Identifying and Sequencing of the Elementary Minor Concepts of the Highest Common Factor (H.C.F.) and the Lowest Common Multiple (L.C.M.) of Numbers

Alam, S. K. Samsul

Research Scholar, Centre for Mathematics Education, Department of Mathematics, Visva-Bharati University, Santiniketan, West Bengal 731235, India; Email: syamal 123@yahoo.co.in

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In this paper some attempts have been made:

- (a) To identify all the elementary minor concepts of the Highest Common Factor (H.C.F.) and the Lowest Common Multiple (L.C.M.), and
- (b) To find the sequential order of these elementary concepts. Total 385 elementary minor concepts have been identified and sequenced.

Keywords: common factor, common multiple, L.C.M., H.C.F.

ZDM Classification: B10, B70, C70 MSC2000 Classification: 97C90, 97C30

0. INTRODUCTION

The subject mathematics is inextricably related with our life. Considering its importance, mathematics is made a compulsory subject up to Grade 10 of schooling in almost all countries. But some inconsistencies are seen in the content of syllabus, text books and question papers. In this regard Singh (2000) said,

"The picture is very heterogeneous over different countries, in respect of both the educational set-up prevalent and the contents of the syllabus in mathematical sciences at different levels."

In the Indian context, "An Analysis of Various Boards' Results" conducted by National Council of Educational Research and Training (NCERT)¹ and edited by Buch (1991, p. 24) shows that the present scenario of this subject is not satisfactory. There are

¹ See http://www.ncert.nic.in/index.htm

many reasons behind it. Syllabus content or subject matter is one important aspect of them. Generally, in the process of framing syllabus, its content selection is made by arranging workshops, seminars or experts' opinion which is an empirical method and, often, it is found to be inconsistent with reality. In this regard Bhopa (2000, p. 44) said,

"The real problem lies in the lack of a successful formulation of a mathematics curriculum and its effective implementation."

In the traditional process of framing syllabus there is a possibility of developing conceptual gaps and it will be difficult for the students to grasp ideas if there is any learning gap in the syllabus. Student may develop apathy to that subject. It will be more harmful especially for the subject mathematics; if in particular, 'learning sequence' is not maintained properly. So in such subject like mathematics the conceptual gaps in the syllabus, text books and in teaching and learning system must be removed.

Moreover proper arrangement of basic concepts according to their hierarchy must have to be maintained to teach mathematics. In this regard, Pushpanadham (1998) said, "Mathematics is the subject which requires proper understanding of the concepts and their interrelations." Chilana (1984) said, "Mathematical concepts should be graded in such a way that there is a proper sequence in their growth and development." Hence to select content of any syllabus one should have to adapt more scientific method.

Only a few works have been done on this problematic area (2, 5, 23), but they are neither sufficient nor complete. But there are no such note worthy work so far to remove the conceptual gaps in teaching and learning methods, syllabus and in text books. Not much works have been done in this direction, especially in the Indian context. This initiates us to deal with this problem. We have adopted micro analysis technique such as "text book scanning processes" and "task analysis technique" for identifying and sequencing of basic concepts of mathematics.

It has been possible for us to identify 49 major concepts and their 10259 elementary minor concepts in arithmetic and algebra up to Grade 10. In this regard, it will be relevant to mention some of the contributions of the present author (such as 30 to 42) towards identification and sequencing of some basic concepts and framing of a model syllabus. In this paper, due to economy of space, a study on identifying and sequencing of the elementary minor concepts of the major concept "H.C.F. and L.C.M. of numbers" is presented.

The objective of the study is to identify different elementary minor concepts and their sequential order of "H.C.F. and L.C.M. of numbers."

2. DEFINITIONS

1. Major Concept and Minor Sub-Concept

A Major concept is an idea which is complete in it and is comprehended through a sequential process of step-by-step partial comprehension of its sub-concepts. Each partial comprehended idea is called a minor sub-concept of the concerned major concept.

Examples of major concepts are "addition of whole numbers", "subtraction of whole numbers", "and measurement of length", "fraction, decimal" etc.

For the major concept such as "Addition of whole numbers", its minor sub-concepts are "addition of one-digit numbers including zero where the sum is one-digit number", "addition of one-digit numbers where the sum is two-digit number" etc.

2. Elementary minor concept

A minor concept which cannot be split further will be called an "elementary minor concept."

For the major concept, say, "Addition of whole numbers", its elementary minor concepts are "vertical addition of two one-digit numbers except zero (through activities)", "vertical addition of two one-digit numbers except zero where the numbers are in numeral form(without activities)", "vertical addition of two one-digit numbers except zero where the numbers are in word form(without activities)" etc.

3. PROCEDURE

Procedure adopted has been discussed below

1. Collection of text books

The prescribed text books of mathematics from Grade 1 to Grade 10 of West Bengal Board of Primary Education (WBBPE)², West Bengal Board of Secondary Education (WBBSE)³, National Council of Educational Research and Training (NCERT) and other available books from the market were collected in the first stage.

(4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28)

See http://www.iers-ptti.com/ See http://www.wbbse.org/

2. Analysis of text books

These books were analyzed to identify "H.C.F. and L.C.M. of numbers" as a major concept and their elementary minor concepts of arithmetic mainly.

3. Task analysis of question papers

In the identifying and sequencing of basic concepts, task analysis technique has also been applied on arithmetical and algebraic problems of mathematics question papers of 10th standard of *Madhyamik Pariksha* (years 1999 and 2000) of WBBSE, Indian Certificate of Secondary Education (ICSE) Examination (years 2005 and 2006) of the Council for the Indian School Certificate Examinations, School Certificate Examination (years 1999 and 2000) of Visva-Bharati University.

4. Sequencing of concepts

The identified major concept and their elementary minor concepts were sequenced keeping in view the logical order of the subject and the psychological order of learners. The gaps in the concepts detected by the researcher were filled in by him at the initial stage.

5. Experts' opinions

The major concept and their elementary minor concepts with examples were given to experts for their comments. The experts were requested to add or omit or alter the sequence of concepts as they felt necessary.

Finally, the sequential form of elementary minor concepts of the major concept "H.C.F. and L.C.M. of numbers" incorporating the experts' opinion was developed.

4. SALIENT POINTS OF STUDY

The major concept "H.C.F. and L.C.M. of numbers" has been divided under 3 sub-concepts which are given in column 2 of Table 1. Each sub-concept has further been divided into different levels of elementary minor concepts. The total number of elementary minor concepts of the sub-concepts is also given in column 3 of Table 1. Total 385 elementary minor concepts have been identified. In this paper the details study of the sub-concept-1 which consists 339 elementary minor concepts will be presented.

Table 1.	H.C.F. a	and L.C.M.	of Numbers
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Column 1	Column 2	Column 3
Sl. no.	Sub-concept	Total number of elementary minor concept
1	H.C.F. and L.C.M. of whole numbers	339
2	H.C.F. and L.C.M. of fractions	23
3	H.C.F and L.C.M. of decimals	23
	Grand Total	385

Now the sub-concept 1 has been broken down into 13 minor sub-concepts which are given in Table 2.

Table 2. H.C.F. and L.C.M. of Whole Numbers

Column 1	Column 2	Column 3
Sl. no.	Minor sub-concept	Total number of elementary minor concepts
1.1	Divisible numbers	1
1.2	Factors	1
1.3	Multiples	3
1.4	Tests for divisibility	9
1.5	Prime numbers and composite numbers	2
1.6	Prime factorization	1
1.7	Common factors and common multiples	2
1.8	H.C.F. of whole numbers	66
1.9	H.C.F. involving compound quantities	90
1.10	L.C.M. of whole numbers	61
1.11	L.C.M. involving compound quantities	90
1.12	Properties of H.C.F. and L.C.M.	6
1.13	Problems involving relationship between H.C.F. and L.C.M. of whole numbers	7
	Grand Total	339

For the economy of space only each of the elementary minor concepts from 1.1 to 1.8 are presented below in detail. For the same reason some of the elementary minor concepts are elaborated and the rest are just mentioned. The notations Q for "Question" and A for "Answer" are used.

Detailing out of the minor sub-concepts

1.1. Divisible numbers	
Q. Fill in the blanks. A number is divisible by another if on dividing, the remainder is. 1.2. Factors	(A: Zero)
Q. Find all the factors of 6. 1.3. Multiples	(A: 1, 2, 3, 6)
1.3.1. Concept of multiple	
Q. Write the next three multiple 3, 6, 9, 12, 1.3.2. Concept of even number	(A: 15, 18, 21)
Q. Write the even numbers 4, 7, 11, 56, 81 1.3.3. Concept of odd number	(A: 4, 56)
Q. Write the numerals for all the odd numbers between 52 to 62	A: 53, 55, 57, 59, 61)
1.4. Tests for divisibility	
1.4.1. Test for divisibility by 2	
Q. Which of the following number is divisible by 2? 241, 302, 303 1.4.2. Test for divisibility by 3	(A: 302)
Q. Which of the following number is divisible by 3? 111, 101, 100 1.4.3. Test for divisibility by 5	(A: 111)
Q. Which of the following numbers are divisible by 5? 120, 121, 123, 125 1.4.4. Test for divisibility by10	(A: 120, 125)
Q. Which of the following number is divisible by10? 65, 210,155 1.4.5. Test for divisibility by 6	(A: 210)
Q. Which of the following number is divisible by 6? 122, 222, 221, 212 1.4.6. Test for divisibility by 9	(A: 222)

Q. Which of the following number is divisible by 9?

1.4.7. Test for divisibility by 4

Q. Which of the following number is divisible by 4?
124, 111, 101
(A: 124)

1.4.8. Test for divisibility by 11

1.4.9. Test for divisibility by other than 2, 3, 5, 10, 6, 9, 4, 11

1.5. Prime numbers and composite numbers

1.5.1. Concept of prime numbers

1.5.2. Concept of composite numbers

1.6. Prime factorization

Q. Find the prime factorization of 12

A:
$$2)\frac{12}{6}$$
2) $\frac{6}{3}$
The prime factors are 2, 2, 3

1.7. Common factors and common multiples

1.7.1. Common factors

Q. Find the common factors of the two given numbers 15, 25

$$\begin{bmatrix} A: & \text{Factors of 15 are 1, 3, 5, 15} \\ & Factors of 25 are 1, 5, 25 \\ & \therefore Common factors are 1, 5 \end{bmatrix}$$

1.7.2. Common multiples

Q. Find the first two common multiples of the numbers 6, 8

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A: Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 48, ...

∴ Common multiples are 24, 48
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- 1.8. H.C.F. of whole numbers its sub-concepts are given below
- 1.8.1. Concept of H.C.F.
- Q. What is the full form of H.C.F.? (A: H.C.F. = Highest common factor)
- 1.8.2. H.C.F. of two whole numbers where both numbers are below or equal to 20
- 1.8.3. H.C.F. of two whole numbers where at least any of the numbers are greater than 20
- 1.8.4. H.C.F. of three whole numbers where all three numbers are below or equal to 20
- 1.8.5. H.C.F. of three whole numbers where at least any of the numbers are greater than 20
- 1.8.6. H.C.F. of more than three whole numbers where all numbers are below or equal to 20
- 1.8.7. H.C.F. of more than three whole numbers where at least any of the numbers are greater than 20
- 1.8.8. When H.C.F. is given and other necessary conditions are given, determine the numbers.
- 1.8.9. Properties of H.C.F.

The concept 1.8.2 and only their elementary minor concepts are given below in the following way. Examples and expected correct answer are omitted to minimize the space. Similar concepts from 1.8.3 to 1.8.7 and their elementary minor concepts are given in the bracket to avoid the repetition.

The minor sub-concepts of the concept 1.8.2: H.C.F. of two whole numbers where both numbers are below or equal to 20

- 1.8.2.1. All factors method (1.8.3.1, 1.8.4.1, 1.8.5.1, 1.8.6.1, 1.8.7.1)
- 1.8.2.2. Prime factors method (1.8.3.2, 1.8.4.2, 1.8.5.2, 1.8.6.2, 1.8.7.2)
- 1.8.2.3. Continued division method (1.8.3.3, 1.8.4.3, 1.8.5.3, 1.8.6.3, 1.8.7.3)

The elementary minor concepts of the concept 1.8.2.1: All factors method

1.8.2.1.1. Finding H.C.F. (1.8.3.1.1, 1.8.4.1.1, 1.8.5.1.1, 1.8.6.1.1, 1.8.7.1.1)

Q. Find the H.C.F. of 8 and 20

A: Factors of 8 are 1, 2, 4, 8

Factors of 20 are 1, 2, 4, 5, 10, 20

Here common factors are 1, 2, 4

Out of these, 4 is the greatest

∴ The H.C.F. of 8 and 20 is 4

1.8.2.1.2. Solution of the given problem and writing proper answer

$$(1.8.3.1.2, 1.8.4.1.2, 1.8.5.1.2, 1.8.6.1.2, 1.8.7.1.2)$$

Q. Find the greatest number that can divide 16 and 20 leaving no remainder in each case.

A: Factors of 16 are 1, 2, 4, 8, 16

Factors of 20 are 1, 2, 4, 5, 10, 20

Here common factors are 1, 2, 4

Out of these, 4 is the greatest

∴ The greatest number 4 that can divide 16 and 20 leaving no remainder in each case

1.8.2.1.3. Formation of the problem and working out its solution

$$(1.8.3.1.3, 1.8.4.1.3, 1.8.5.1.3, 1.8.6.1.3, 1.8.7.1.3)$$

Q. Make the problem involving H.C.F. using the following numbers and then solve. 6, 18

A: Find the greatest number that can divide 6 and 18 leaving no remainder in each case.

Factors of 6 are 1, 2, 3, 6

Factors of 18 are 1, 2, 3, 6, 9, 18

Here common factors are 1, 2, 3, 6

Out of these, 6 is the greatest H.C.F. of 6 and 18 is 6

.. The greatest number 6 that can divide 6 and 18 leaving no remainder in each case

The elementary minor concepts of the concept 1.8.2.2: Prime factors method

1.8.2.2.1. Finding H.C.F. (1.8.3.2.1, 1.8.4.2.1, 1.8.5.2.1, 1.8.6.2.1, 1.8.7.2.1)

1.8.2.2.2. Solution of the given problem and writing proper answer

$$(1.8.3.2.2, 1.8.4.2.2, 1.8.5.2.2, 1.8.6.2.2, 1.8.7.2.2)$$

1.8.2.2.3. Formation of the problem and working out its solution

$$(1.8.3.2.3, 1.8.4.2.3, 1.8.5.2.3, 1.8.6.2.3, 1.8.7.2.3)$$

The elementary minor concepts of the concept 1.8.2.3: Continued division method

- 1.8.2.3.1. Finding H.C.F. (1.8.3.3.1, 1.8.4.3.1, 1.8.5.3.1, 1.8.6.3.1, 1.8.7.3.1)
- 1.8.2.3.2. Solution of the given problem and writing proper answer

$$(1.8.3.3.2, 1.8.4.3.2, 1.8.5.3.2, 1.8.6.3.2, 1.8.7.3.2)$$

- 1.8.2.3.2.1. Simple sum (1.8.3.3.2.1, 1.8.4.3.2.1, 1.8.5.3.2.1, 1.8.6.3.2.1, 1.8.7.3.2.1)
 - Q. Find the greatest number that can divide 4 and 14 leaving no remainder in each case.
- 1.8.2.3.2.2. Complex sum (1.8.3.3.2.2, 1.8.4.3.2.2, 1.8.5.3.2.2, 1.8.6.3.2.2, 1.8.7.3.2.2)
 - Q. Find the greatest number that can divide 5 and 19 leaving 1 as remainder in each case
- 1.8.2.3.3. Formation of the problem and working out its solution

$$(1.8.3.3.3, 1.8.4.3.3, 1.8.5.3.3, 1.8.6.3.3, 1.8.7.3.3)$$

The elementary minor concepts of the concept 1.8.8: When H.C.F. and other necessary conditions are given, determining the numbers

- 1.8.8.1. H.C.F. of two numbers is given and their sum is given, determining the numbers.
- 1.8.8.2. H.C.F. of two numbers is given and their difference is given, determining the numbers
- 1.8.8.3. H.C.F. of two numbers is given and their product is given, determining the numbers

The elementary minor concepts of 1.8.9: Properties of H.C.F.

- 1.8.9.1. H.C.F. of two or more numbers is not altered if the individual number is multiplied by different multipliers which have no common factor.
- 1.8.9.2. H.C.F. of two numbers is same as the H.C.F. of smaller number and the number which is the remainder when greater number is divided by the smaller number.

5. CONCLUSION

- i) This methodology will help to mark different types of gaps. Total 385 elementary minor concepts have been identified which is sufficient in the curriculum. It has been noticed that the significant numbers of conceptual gaps are present in the different Boards' syllabus and in their prescribed text books. This method can also be applied to find gaps in any syllabus or any text book.
- ii) As the subject matter is gapless, a proper syllabus can be prepared.
- iii) It will also help to write good text books,
- iv) The quality of teaching and learning process will be more improved.
- v) One can judge learning gaps in any syllabus, text book and in teaching and learning process.
- vi) Backward learners can be found easily and remedial method for them can be applied.
- vii) Special interest for mathematics can be enhanced which will be helpful for entire science education.

6. FURTHER STUDY

- i) This study may be taken on all educational subjects like English, Bengali, Geography, History and other Science subjects.
- ii) This study may also be taken on other mathematical sciences such as Statistics, Computer Science etc.

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