# 회전행렬과 쿼터니언에 근거한 비행체 제어기 설계 <br> Controller Design for Aircraft Based on Rotational Matrix and Quaternion 

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#### Abstract

In this paper, we present a linear controller for attitude of aircraft. We use a rotational matrix in one approach and a quaternion in the other approach. We also find some interesting mathematical properties concerning a symmetric rotational matrix and we use these properties to analyze the stability of the proposed control law. We find that the quaternion approach is better than rotational matrix approach because there exists no singular region problem in quaternion approach. On the other hand, singular region problem may happens in rotational matrix approach. The controller structure of the quaternion is also very simple compared with the one proposed by using a rotational matrix approach. We make use Matlab Simulink to simulate and illustrate the theoretical claims. The graphic animation program is developed based on Open-GL for the computer simulation of the proposed control algorithm.


Keywords:3D modeling, Quaternion, Rotational Matrix, Aircraft, Stability, Singular Region

## 1. Introduction

Many papers has been published for the attitude control of fight vehicles such as glider and helicopter. We also proposed a new control strategy for attitude control of aircraft under the assumption that all the parts are rigid ${ }^{[1]}$. In this paper, we address one solution for this control problem, in particular, the task of forcing the attitude of a rigid body fight vehicle to asymptotically track a desired reference model. We focus only on the regulator problem because of its easiness compared with model following problem. In real system in aircraft, we use the GPS and gyro as a sensor for measuring the location and angular velocity. There has been many research $^{[2,8]}$ for reducing the bias error by using least-squares approach or Kalman filter even thoughthis filtering strategy is originally derived based on the

[^0]assumption that the dynamics of control object are expressed or approximated by linear system disturbed by random noise the characteristic of which we know such as mean and variation etc. There are been also many research works in which the rotational motions are described by using quaternion. R.M. Sanner ${ }^{[3,4]}$ and T.I Fossen ${ }^{[5,6,7,10]}$ also proposed a nonlinear controller and observer based on quaternion kinematics. Recently there has also been many research to apply adaptive observer and control theory to aircraft ${ }^{[9]}$. S. Di. Gennaro ${ }^{[11]}$ proposed error modeling based on quaternion to describe the dynamics of a flexible spacecraft. There has been also some research works in which a quaternion feedback control was used based on the Euler angle ${ }^{[12,13]}$.

Here, in order to make a rigid fight object to track a reference attitude, we propose linear control law for angular velocity based not only on rotational matrix approach but also on quaternion approach. We also review briefly on the operations of quaternion such as product, inverse and its kinematics etc. The graphic animation program is developed based on Open-GL for
the computer simulation of the proposed control algorithm. The original graphicdata for helicopter are derived by using 3ds Max software. This simulation program is now developing in our laboratory and will be used for the tracking and stabilization of helicopter with the considering the robustness for the wind gust, dynamic uncertainty and etc.

## 2. Mathematical Preliminary

In this section, we will first present basic mathematical description of aircraft attitude in terms of quaternion. Then the error model of the attitude is described based on the quaternion.

For simplicity we refer to orthogonal matrices with determinant +1 as rotation matrices and we refer the set of all $3 \times 3$ rotation matrices by the symbol $\mathrm{SO}(3)$. We also denote the set of all $3 \times 3$ skew symmetric matrices by $S S(3)$ The attitude of a spacecraft can be represented by a quaternion, consisting of a unit vector e, known as the Euler axis and rotation angle $\varphi$ about this axis, so that

$$
\begin{equation*}
q=\left[\frac{e \sin \varphi}{\cos \varphi}\right]=\left[\frac{\varepsilon}{\eta}\right] \tag{1}
\end{equation*}
$$

where q is the quaternion, partitioned into a vector part, $\varepsilon$ and a scalar part, $\eta$. In the mathematical description of spacecraft attitude, the quaternion represents the rotation of spacecraft body coordinate system in respect to the inertial coordinate system. Note that the quaternion discussed in here is an element of the set Q of unit quaternions defined as

$$
\begin{equation*}
Q=\left\{q \mid q^{T} q=1, q=\left[\varepsilon^{T} \eta\right]^{T}, \varepsilon \in \mathfrak{R}^{3}, \eta \in \mathfrak{R}\right\} \tag{2}
\end{equation*}
$$

The rotation matrix corresponding to a quaternion is given as

$$
\begin{align*}
R(q) & =I-2 \varepsilon^{T} \varepsilon I+2 \varepsilon \varepsilon^{T}+2 \eta S(\varepsilon) \\
& =I+2 \eta S(\varepsilon)+2 S^{2}(\varepsilon) \tag{3}
\end{align*}
$$

where $I$ is an $3 \times 3$ identity matrix and $S(\varepsilon)$ is a $3 \times 3$
skew symmetric matrix defined as

$$
S(\varepsilon)=\left[\begin{array}{ccc}
0 & -\varepsilon_{z} & \varepsilon_{y}  \tag{4}\\
\varepsilon_{z} & 0 & -\varepsilon_{x} \\
-\varepsilon_{y} & \varepsilon_{x} & 0
\end{array}\right]
$$

An important property of $S(\varepsilon)$ is that for any vector $\mathrm{p}=[\mathrm{px}, \mathrm{py}, \mathrm{pz}] \mathrm{T}$

$$
\begin{equation*}
S(\varepsilon) p=\varepsilon \times p \tag{5}
\end{equation*}
$$

Suppose that a rotational matrix $R_{b}^{l}$ is time varying, then the time derivative $\dot{R}_{b}^{l}$ of $R_{b}^{l}$ is given by

$$
\begin{equation*}
\dot{R}_{b}^{l}=S\left(\omega_{l b}^{l}\right) R_{b}^{l}=R_{b}^{l} S\left(\omega_{l b}^{b}\right) \tag{6}
\end{equation*}
$$

In here, we use the following mathematical fact that

$$
\begin{equation*}
R S(\varepsilon) R^{T}=S(R \varepsilon) \tag{7}
\end{equation*}
$$

where $R$ is any rotational matrix.

## 3. Controller Design: Rotational Matrix Approach

### 3.1 Control Law and Stability Analysis

We state the invariant property of the rotational matrix in the following theorem.

Theorem 1. Let $S(t)$ be sew symmetric matrix. Then the solution matrix $R(t)$ such that

$$
\begin{equation*}
\dot{R}=S(t) R(t) \tag{8}
\end{equation*}
$$

satisfies the following property of the rotational matrix for any given $R(0) \in S O(3)$.

$$
\begin{equation*}
R(t) R(t)^{T}=I \tag{9}
\end{equation*}
$$

Proof: We can derive the solution matrix $R(t)$ as follows.

$$
\begin{equation*}
R(t)=e^{\int S(\tau) d \tau} R(0) \tag{10}
\end{equation*}
$$

From the fact that $S$ is skew symmetric matrix, we obtain following mathematical property

$$
\begin{equation*}
S S^{T}=S^{T} S \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
e^{\int S(\tau) d \tau} \cdot e^{\int S(\tau)^{T} d \tau}=e^{\int S(\tau)+S(\tau)^{T} d \tau}=I \tag{12}
\end{equation*}
$$

Therefore, we obtain

$$
\begin{equation*}
R(t)\left(R(t)^{T}=R(0)\left(R(0)^{T}=I\right.\right. \tag{13}
\end{equation*}
$$

Now we propose a stable control law as a theorem.

Theorem 2. For a dynamic equation of a helicopter given as equation (8) if we select the control law given as

$$
\begin{align*}
& \omega_{x}(t)=-k_{x}\left(R_{32}(t)-R_{23}(t)\right) \\
& \omega_{y}(t)=-k_{y}\left(R_{13}(t)-R_{31}(t)\right)  \tag{14}\\
& \omega_{z}(t)=-k_{z}\left(R_{21}(t)-R_{12}(t)\right)
\end{align*}
$$

for any positive constants $k_{x}, k_{y}, k_{z}$, then $\lim _{t \rightarrow \infty} R(t)=M_{s} I$ where $M_{s}$ is a symmetric rotational matrix.
In the above equation, $\omega x(\mathrm{t}), \omega y(\mathrm{t})$, and $\omega \mathrm{z}(\mathrm{t})$ are the angular velocities of the helicopter represented by projections on the axes of the body attached coordinate system.

Proof: Let us choose the performance index J as follows.

$$
\begin{equation*}
J=\operatorname{tr}(R(t)-I)(R(t)-I)^{T} \tag{15}
\end{equation*}
$$

The time derivative of J can be expressed as

$$
\begin{equation*}
\dot{J}=\operatorname{tr}\left[\frac{d}{d t}\left[(R(t)-I)(R(t)-I)^{T}\right]\right]=-2 \operatorname{tr}[(S(t) R(t)] \tag{16}
\end{equation*}
$$

We obtain time derivative of J as follows.

$$
\begin{gather*}
\dot{J}(t)=\omega_{x}\left(R_{32}(t)-R_{23}(t)\right) \\
+\omega_{y}\left(R_{13}(t)-R_{31}(t)\right)  \tag{17}\\
+\omega_{z}\left(R_{21}(t)-R_{12}(t)\right)
\end{gather*}
$$

Therefore if we choose control law (14), we obtain

$$
\begin{gather*}
\dot{J}(t)=-k_{x}\left(R_{32}(t)-R_{23}(t)\right)^{2} \\
-k_{y}\left(R_{13}(t)-R_{31}(t)\right)^{2}  \tag{18}\\
-k_{z}\left(R_{21}(t)-R_{12}(t)\right)^{2}
\end{gather*}
$$

and we can see that $R(t)$ converges to symmetric rotation matrix one of which is identity matrix which we want. Q.E.D

Theorem 3. There exist only 8 kinds of rotational matrix that is symmetric and 4 kinds of them are in $S O(3)$ and one of them is $I_{3 x 3}$.

Proof: From the basic knowledge of linear algebra, we already know that symmetric matrix has only real eigenvalues. We also know that the absolute values of rotational matrix's eigenvalue are all equal to 1 . Therefore the eigenvalue set S1of symmetric rotational matrix is given as

$$
S_{1}=\{( \pm 1, \pm 1, \pm 1)\}
$$

From the definition of $\mathrm{SO}(3)$ in section 2, it is impossible for a symmetric rotational matrix has odd number of eigenvalue -1 . The possible combination of eigenvalue of the symmetric rotation matrices with determinant are as follow.

$$
\{(1,1,1),(1,-1,-1),(-1,1,-1),(-1,-1,1)\}
$$

The common property of the symmetric rotational matrix is that the sum of the eigenvalue is either 3 or -1 . I3x3 is the only symmetric rotational matrix whose sum of eigenvalues is 3 and we use this fact to derive the stable control law in this section.

Remark 1. From the above theorem 2,

$$
\dot{J}=-\frac{1}{2}\left(\dot{R}_{11}(t)+\dot{R}_{22}(t)+\dot{R}_{33}(t)\right) \text { and from this we know }
$$ that the fact $\hat{J} \leq 0$ means that the summation of the eigenvalues are increasing for all time. So we can guess that if we set the initial condition of rotational matrix satisfy the following equation

$$
\begin{equation*}
-1<\left(R_{11}(0)+R_{22}(0)+R_{33}(0) \leq 3\right. \tag{19}
\end{equation*}
$$

$R(t)$ converges to $I_{3 x 3}$.
From the above theorem 2, we can not guarantee that rotation matrix $\mathrm{R}(\mathrm{t})$ converse to identity matrix, because there are 4 symmetric matrices in $\mathrm{SO}(3)$. But if we can restrict the initial condition of the rotational matrix in a certain domain, we can find a global stable control law and so we now propose a global stable control law as a theorem.

Theorem 4. If we use the same a control law proposed in theorem 2 and initial condition of rotational matrix satisfies equation(19) then $\lim _{t \rightarrow \infty} R(t)=I$.

Proof: From the theorem 2, we know that the $R(t)$ converges to symmetric matrix. We also know by using the fact of remark 1 that the only symmetric rotational matrix which can be converged to is $\mathrm{I} 3 \times 3$ and this proves the theorem.

Now we try and discuss the same problem with the mathematical tool of quaternion in the next section.

### 3.2 Simulation

We use the Matlab Simulink to simulate the control law proposed in the previous section. We consider two case of initial conditions of rotational matrix as shown in table 1 . We set the control parameters $k_{x}, k_{y}, k_{z}$ as 1,1 , 1 respectively in both case and the simulation time is set 5 seconds. Initial conditions of case 1 and case 2 are obtained by $R_{z, 120}$ and $R_{z, 30} R_{y, 30} R_{x, 0}$ respectively where $R_{z, \mu}$ denote the rotational matrix which we obtain when we rotate inertial coordinate system $\psi$ degree about $z$ axis. We observe that the rotational matrix converges to the $I_{3 x 3}$ in both case. We emphasize that the sum of the eigenvalue of the initial rotational matrix in both case is large than -1 . The wave forms of each element of the

Table 1. Initial and final value of rotational matrix
(a) Attitude of small

| Case1 | Initial | Final |
| :---: | :---: | :---: |
| $(R 11, R 12, R 13)$ | $(-0.50,-0.86,+0.00)$ | $(1,0,0)$ |
| $(R 21, R 22, R 23)$ | $(+0.86,-0.50,+0.00)$ | $(0,1,0)$ |
| $(R 31, R 32, R 33)$ | $(+0.00,+0.00,+1.00)$ | $(0,0,1)$ |

(b) Attitude of large deviation

| Case1 | Initial | Final |
| :---: | :---: | :---: |
| $(R 11, R 12, R 13)$ | $(+0.75,-0.50,+0.43)$ | $(1,0,0)$ |
| $(R 21, R 22, R 23)$ | $(+0.43,+0.86,+0.25)$ | $(0,1,0)$ |
| $(R 31, R 32, R 33)$ | $(-0.50,+0.00,+0.86)$ | $(0,0,1)$ |

(c) Attitude of deviation in Singular Region

| Case1 | Initial | Final |
| :---: | :--- | :--- |
| $(R 11, R 12, R 13)$ | $(+0.77,+0.64,+0.00)$ | $(+0.77,+0.64,+0.00)$ |
| $(R 21, R 22, R 23)$ | $(+0.64,-0.77,+0.00)$ | $(+0.64,-0.77,+0.00)$ |
| $(R 31, R 32, R 33)$ | $(+0.00,+0.00,-1.00)$ | $(+0.00,+0.00,-1.00)$ |

rotational matrix and attitude of the helicopter corresponding to the kinematic parameters are shown in Fig. 1 and Fig. 2 for case 1 and case 2, respectively.

We observe from the computer simulation that there exist some domain of initial condition for rotational matrix where eigenvalue of the initial rotational matrix has $1,-1,-1$ and we can't guarantee the stability when helicopter is located in this region initially. So we call this region as "Singular Region" of the proposed control law. We simulate this case as shown in case 3, where we obtain the initial rotational matrix as follow.

$$
R(0)=R_{z, 20} \operatorname{diag}(1,-1,-1) R_{z, 20}^{T}
$$

From this discussion, we can see that if the initial condition of rotational matrix is belong to Rs defined bellow, we can not guarantee the stability but the size of this space is not so large compared to the whole space of rotational space.

$$
\begin{equation*}
R_{s}=M_{1} \cup M_{2} \cup M_{3} \tag{20}
\end{equation*}
$$


(a) Wave form of rotational matrix : case 1.

(b) Kinematic parameter of the initial attitude: case 1.

(c) Kinematic parameter of the final attitude: case 1.

Figure 1. Control based on rotational matrix approach.(case 1.)

(a) Wave form of rotational matrix : case 2.

b) Kinematic parameter of the initial attitude: case2.

(c) Kinematic parameter of the final attitude: case2.

Figure 2. Control based on rotational matrix approach.
where

$$
\begin{aligned}
& M_{1}=R \operatorname{diag}(1,-1,-1) R^{T} \\
& M_{2}=\operatorname{Rdiag}(-1,-1,1) R^{T} \\
& M_{3}=R \operatorname{diag}(-1,1,-1) R^{T}
\end{aligned}
$$

$R$ is any rotational matrix and $M_{1}, M_{2}, M_{3}$ are initial conditions of symmetric rotational matrix in the Singular Region.

## 4. Controller Design: Quaternion Approach

### 4.1 Control Law and Stability Analysis

In this section, we review research works done on the aircraft attitude control base on quaternion. Quaternion differentiation's formula can be represented as

$$
\begin{equation*}
\frac{d q(t)}{d t}=\frac{1}{2} \Omega(t) \otimes q(t)=\frac{1}{2} q(t) \otimes \omega(t) \tag{21}
\end{equation*}
$$

where

$$
\begin{gathered}
q(t)^{-1} \otimes \Omega(t) \otimes q(t)=\omega(t) \\
\Omega(t)=\left(0, \Omega_{x}(t), \Omega_{y}(t), \Omega_{z}(t)\right) \\
\omega(t)=\left(0, \omega_{x}(t), \omega_{y}(t), \omega_{z}(t)\right)
\end{gathered}
$$

In the above equation, $\otimes$ denotes quaternion product ${ }^{[4,}$, 14] operator, $\Omega(t)$ is the angular velocity represented by projections on the axes of inertial coordinate system and $\omega(t)$ is same angular velocity represented by projections on the axes of the body attached coordinate system. The above equation for the quaternion can be expressed as

$$
\dot{q}(t)=\left[\begin{array}{c}
\dot{e}  \tag{22}\\
\dot{\eta}
\end{array}\right]=\frac{1}{2} Q(q(t)) \omega(t)
$$

where

$$
Q(q(t))=\left[\begin{array}{c}
\eta(t) I+S(\varepsilon(t)) \\
-\varepsilon(t)^{T}
\end{array}\right]=\left[\begin{array}{c}
Q_{1}(q(t)) \\
-\varepsilon(t)^{T}
\end{array}\right]
$$

In here, $Q 1(q(t))=\eta(t) I+S(\varepsilon(t))$ by inspection. The relative orientation error $\widetilde{q}$ between coordinates frames represented by $\mathrm{qd}(\mathrm{t})$ and $\mathrm{q}(\mathrm{t})$ is defined and computed as

$$
\begin{align*}
& \widetilde{q}(t)=\left[\begin{array}{l}
\widetilde{e} \\
\tilde{\eta}
\end{array}\right] \equiv q(t) \otimes\left(q_{d}(t)^{-1}\right.  \tag{23}\\
& {\left[\begin{array}{cc}
\eta_{d} I-S\left(\varepsilon_{d}\right) & -\varepsilon_{d} \\
-\varepsilon_{d}^{T} & \eta_{d}
\end{array}\right]\left[\begin{array}{l}
\varepsilon_{1} \\
\eta_{1}
\end{array}\right]}
\end{align*}
$$

where $q_{d}$ is a desired target attitude represented by quaternion. The control object is to make spacecraft be aligned with the desired attitude represented by quaternion $\mathrm{q}_{\mathrm{d}}$, that is , to make that $\|\widetilde{\varepsilon}\|=0, \widetilde{\eta}= \pm 1$.

The dynamic equation for the relative orientation error $\tilde{q}$ is given as

$$
\begin{equation*}
\dot{\vec{q}}(t)=\frac{1}{2} Q(\widetilde{q}) \widetilde{\omega} \tag{24}
\end{equation*}
$$

where

$$
\widetilde{\omega}=\omega-R(\widetilde{q}) \omega_{d}
$$

and $R(\widetilde{q})$ is rotational matrix corresponding to a quaternion error $\widetilde{q}, \omega_{d}$ is the desired angular velocity for a desired quaternion $q_{d}$. In the above mathematical derivation we use the following facts concerning quaternion

$$
\begin{gathered}
\widetilde{q} \otimes \widetilde{q}^{-1}=1 \\
\frac{d \widetilde{q}^{-1}}{d t}=-\widetilde{q}^{-1} \otimes \frac{d \widetilde{q}}{d t} \otimes \widetilde{q}^{-1}
\end{gathered}
$$

Therefore the essential control object is to find $\omega(t)$ that stabilizes above error dynamics for quaternion and makes that $\|\tilde{\varepsilon}\|=0, \tilde{\eta}= \pm 1$. In this paper, we propose control law in case of regulator problem, that is to consider only the case when the desired quaternion $q_{d}(t)$ is a constant quaternion $(0,0,0,1)$.

Therefore the control problem in this case is to find $\omega(t)$
such that $\widetilde{q}(t)=q(t)$ converge to a constant quaternion $(0,0,0,1)$. In near future, we will tackle the general model following problem where the desired quaternion is obtained by desired angular velocity $\omega_{d}(t)$ from the model such as follow.

$$
\dot{q}_{d}(t)=\frac{1}{2} Q\left(q_{d}(t)\right) \omega_{d}(t)
$$

We now propose control law based on the quaternion as a following theorem and we can see that the control law is very simple compared with the one proposed by using a rotational matrix approach.

Theorem 5. For the kinematics of quaternion given by equation (22), if we select the control law given as

$$
\begin{gather*}
\omega(t)=-k_{\omega}\left(Q_{1}(q(t))^{T}+(1-\eta(t)) I\right) \varepsilon(t) \\
=-k_{\omega}\left(S(\varepsilon(t))^{T}+I\right) \varepsilon(t) \\
=-k_{\omega} \varepsilon(t) \tag{25}
\end{gather*}
$$

for any positive constant $k_{\omega}$, then $\lim _{t \rightarrow \infty} q(t)=(0,0,0,1)$ Proof : Let us choose the performance index J as follows.

$$
\begin{equation*}
J=\varepsilon^{T}(t) \varepsilon(t)+(1-\eta(t))^{2} \tag{26}
\end{equation*}
$$

We obtain time derivative of J as follows.

$$
\begin{align*}
& \dot{J}(t)=2 \varepsilon^{T} \dot{\varepsilon}-2(1-\eta(t)) \dot{\eta}(t) \\
& \quad=\left(\varepsilon^{T} Q_{1}+(1-\eta) \varepsilon^{T}\right) \omega \tag{27}
\end{align*}
$$

Therefore if we apply the control law (25), we obtain

$$
\begin{equation*}
\dot{J}(t)=-2 \varepsilon^{T}\left[Q_{1}-(\eta-1) I\right]\left[Q_{1}^{T}-(\eta-1) I\right] \varepsilon \leq 0 \tag{28}
\end{equation*}
$$

From the property of the quaternion kinematics, that is $\mathrm{Q} 1=\eta \mathrm{I}+\mathrm{S}(\varepsilon)$, we obtain

$$
\begin{gather*}
\dot{J}(t)=-k_{\omega} \varepsilon^{T}[I+S(\varepsilon)]\left[I+S(\varepsilon)^{T}\right] \varepsilon \\
=-k_{\omega} \varepsilon^{T}\left[I+S(\varepsilon) S(\varepsilon)^{T}\right] \varepsilon \leq 0 \tag{29}
\end{gather*}
$$

We can see that the matrix $I+S(\varepsilon) S(\varepsilon)^{T}$ is positive definite matrix for any $\varepsilon$ and therefore we can guarantee that $\varepsilon(t)$ and $\eta$ converge to zero vector and constant 1 as time goes to infinity. This prove the theorem. Q.E.D

### 4.2 Simulation

We set the control parameter $k_{\omega}$ to 5 and simulation time is set to 5 . We obtain set the initial quaternion $q(0)$ $=(0.1619,0.0754,0.3830,0.9063)$ by choosing e and $\eta$ as $(0.383,0.1786,0.9063)$ and 25 degree. We observe that the quaternion $q(t)$ converges to $(0,0,0,1)$ within in 2.5 second and this fact verifies the validity of the proposed control law based on quaternion approach. The most advantage of this control law based on quaternion

(a) Trajectories of quaternion

(b) Coordinate reference frames of model helicopter

Figure 3. Control based on quaternion approach.


Figure 4. Animation graphic for the proposed control.
compared with the one based on rotational matrix is that it guarantees the global stability for any initial attitude of craft. The trajectories of each element of quaternion $q(t)$ and coordinate reference frames of model helicopterare shown in figure Fig3. The following figure Fig4 is a graphic animation program based on OPEN-GL program for the computer simulation of the proposed control algorithm. The original graphic data for helicopter are derived by using 3ds Max software

## 5. Conclusion

A linear controllers designed based on both rotational matrix and quaternion are presented for aircraft attitude application. We also analyze the stability in each case by using Lyapunov-like method prove the global stability of the proposed linear controller. In this paper, we also review and comment onthe error dynamic for the quaternion approach for helping those who are interested in the quaternion for aircraft or robotics. From the results of computer simulation, we can see that the proposed control algorithm can be applied to solve the self stabilizing problem such as hovering problem in helicopter etc.

In near future, we will try to tackle for the robust control problem even in the presence of noise in sensor such as gyro, GPS etc.

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