (4). The number of thermally excited electrons above the Fermi energy level increases as the work function decreases. The electrons above the Fermi energy level, which have an exponentially higher tunneling probability, cause the field emission to increase.

The current density of the field emission is now shown, and equation (7) is simplified into the Fowler-Nordheim equation with a zero temperature limit and integration approximation. The total current density of the electron emission does not match that of the Fowler-Nordheim equation even at a low temperature. The reason for such discrepancy is that two kinds of approximations were used.

The first approximation is the zero temperature limit.

The factor $\ln(1 + \exp(-\frac{E - E_F}{k_B T}))$ becomes $\frac{E_F - E}{k_B T}$, which is a very good approximation as the electron energy is lower than the Fermi energy level in the zero temperature limit. This approximation shows a deviation when the electrons appear above the Fermi energy level for most field emis-

sion cases, in which the tunneling current of the electron causes heating at the tip even in low-temperature environments. The electron energy can be considered up to the Fermi energy level at the zero temperature limit. Considering that the electrons reach the Fermi energy level in the zero temperature limit, equation (7) can be written as

$$J_F = \frac{16\pi mek_B T}{h^3(E_F + \phi_w)} \int_{E_F + \phi_w}^{E_F} dE \sqrt{E} \sqrt{E_F + \phi_w - E} (E_F - E) e^{-4k(E_F + \phi_w - E)^{3/2}/3F}. \quad (8)$$

As electron tunneling mainly comes from around the Fermi energy level, the electron energy can be expanded as $E = E_F - x$. Equation (8) can be expressed in terms of x .

$$J_F = \frac{16\pi mek_B T}{h^3(E_F + \phi_w)} \sqrt{E\phi_w} e^{-4k\phi_w^{3/2}/3F} \int_0^\infty dx x e^{-2xk\sqrt{\phi_w}/F}, \quad (9)$$

where the integration range, which is from zero to the Fermi energy level, is approximated from zero to infinity.

The second approximation comes from the fact that the exponential term in equation (9) is big enough to cause integration approximation. The analytical solution was found from equation (9), via first-order approximation, as [12]

$$J_F = \frac{e}{2\pi h} \frac{\sqrt{E_F}}{(\phi_w + E_F)\sqrt{\phi_w}} F^2 e^{-4k\phi_w^{3/2}/3F}. \quad (10)$$

This is the well-known Fowler-Nordheim equation.

The integration approximation becomes better as the exponential term in equation (8) assumes a bigger value. When the exponential term in equation (8) becomes small, the deviation becomes large; as the Fermi energy and work function become small and the electric field becomes large, the deviation becomes large. The electric-field range of the field emission is usually from 3×10^7 to 6×10^7 V/cm.

Electrons above vacuum cause thermionic emission, as shown in Fig. 1. The current density of thermionic emission is

$$J_T = \int_{E_F + \phi_w}^\infty en(E)v_x(E)dE, \quad (11)$$

where the integration range is considered from vacuum to infinity, and the electron tunneling coefficient is approximated to 1. The current density of the thermionic emission, equation (11), can be written as

$$J_T = \frac{4\pi mek_B T}{h^3} \int_{E_F + \phi_w}^\infty \int_0^\infty \frac{dy}{e^{(\frac{E-E_F}{k_B T})+y} + 1} dE. \quad (12)$$

Equation (12) then becomes

$$J_T = \frac{4\pi mek_B T}{h^3} \int_{E_F + \phi_w}^\infty \ln(1 + \exp(-\frac{E-E_F}{k_B T})) dE. \quad (13)$$

As $\ln(1 + \exp(-\frac{E-E_F}{k_B T}))$ becomes $\exp(-\frac{E-E_F}{k_B T})$ within the aforementioned integration range, in which the electron energy level is much higher than the Fermi energy level, equation (13) can be simplified into

$$J_T = \frac{4\pi mek_B T}{h^3} \int_{E_F + \phi_w}^\infty \exp(-\frac{E-E_F}{k_B T}) dE, \quad (14)$$

where the electrons with a higher energy level than vacuum can cause thermionic emission. Equation (14) can be solved analytically to yield

$$J_T = \frac{4\pi mek_B^2}{h^3} T^2 e^{-\phi_w/k_B T}. \quad (15)$$

This result, as expected, is the same as the well-known thermionic emission, which starts from the temperature of 1500 K in the case of tungsten and increases exponentially as the work function decreases. To come up with a more accurate theory, the electron reflection, the existence of a negative space charge (which makes electron emission harder), and the actual surface area (which includes the surface roughness), and the absorbed molecules on the surface must be considered [11].

In this paper, the current density of the total electron emission consists of the current density of the field emission [equation (7)] and the current density of the thermionic

emission [equation (15)], in which equation (7) can be calculated numerically. Fig. 2 shows the total current density of the electron emission as a function of the electric field at the temperature of 2000 K for the work function, and the Fermi energy levels are 5 and 4 eV, respectively. As the thermionic emission due to 2000 K is constant, it can be known from Fig. 2 that the thermionic emission is dominant below the electric field of 3×10^7 V/cm, and that the field emission becomes more important above the electric field of 3×10^7 V/cm. The current density increases exponentially from the electric field of 3×10^7 V/cm as the electric field increases.

Fig. 3 shows the total current density of electron emission as a function of the temperature at the electric field of 4×10^7 V/cm. The total current density of the electron emission comes mainly from the field emission effect below the temperature of 2300 K and is added by the thermionic-emission effect as the temperature increases to above 2300 K. The current density of the total electron emission increases rapidly from about 2300 K as the temperature increases because the number of electrons that are thermally excited above vacuum becomes much higher than that of the tunneling electrons. As the temperature increases to

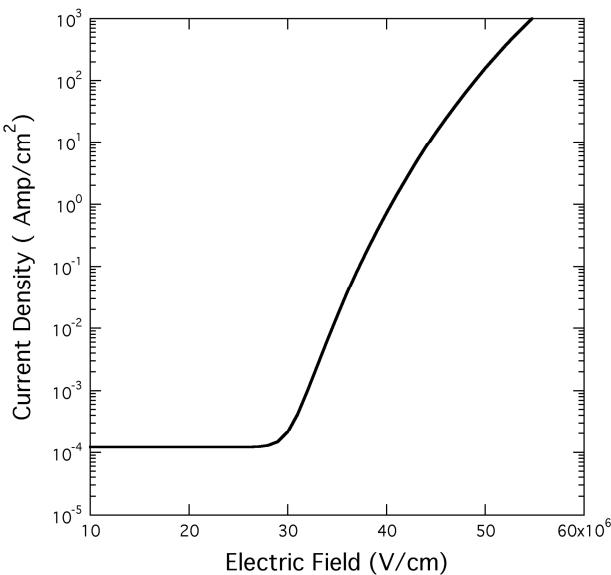


Fig. 2. The total current density of electron emission is shown as a function of the electric field at the temperature of 2000 K. Below the electric field of 3×10^7 V/cm, the thermionic emission is dominant, and at above 3×10^7 V/cm, the field emission becomes more important. The current density increases exponentially from the electric field of 3×10^7 V/cm. Here, the work function and Fermi energy are 5 and 4 eV, respectively.

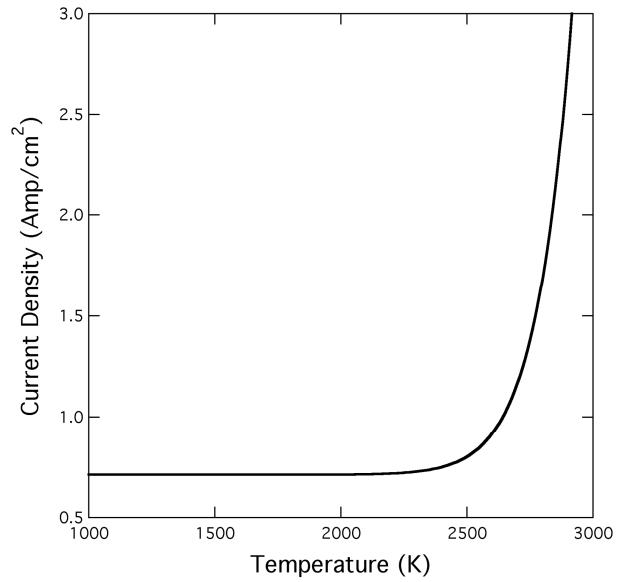


Fig. 3. The total current density of electron emission is shown as a function of the temperature at the electric field of 4×10^7 V/cm. The total electron emission current density below the temperature of 2300 K comes mainly from the field emission effect and increases according to the thermionic-emission effect as the temperature increases to above 2300 K.

above 1500 K, the discrepancy between Fowler-Nordheim tunneling and the unified-emission theory increases, and the temperature correction for the electron emission becomes important.

The unified theory of field and thermionic emission can be applied to the electron emission of metal, which depends on the temperature as well as the electric field. This unified theory will be useful for low-work-function materials, in which the Fower-Nordheim equation shows a deviation. It will also be useful for calculating the electron emission from CNTs, whose temperature rises to above 1500 K.

3. Conclusions

In this study, the unified theory of field and thermionic emission was constructed with a free electron gas model. The electrons in a metal follow the Fermi-Dirac distribution for a given temperature. The unified theory of field and thermionic emission shows that the electrons with an energy level higher than vacuum have enough thermal energy to overcome the work function and to cause thermionic emission, and that the electrons with an energy level lower than vacuum tunnel through the thin barrier caused by the

applied electric field, and cause field emission. The total current density of electron emission consists of that of the field emission [equation (7)] and of the thermionic emission [equation (15)]. The total current density of electron emission increases abruptly at a temperature above about 2300 K at the electric field of 4×10^7 V/cm. The total current density of electron emission increases as the electric field increases to above 3×10^7 V/cm at the temperature of 2000 K.

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