Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 13, No. 1, March 2009, 23–30

韓國數學敎育學會誌 시리즈 D:
<數學敎育硏究>
제 13 권 제 1 호 2009 년 3 월, 23–30

Open-Ended Questions and Creativity Education in Mathematics[1]

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(Received July 12, 2008. Accepted February 6, 2009)

How to promote creativity for all students in mathematics education is always a hot topic for mathematics educators. Based on the theory study and practice in the project "Open-ended Questions in Mathematics" granted by Ministry of Basic Education Curriculum Study Center in China, the paper reported the effect of "Open-ended Questions in Mathematics" on the way to change the development of thinking ability, to inspire students to develop thinking flexibility, to expand their imagination, to stimulate their interest in learning, and to foster students' creativity.

Keywords: open-ended questions in mathematics, quality education, stimulate interest, creativity
ZDM Classification: D30, D50
MSC2000 Classification: 97D30, 97D50

1. INTRODUCTION

I attended the ICMI-China Regional Conference on Mathematical Education held at the East China Normal University, Shanghai, during Aug. 15–21, 1994. I was attracted by Dr. Pingtiangensan's presentation and discussed with him in detail after his presentation.

[1] This article is an extended version of the paper (Li, 2008) presented at Topic Study Group 6 (Activities and Programs for Gifted Students) of the 11th International Congress on Mathe matical Education (ICME-11) held at the Universidad Autonoma de Nuevo Leon (UANL), Monterrey, Mexico, July 6–13, 2008.

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We exchanged our latest books after the discussion. There are many interesting topics in his book. I introduced some topics in his book "For Primary School Teacher" into China's Primary School Mathematics Education Community and attracted a lot of attention.

The Basic Education Curriculum Center in China provided me a grant for studying "Open-ended Questions in Mathematics for Primary School Students." As the leader of Mathematics Education Research Institute in Dezhou College, I focused on the open-ended mathematics questions study for more than two years. As a reward, we published two books titled "Mathematics Questions for Inspiring Interests in Mathematics." One book is for primary school students from first grade to third grade, published in 2004. The other book is for primary school students from forth grade to sixth grade, published in 2005.

2. REQUIREMENT FOR OPEN-ENDED QUESTION STUDY

- Dr. Zhang presented me one of the latest editions of his book "Mathematics Quality Education Design" (*cf.* Zhang, 1996) when I visited him in summer 1996. There is one paragraph talking about how to use open-ended questions to promote creativity in mathematics and gives some examples of open-ended questions used in middle school mathematics education in foreign countries. For example, "If the distance between A and a school is 5,000 meters, the distance between B and the school is 10, 000 meters, what is the distance between A and B?"
- This question appears as simple as an arithmetic question for primary school students. However, it actually includes many contents such as adding nature numbers, subtracting rational numbers, geometry orbit of a circle, point distance, circle parameters, complex number subtraction and etc. The question is open-ended and also can be calculated. The conditions for this open-ended question can be arbitrarily added according to your mathematics knowledge. The open-ended questions usually leave a lot of imagine space for readers. At the same time, they can also actively involve the readers and inspire them thinking from different points of view.

On the contrary, the mathematics questions in China are restricted to exam questions like mathematics questions in College Entry Exam and all kinds of Mathematics Competition exams. The questions in those exams often have common characteristics such as sufficient conditions, specific process and unique conclusions. This type of questions is necessary for mathematics education, but it is not enough to promote students' creativity and open thinking style.

As Dr. Zhang said, in the 1980's, the basic science education attracted a lot of attention with the development and application of science and technology in Japan. They paid a lot of attention to its own creativity instead of just imitate the West in science and technology. In education, they highly advocate creative thinking training and promote creativity. The emergence of Open-ended Questions is the natural fruit of this kind of development.

The economy in China developed greatly after the reform and open policies took action. The talent competition is very fierce in the market, which gives new requirements for educators. In 1990, China proposed "Quality Education." The Quality Education marked the new era of Education in China. With the development of teaching reform in education, the reform of education material is natural based on the requirement of talents who can express their opinion and have creativity ability. In this situation, the study of open-ended questions is very necessary.

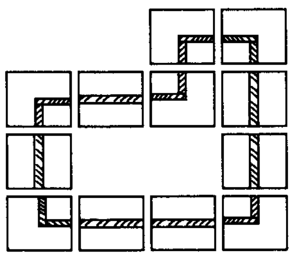
I mailed our newly published book "Mathematics Questions for Inspiring Interests in Mathematics" to Zaiping Dai, an expert in the area of Mathematics Education in China. He thought the study of Open-ended Questions is valuable for Quality Education especially for promoting creativity for Chinese students.

3. STUDIES IN OPEN-ENDED QUESTION

One characteristic of Open-ended Question is multiple answers for the same question. To have perfect answers for Open-ended Questions, one need to both master the knowledge from textbook and think actively and imagine boldly. Therefore, open-ended questions are helpful to promote students' creativity and make them smarter.

An example of Open-ended Question: There are many square cards with two different strips like the following. Take 12 cards from them and see what kind of closed graphics can be made by the strips on those cards?

Answers:
For example：

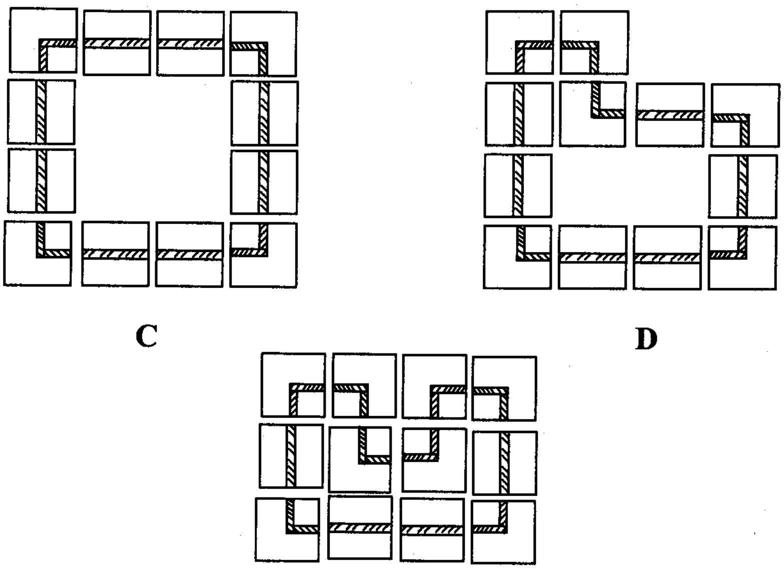


First, we should let our students understand that the length of strip in each of those cards is fixed. Thus, it is an activity of making a closed graphics using square cards with same length strips. As Figure1 shows, the strip in card A is a straight line. Assume the length of strip on A is one unit, then the acute angle of strip on card B is composed of "half unit + half unit = one unit". According to above reasoning, we will know that the strip length on card A is the same as the strip length on card B. It also means the strip length is always one unit no matter which card you are using. The circumference of the closed graphics is fixed value 12 when you use 12 such cards to make a "Closed Graphics." However, the areas are different when the closed graphics are different. Alternatively, the closed graphics could be different even the areas are the same. The purpose of this activity is making students realize above differences.

A B

Figure 1. Two square cards with same length strips

It is shown in Figure 2 that the areas are all different for different closed graphics.



C D

E

Figure 2. Three closed graphics with different areas

The areas are given in Table 1 for different closed graphics in Figure 2.

Table 1. The areas for closed graphics in Figure 2.

Graphics	C	D	E
Area (units)	9	7	5

Figure 3 shows different closed graphics with the same area. Closed graphics E and closed graphics F have the same area, which is 5, although they are different closed graphics.

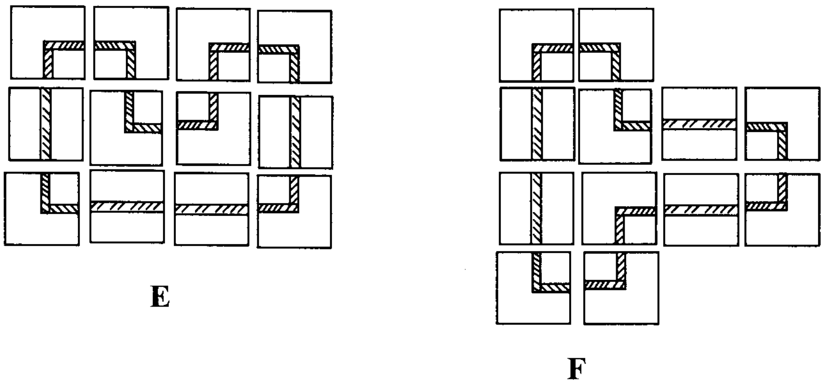


Figure 3. Different closed graphics with same area

In addition, it is necessary to let students have the ability to distinguish "closed graphics" and "unclosed graphics" for first grade students at the beginning of teaching. Figure 4 shows examples of closed graphics and unclosed graphics.

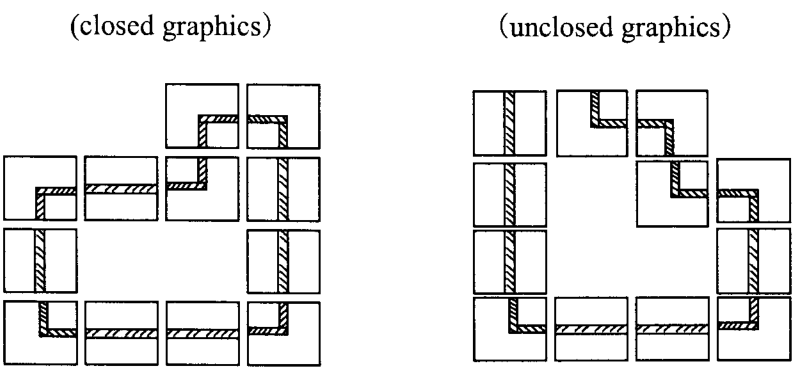


Figure 4. Examples for closed graphics and unclosed graphics

These hands-on activities not only increased students' interest in learning mathematics but also made them realize many mathematics principles. These activities will greatly benefit the development of students' mathematics thinking skills.

Another open-ended question for second grade student:

Please fill in appropriate numbers in □ and do the calculation.

$$\begin{array}{r} \square\square \\ +\square\square \\ \hline \square\square\square \end{array}$$

It is a question about adding two double-digit numbers with the result of a triple-digit number. There are 8,100 formulas if you list all the sum of two double-digit numbers from "10 + 10" to "99 + 99." Among those formulas, there are 4,860 formulas can get a triple-digit number results from "10 + 90 = 100" to "99 + 99 = 198."

If the students can write down all of those formulas, they actually practiced the sum calculation many times without awareness.

When listing those formulas, the students can list all of them without losing one if they order those formulas by certain criteria.

The activity itself is a mathematics activity. At the beginning, students might list those formulas at random. However, they will find the law gradually.

Students generally find many formulas such as "10 + 90, 11 + 89, … , 90 + 10" etc. They all have the same answer 100 in the formula. Some students can find the law after listing a lot of formulas: Fix the first number in the additive formula, from "10 + 90, 10 + 91, 10 + 92, … , 10 + 98, 10 + 99" to "89 + 11, 89 + 12, … , 89 + 98, 89 + 99", the number of all types formulas are 10, 11, … , 89. From " 90 + 10, 90 + 11, … , 90 + 98, and 90 + 99 " to " 99 + 10, 99 + 11, … , 99 + 98, and 99 + 99 ", the number of formulas are all 90 for those formulas belong to these 10 categories. Therefore, it is easy to get the total number of all formulas: $(10+89)\times 80\div 2+90\times 10=4860$.

To answer this type of questions, students usually can find out some formulas that satisfy the condition. At this time, if teacher can ask them to find out the total number of formulas that satisfy the condition and list all of them, students can think harder from different point of view and apply the "classification theory" and the knowledge about serial adding unwittingly.

Another example for third grade students: Team Marathon.

Table 2 is the result of team marathon with four persons in each team.

Please give the rule for determining the rank of each team.

Different rules will give different ranks for each team. The purpose of this activity is applying the arithmetic knowledge to figure out the determining rule.

Table 2. The result of team marathon

Team	**Rank of team members**			
Team A	4	7	10	13
Team B	2	6	12	14
Team C	1	8	9	16
Team D	3	5	11	15

For example, if we use the best rank in each team as the rule for ranking the teams, then team C ranks first, team B ranks second, team D ranks third and team A ranks fourth. If using the sum of all players' ranks in each team as the rule for ranking the team, then these four teams have the same rank, and so on. Table 3 summaries some rank methods and corresponding ranks for each team.

Table 3. Summary of some rank methods and corresponding ranks for each team

Rules for determine ranks	**Rank of each team**
Best player's rank in each team	Team C 1st, Team B 2nd, Team D 3rd, Team A 4th
Worst player's rank in each team	Team A 1st, Team B 2nd, Team D 3rd, Team C 4th
Sum of all player's rank in each team	Same rank for all teams
Median player's rank in each team	Team D 1st, Team A and C 2nd, Team B 3rd

Finding out the answers from different point of view, one can think out many ways to determine the ranks.

4. CONCLUSIONS AND FUTURE WORK

For a long time, the exercises in mathematics textbooks are designed for the purpose that students can remember the mathematics theorems and conclusions, and familiar with the arithmetic process. Therefore, students only focus on remembering things instead of involving in the learning process and thinking critically. From the examples of those Open-ended Questions, it can be seen that designing the Open-ended Questions according to the age and cognitive level of primary school students can greatly inspire their wisdom, improve their critical thinking and reasoning skills, challenge their imagination, stimulate their interest in learning mathematics and foster their creativity. In the future, we will continue our efforts in the study of Open-ended Questions in mathematics find out more interesting and inspiring open-ended questions for different levels of students.

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