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Who are the Mathematically Gifted? Student, Parent and Teacher Perspectives[1]

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This paper reports on student, parent, and teacher perspectives of the characteristics of the mathematically gifted. The data are extracted from a two-year qualitative study that examined multiple perspectives, school policy documents and program provision for 15 mathematically gifted and talented students aged from 10 to 13 years. The findings have implications for identification and program provision.

Keywords: mathematically giftedness, characteristics, student, parent, teacher perspectives.
ZDM Classification: C22, C42, D62
MSC2000 Classification: 97C20, 97C40, 97D60

INTRODUCTION

There is no single definition of mathematical giftedness. The literature suggests that mathematically gifted students could be thought of as those who have special mathematical abilities or those who engage in qualitatively different mathematical thinking. Most studies on students who are gifted in mathematics draw on the enduring work of a Russian psychologist Krutetski (1976) who originally published his findings in 1968. His comprehensive investigation of mathematical ability was designed to explore the nature and structure of mathematical abilities over a twelve-year period. He defines ability as a personal trait that enables one to perform a given task rapidly and well, and contrasts this to a habit or skill, which relates to the qualities or features of the activity a person is carrying out. Students gifted and talented in mathematics tend to view the world

[1] This article is an extended version of the paper (Bicknell, 2008) presented at Topic Study Group 6 (Activities and Programs for Gifted Students) of the 11th International Congress on Mathematical Education (ICME-11) held at the Universidad Autonoma de Nuevo Leon (UANL), Monterrey, Mexico, July 6–13, 2008.

through a mathematical lens. Krutetski uses the term a "mathematical cast of mind" to describe this characteristic. According to Krutetski (1976, p. 77) "mathematical giftedness" is the name given to "a unique aggregate of mathematical abilities that opens up the possibility of successful performance in mathematical activity."

Gardner (1983) also sought to identify the characteristics of those with mathematical gifts by examining Piaget's development of logical mathematical thought and the thinking of eminent mathematicians (*cf.* Piaget & Inhelder, 1969). Students may engage in other curriculum activities using mathematical skills and predictive logic as described by Gardner's Multiple Intelligences. These students choose to represent information in a mathematical or quantitative way. Some of these characteristics resemble those identified by Krutestki. Gardner described abilities such as using mathematical notation, sustaining long chains of reasoning, abstracting general features from mathematical material, and using mathematical reasoning. There were also features of Gardner's "Spatial Intelligence" that reflected capacities described in Krutetski's "geometric mind."

Published lists of indicators of mathematical giftedness support those characteristics provided by the research cited above. The basis of the development of these lists is not always clear but further to Krutetski's work a useful contribution is made from the writings of Diezmann & Watters (2000), House (1987), Johnson (1983), and Straker (1983). The mathematical characteristics provided by all of these writers reflect similarities. Their descriptions include aspects such as an early curiosity and understanding about the quantitative aspects of things; an ability to think logically and symbolically about qualitative and spatial relationships; to perceive and generalize about mathematical patterns, structures, relations, and operations; to reason analytically, deductively, and inductively; to abbreviate mathematical reasoning; and energy and persistence in solving mathematics problems.

Wieczerkowski, Cropley & Prado (2000, p. 415) proposed an approach that regarded mathematical giftedness as "involving special ways of looking at and attempting to solve mathematical problems." Similarly, Winner (1996, p. 3) asserted that gifted children insist on "marching to their own drummer." These children, she claimed, independently invented rules and devised novel, idiosyncratic ways of solving problems. This reflects a more qualitative conceptual view of mathematical giftedness. Cropley (1994) argued that a combination of quantitative and qualitative approaches defined true mathematical giftedness.

The usefulness of these descriptions is as a tool for teachers and parents to help with the identification of mathematically gifted and talented students and subsequent program provisions.

RESEARCH DESIGN

The part of the study reported here was designed to examine multiple perspectives on the characteristics of the mathematically gifted. The theoretical viewpoint of this paper is drawn from social constructivist and socio-cultural perspectives. These perspectives belong within the interpretive paradigm. A purposive sampling method was utilized, the criterion being that students were identified by their schools as gifted and talented in mathematics and were in a special class or program for mathematics. There were 15 students, originating from three schools. The schools' identification processes differed in some particular aspects but all used standardized test results and teacher nomination. There were 10 students (2 girls, 8 boys) from two regular Grade 6 classes (10 and 11 year olds) and five Grade 8 students (2 girls, 3 boys) from a fulltime class for gifted students at an intermediate school (12 and 13 year olds). The students all transferred in the following year to other schools. Hence, the data collection occurred over a two year period.

The researcher used predominantly qualitative data gathering methods. The data sources for the study included questionnaires, semi-structured interviews, classroom observations, and documents (school policies, teacher plans and student workbooks). Students and parents were interviewed once during 2005 and again in 2006. Each teacher (n = 13) was interviewed once. Data were analyzed and coded; these codes were derived from a variety of conceptual frameworks that related to the research questions of the larger study. The study adhered to the University Code of Ethics and pseudonyms are used for participants.

RESULTS

This section presents the perspectives of the parents, students, and teachers regarding the characteristics of mathematical giftedness. The data presented here are extracted from the interview material.

Mathematics at an early age

Most of the parents recognized their child's abilities in mathematics at an early age. The characteristics of pre-school children described by parents included "impressive concentration" and the ability to work independently for a sustained period on a particular task. The types of activities the parents observed in their children at an early age indicated an interest in things mathematical, and included building with construction blocks, creating symmetrical patterns, ordering objects, completing puzzles (in unconventional

ways) and designing challenging games that involve numbers, problem solving, and operating on numbers. The children showed a relatively sound concept of number and in some cases an interest in large numbers and concepts such as time and space. Parents described some of these pre-school self-initiated tasks:

> They would make a circus and [Ryan] would go off and make tickets for everyone and they would have numbers on the top and they would all be consecutively numbered... He used to read the numbers off the letterbox all the way to school. It started with the easy ones...we got to 356 and this is a two and half to three year old. He used to read all the way there and back. I obviously thought this child has an interest in numbers... He spent an entire morning at morning kindy so he must have been four, sitting at the front of the gate with a chart he's asked the teachers to make for him, counting cars, marking off the cars and what colour they were in what box and then added up the different colours. The whole three hours he spent counting cars. (Ryan's mother)

Whilst Ryan showed early signs of confidence and competence in number, Mia and Lily were more interested in the spatial or geometric view of the world.

> I think my ballet teacher helped me a lot with it, with angles and things, you know like getting feet at the right angle and things. So I've been good at the angles and things since I was quite little because of dancing. (Mia)

> She has always had an impressive concentration, she would sit for hours, literally when she was two/two and a half with Lego - big kids Lego and put it together. Everything was symmetrical and all colour coded... Mathematics was always a big part... She always loved jigsaws. The first time she did it was turn it upside down, then turn it back so it was the mirror image and then put it all back. The second time she would do it she would do the same and then she would break them up into little breaks. She never did all around the outside and then fill in the middle. She goes for a patterny bit and then a bit more. (Lily's mother)

Martin, on the other hand, had a more logical, analytic and arithmetic approach to mathematics that was evident from an early age. He showed a real fascination for numbers.

> I think from the start, anything that came to numbers, he could recognize numbers and understand their meaning quite quickly. He would say there are two apples and three oranges... his perception of things adding up and subtracting from each other, he had that from an early age. When he was three or four, one of the uncles gave him a time table's poster and he insisted on having it next to his bed and he would just pop up one day and say "Did you know that 4 x 6 is 24?" ... he was very interested in money when he was a preschooler and when we started giving him little bits of money to spend he would work out how much he wanted to put away for a bigger present at a preschool age and how much he would spend on, you know, a lolly or something. (Martin's mother)

Three students in the study showed an early interest in all things mathematical, both geometric and numeric.

> He did a lot of hands on like blocks and knex, he made these angles... [Victor] just knew how to manoeuvre the magnets around with angles, mathematical shapes, geometric

> objects and stuff like that. I think it's more recognition with how things went that's how he thought about it. But then when it came to mathematical problems, 2 + 3 = 5 or something like that he just tended to understand that. He says "give me something with figures".... He just solves problems, sees things and knows like that one goes there and that may go there. ... that was noticeable even at that early age. (Victor's mother)

> He just had a tremendous desire of learning, right from day one and even pre-school, it was amazing what he, he had a tremendous memory, and you could tell him one thing and he'd have it the second time ... what I sort of saw was that he grasped concepts very, very quickly and even when he was very young if you said 2 + 2 it was just like that, he knew it. He was just very quick at grasping concepts very early on and division questions and all that sort of thing. (Eric's mother)

> I just think he has a brain which is mathematical, he sees things as mathematical, and sometimes he is too mathematical. His realization that numbers and mathematics are important for life which is something that we did not have.... He loves it and has a passion for it. If things can come down to numbers he's quite happy.... The first thing he told me, "Ma it turns" "Look Ma, it turns"... Then he got very interested in space, so for him that became big, like mathematics could go into the millions, kilometers, large numbers. (Nardu's mother)

Characteristics as evidenced in school mathematics

Once the students began school, their awareness of a differing level of interest and ability in mathematics compared to their peers became more apparent. This was reinforced for those parents who suspected that their child might be gifted and talented in mathematics. Take Martin for example:

> When he went to school, he'd only been at school a few months and he got a principal's award and it basically read something like: "For knowing more about money and numbers than your teacher or your principal put together" and that was just their way of acknowledging that, yes, he had a special interest in it. (Martin's mother)

For some students this recognition came in Grade 2, their second year of formal schooling.

> Probably since Grade 2, I began to like it more and even when I wasn't that good, I liked it and I became better. I liked the class games and competitions and I liked that so I then became better at mathematics. In Grade 3 it was the same and in Grades 5 and 6 we went off in groups and I got in better groups and got to do more and then at intermediate I got in the extension room. (Amir)

> The further I can think back would be Grade 2. I learnt about borrowing with subtracting. I just started liking it, the subject. Then I went to [named teacher] class and then I carried on from there. I started learning my times tables. (Eric)

It was repeatedly mentioned by some of the students and parents that Grade 3 was a 'significant' year. This also seemed to be the time when students become aware of their ability because of the type of groupings being used in the mathematics classroom. One of the students explained:

> I first knew at about Year 3 when I was more above my age level but I also kind a noticed it in Year 1 and 2. I knew a few skills, half way through Grade 1. I already knew how to add like 5 plus 4 whereas others were just learning about addition. In Grade 3, I was above my level and I got reinforced with that in Year 4. (Lewis)

This awareness of ability for the students is based on evidence when comparing themselves with others. Two of the students articulated this viewpoint:

> I'm a child with a special ability; I'm above average in my ability in mathematics compared to others. I like problem solving and I like algebra as well. I'm also quite a fast learner, at first it's a little bit difficult to overcome a new topic but then a bit more into it I'm able to remember it; I remember new stuff really easy. (Nina)

> I was brighter than other kids and realized this in my first year at [Named School]. I was ahead of the other students. In Year 6, I really stood out as I got High Distinction in New South Wales mathematics thingy, so that was when I realized that I was actually good, really good. (Paul)

The teachers believed that the characteristics that stood out the most initially for them were the keen interest in mathematics, the sense of humor, and the viewing of the world through a mathematical lens.

> A lot of them are bright but these gifted and talented ones are different. (Teacher G)

> He's got a real thirst for mathematics... mathematics is his passion. (Teacher H)

> They're like little sponges... if you say you're going to be doing algebraic equations they've got to know how and why they're going to be using these in the future. (Teacher A)

> They have a sense of humor... creative thinkers as well as logical thinkers. They are hugely enthusiastic about mathematics... they play with numbers, a lot!... One or two of them seem to be almost instinctively mathematical, they would know the answer almost as though it happened unconsciously... that was quite a new experience for me to have children who would just come up with an answer which was in their head like a flash and yet initially they couldn't say how they got it. (Teacher C)

> They have flexibility in their thinking... they think logically. (Teacher D)

Two of the teachers recognized the advanced thinking skills and ability to think in more abstract terms than their age peers.

> They usually grasp new concepts more quickly than other children in your class. (Teacher B)

> [Named student] would regularly be able to talk at quite a complex level about the question... if I had talked with other students they would not have been as abstract. In Piagetian terms she is at that abstract level already; she can talk like that. (Teacher K)

The difference in ability and achievement across the domains was recognized by both students and teachers. Two students noted that they did not achieve as well in geometry as in other strands and two teachers also recognized the differing types of mathematical

giftedness. The teachers explained:

> They usually do stand out in one way or another, some of them are gifted in only one area of mathematics; others are able to do a whole lot of mathematics that they enjoy. You'll get children who are amazing with visual patterns and some of the geometry things like rotation, translation etcetera but they may not be very good at addition, subtraction, multiplication and division. (Teacher B)

> There are those that have very good spatial skills and those that have very good mathematical computational skills. (Teacher K)

Perseverance with mathematical problems was regularly observed by one of the teachers who commented: "I'd come into class or if I was working with another group they'd have a whiteboard covered with numbers and diagrams, they worked out things, laughed and were so excited about mathematics." This perseverance with problems would continue through morning tea break or they would take a problem away and go on thinking about it and return the next day with other possibilities. "Even when we'd solved a problem they would be carrying on, working at it, thinking about it in a number of different ways." Another teacher also observed students returning to class at morning or afternoon tea to play mathematics games or puzzles or returning to work on their mathematics projects.

Three of the teachers noted the behavioral challenges that these students may present. For two of the three teachers it was not really an issue, whereas for another it was problematic.

> A lot of these gifted and talented kids are not very well behaved or hard workers. They have a sense of humor; you have to have your wits about you. (Teacher A)

> They're not sit-still kids. They're not kids who are going to sit still very long unless their brains are actually engaged. (Teacher I)

> Other teachers have identified them and said "I can't stand that kid", the kids who are challengers... it's definitely not always recognized as people see them as cheeky, rude and precocious, inappropriate and their jokes not a smart thing to say. But really, it's a very smart thing. (Teacher C)

> The majority of those kids can be quite demanding almost to the point where they are a little bit arrogant. If they don't rate the teacher, good kids will turn and blame the teacher for not doing well. (Teacher F)

Another, perhaps minor, characteristic observed by the teachers was in regards to presentation of student work. Overall these gifted and talented students did not present their work to a particularly high standard (work samples were collected from all students in the study). Some of the students commented that it was not important to them. They saw it as a teacher problem and were usually frustrated by teachers who demanded neat layout. A few of the Year 6 students cynically mentioned the teachers who like borders around project work. As one teacher stated; "generally their handwriting is terrible but

they are accurate too." The teachers in this study said that they were more interested in gaining insight to the students' thinking. However, some of the students expressed frustration at having to write down their thought processes and explained that answers sometimes just came into their heads and they really were not interested in how they got there but "it was right so who cares?" One parent explained that she tried to convince her son about the importance of being able to record his thinking; "he does know what he's doing but he's too lazy to write it down." Her child's response to that expectation was: "What's the big deal I know the answer. It didn't say 'show'. "

Other aspects of mathematics at school that, according to the students, confirmed their mathematical giftedness included: success in competitions; competence with basic facts; speedy computational skills; problem solving abilities; being able to work on "special projects"; or being given more or different work (than their class mates) to complete independently.

It is interesting to note that not all of the students would categorize themselves as gifted. Two of the students did not rate mathematics as their favorite subjects, yet, they had been identified by their schools as gifted and talented in mathematics. They were aware, based on test results and the students that they were working with, that they had been given this classification but did not identify themselves as gifted and talented in mathematics.

Conclusion, Discussion, and Implications

Characteristics of the mathematically gifted and talented students were described from the multiple sources of data. The parents described early signs of mathematical giftedness based on their experiences with other children in the family or their own knowledge and experience. They gave detailed accounts of the mathematical behaviors they observed in their children's play and talk. These early signs included aspects such as 'playing with numbers', a sense of symmetry in pattern making, and viewing the 'world through a mathematical lens' which are described in the literature (see for example, Gardner, 1983; House, 1987; Johnson, 1983; Straker, 1983). The parents described situations in both the home and early childhood education environments. In the home environment, the parents supported and encouraged that interest and fascination with things mathematical. Descriptions of atypical instances of mathematical play were passed on to the parents by the early childhood educators. However, none of the parents was aware of information about their children's special interests and abilities in mathematics being passed on, or taken into account as part of the transfer process to school.

The students showed an awareness of their mathematical giftedness when they were able to compare themselves with peers at school. The teachers articulated some

characteristics but no teacher described in depth the range of characteristics associated with the mathematically gifted and talented. Collectively, the teachers mentioned most indicators but their contributions were based on anecdotal evidence gained from teaching experience. None of the school policy documents supported the teachers in providing indicators of mathematical giftedness.

The notion of an appreciation for different types of mathematical giftedness was evident from student, parent, and teacher data. The parents were able to differentiate between early signs of spatial reasoning and arithmetic reasoning. The students, on the other hand, associated mathematical abilities with numeric reasoning and computational ability, although later in their schooling, especially when trialling for mathematics competition teams, they realised the importance of problem solving skills. These different types of mathematical giftedness were most clearly defined in the literature by Krutetski (1976). The focus from the students and teachers in this study was not on these differences. However, these are differences that had been apparent to some of the parents in their children's early childhood experiences and to two of the teachers. Recognition that students display differing types of mathematical giftedness has implications for the identification process and provision. With so many countries placing an increased emphasis on number, and that spatial giftedness has often been overlooked (Webb, et al., 2007) in the identification process, it is important that this view of mathematical giftedness is given attention.

So what happens to these children when they enter the formal educational system? There is a need for early and ongoing identification of mathematical giftedness that is inclusive of parent, student, and teacher nominations. Related to this, it is important that, there are opportunities for input from parents and early childhood educators regarding children's early development and interests in mathematics when they begin school. Similarly, as students' transition from one level of schooling to the next there needs to be opportunities for communication between teachers and parents. The teachers appreciated and recognized some of the characteristics of mathematical giftedness. Parents and students seemed content to take a "wait and see" approach in terms of identification and consequent provision of special programs. Early identification and acceleration did not occur for any of the students in the study. These two issues of early identification and acceleration deserve attention and debate; potentially young mathematically gifted students could be left isolated, frustrated, and bored.

Identification of mathematical abilities should be based on rating scales or observation checklists, inclusive of specific characteristics (as opposed to more generalized sets of behaviors). However, given the diversity of their individual behaviors, it is essential that we always remember that mathematically gifted and talented students are not a homogenous group. They share a cluster of similar behaviors, but as their teachers',

parents' and self-perceptions demonstrate, these are not universally shared by all. The information gathered during the identification process should be used as a means to an end, not an end in and of itself. In other words, the individual student's abilities and skills should create a baseline from which appropriate educational provisions are designed.

Given their mathematical views of the world, coupled with their unique abilities, gifted and talented students require qualitatively differentiated mathematics programs, not merely 'more of the same' learning opportunities. Opportunities to work alongside like-minded peers, for extended periods of time, in both enriched and accelerated programs should be provided.

Teachers of mathematically gifted and talented students may be aware of some of the characteristics of the mathematically gifted. However, there is a need for provisions of professional development and support, to ensure that teachers are able to identify and then develop these students' unique characteristics and abilities with an appropriate curriculum and learning environment.

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