Performances of Simple Option Models When Volatility Changes

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ABSTRACT

In this study, the pricing performances of alternative simple option models are examined by creating a simulated market environment in which asset prices evolve according to a stochastic volatility process. To do this, option prices fully consistent with Heston[9]'s model are generated. Assuming this prices as market prices, the trading positions utilizing the Black-Scholes[4] model, a semi-parametric Corrado-Su[7] model and an ad-hoc modified Black-Scholes model are evaluated with respect to the true option prices obtained from Heston's stochastic volatility model. The simulation results suggest that both the Corrado-Su model and the modified Black-Scholes model perform well in this simulated world substantially reducing the biases of the Black-Scholes model arising from stochastic volatility. Surprisingly, however, the improvements of the modified Black-Scholes model over the Black-Scholes model are much higher than those of the Corrado-Su model.

Key words : Option Models, Skewness, Kurtosis, Stochastic Volatility

1. Inroduction

Over the three decades after the seminal paper by Black and Scholes[3], alternative option models have been put forth. The driving force of these theoretical developments is empirical biases associated with the Black-Scholes model[15].

Perhaps the most widely recognized violation of the assumption of the Black-Scholes model is that the price volatility of the underlying asset is not constant[6]. As a result, a number of researchers have developed option-pricing models that allow changing volatility[5][10]. A notable advance in this vein of research is the development by Heston[9] of a closed-form option pricing formula for the case in which volatility follows a square-root diffusion. Building on Heston[9], option pricing formula that allow price jumps and stochastic interest rates have been developed[1][2]. In addition, in the discrete time framework, generalized autoregressive conditional heteroskedasticity option models (GARCH) have also been proposed[8].

However, a major impediment to using an option model that allows changing volatility comes from the difficulty of estimating and calibrating parameters required for such a model. For example, the Heston's model[9] requires five parameters to fully specify the volatility process, whereas the Black-Scholes model, by assuming constant volatility, has only one. Therefore, to make use of the Heston model, one faces the

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problem of estimating these five parameters and incurring the resulting estimation risk. Bakshi, Cao, and Chen[1] show that estimating the parameters of a volatility process often results in unrealistic and unstable values.

On the other hand, one method preferred by option professionals involves using the Black-Scholes formula to back out implied volatilities and then pricing making use of these implied volatilities. Often, the implied volatilities are smoothed by fitting to a (typically quadratic) polynomial of strike prices. Though ad hoc and without theoretical foundation, this modified version of the Black-Scholes model has naturally come into practice to adjust non-normal characteristics of the underlying asset returns revealed by implied volatilities.

An alternative approach offering the virtue of simplicity, but with theoretical foundation has been suggested by Jarrow and Rudd[11] and Corrado and Sue[7]. They propose option models that assume that the underlying asset price has a distribution known only through its moments. Specifically, they directly adjust Black-Scholes formula for non-normal skewness and kurtosis of the underlying asset distribution. Their models require only two additional parameters beyond volatility.

While an ad-hoc modified version of Black-Scholes model and a skewness and kurtosis adjusted semi-parametric model are simple and easy-to-implement, an interesting issue is whether and to what degree these two alternatives provide a substitute for a complete stochastic volatility model. The existing literature has shown that either a modified Black-Scholes model or a skewness and kurtosis adjusted model can effectively reduce the original Black-Scholes model's price deviations and be used as a workable solution even under stochastic volatility[7][12][13][14]. However, their comparative performances have not yet been examined.

In this study, the pricing performances of the modified Black-Scholes model and the Corrado-Su model are examined by Monte Carlo simulation. The Monte Carlo approach is a valuable and flexible computational tool in modern finance. With the aid of IT revolution, epitomized by the increased availability of powerful computers, this method has allowed a variety of numerical methods extensively used in financial industry. This study utilizes this method to examine the validity of using alternative option models when asset volatility is stochastic. Indeed, if an ad-hoc model based on the Black-Scholes formula can effectively performs on par with an exact stochastic volatility model or a skewness and kurtosis adjusted model, we might then understand why market participants still overwhelmingly prefer such a method to value and hedge options. Therefore, the results of the study may have some important implications for options trading and valuations as well as its related digital financial industries.

The simulation proceeds as follows:

First, simulated security prices consistent with the stochastic volatility process specified by Heston[12] are generated. At the same time, option prices are computed using the Heston[12] formula to produce true option prices. Market participants accept these prices as given, but do not know the exact process generating these prices.

Second, option traders are assumed to price their positions utilizing either the Black-Scholes model, the modified Black-Scholes model and the Corrado-Su model until option maturities. Their trading positions are then evaluated with respect to true prices obtained from Heston's stochastic volatility model to examine to what degree these models approximate the performance of the Heston model in this simulated world.

This study is organized as follows. Section 2 introduces the alternative option pricing models employed in this study. Section 3 briefly examines the empirical performance of these models utilizing a data set published in Rubinstein[16]. Section 4 explains Monte Carlo simulation design and presents simulation results. Conclusions are presented in the final section.

2. Option Models

2.1 The Black-Scholes model

The Black and Scholes (BS) model assumes that the underlying security price St, follows a geometric Brownian motion diffusion process as specified in equation (1). The parameters μ and $\sqrt{v}$ designate the constant drift and volatility assumptions of the model, and dz represents a Wiener process for which $E(dz)=0$ and $E(dz^2)=dt$.

$$dS=\mu S\,dt+\sqrt{v}S\,dz \qquad (1)$$

Black and Scholes[4] then derive their famous formula for the arbitrage-free price of a European call option on a non-dividend paying security:

$$C_t=S_tN(d_1)-Ke^{-r(T-t)}N(d_2)$$
$$d_1=\frac{ln(S_t/K)+(r+v/2)(T-t)}{\sqrt{v(T-t)}} \qquad (2)$$
$$d_2=d_1-\sqrt{T-t}$$

2.2 Heston's stochastic volatility model

Heston's stochastic volatility (SV) model generalizes the Black-Scholes formula to allow a stochastic volatility process, in which the security price S_t and the return variance v_t are assumed to follow this joint diffusion process:

$$dS_t=\mu S_tdt+\sqrt{v_t}dz_1$$
$$dv_t=k(\theta-v_t)dt+\sigma\sqrt{v_t}dz_2 \qquad (3)$$

The security price diffusion in equation (3) is similar to that specified in equation (1) except that variance v_t itself follows a diffusion process driven by the Wiener process dz_2 with a "volatility of volatility" parameter σ, with reversion to a mean value of θ at the rate κ. The joint diffusion process in equation (3) allows a correlation between the Wiener diffusions dz_1 and dz_2, such that $E(dz_1dz_2)=\rho$.

Heston[9] derives the price of a European call option on a non-dividend paying security when the underlying price and its return variance are driven by the joint diffusion process specified in equation (3). His closed-form formula is:

$$C_t=S_tP_1-Ke^{-r(T-t)}P_2 \qquad (4)$$

where $P_j(j=1,2)$ are conditional probabilities:

$$P_j(x,v,t;ln(K))=\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}Re[\frac{e^{i\phi ln(K)}f_j(x,v,t;\phi)}{i\phi}]d\phi$$

and where

$$f_j(x,v,t;\phi)=e^{C_j(T-t;\phi)+D_j(T-t;\phi)v+i\phi x}$$
$$C_j(T-t;\phi)=r\phi i(T-t)+\frac{a}{\sigma^2}\times\{(b_j-\rho\sigma\phi i+d_j)(T-t)-2ln(\frac{1-g_je^{d_j(T-t)}}{1-g_j})\}$$
$$D_j(T-t;\phi)=\frac{b_j-\rho\sigma\phi i+d_j}{\sigma^2}(\frac{1-e^{d_j(T-t)}}{1-g_je^{d_j(T-t)}})$$
$$g_j=\frac{b_j-\rho\sigma\phi i+d_j}{b_j-\rho\sigma\phi-d_j}$$
$$d_j=\sqrt{(\rho\sigma\phi i-b_j)^2-\sigma^2(2u_j\sigma i-\phi^2)}$$

In the above equation, $x=ln(S)$, $u_1=1/2$, $u_2=-1/2$, $a=\kappa\theta$, $b_1=\kappa+\lambda-\rho\sigma$, $b_2=\kappa+\lambda$ where λ is a volatility risk premium. Note that, in addition to the Black-Scholes model, the Heston formula has four additional unknown parameters: mean of variance θ, reversion rate κ, volatility of volatility σ, and correlation ρ.

2.3 Modified Black-Scholes Model

The modified Black-Scholes (MBS) model is an ad-hoc method widely used among options professionals in various forms. Essentially, the method accepts market option prices as given and then calculates an implied volatility for each observed option price. Specifically, the implied volatility, IV(K) for a call option with strike price K is the volatility value that equates the observed market price of the call C(K) to the Black-Scholes formula price given in equation (2) when IV(K) is substituted for the volatility parameter $\sqrt{v}$. The specific volatility smile method followed here involves estimating the structural form of the implied volatility using a quadratic regression. The volatility value for a call option with strike price K is then obtained from the regression equation to be used as an input for the Black-Scholes model.

2.4 Corrado-Su Model

Corrado and Su (CS) model computes option prices when the distribution of the underlying asset returns at option expiration departs from a normal distribution by directly adjusting non-noraml skewness and kurtosis to the Black-Scholes formular as follows:

$$C_t = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3) Q_4 \qquad (5)$$

where CBS is the Black-Scholes option price in equation (2) and

$$Q_3 = \frac{1}{3!} S_t \sqrt{v(T-t)}[2\sqrt{v(T-t)}n(d) + vN(d)]$$

$$Q_4 = \frac{1}{4!} S_t \sqrt{v(T-t)}[d^2 - 1 - 3\sqrt{v(T-t)} \times (d - \sqrt{v(T-t)})n(d) + (v(T-t))^{3/2}N(d)]$$

In equation (5), all parameters are equally defined as those of the Black-Scholes model except for the additional parameters μ_3 and μ_4 which represent non-normal skewness and kurtosis of unknown distribution of the underlying asset returns. The Corrdo-Su model has three price distribution parameters: $\sqrt{v}$, μ_3 and μ_4

3. Empirical Analysis

3.1 Call option data and Implied volatility

As an initial illustration of deviations of Black-Scholes option prices from actual market prices, I make use of S&P 500 index call options data originally published and presented in Figure 4 of Rubinstein[16]. These data represent observed June 1990 prices for call options on the S&P 500 index expiring in 164 days.

These data are presented here in <Table 1>, in which the first column lists strike prices, while the second and third columns report the corresponding bid and ask option prices. The fourth column contains averages of these bid-ask prices. Finally, the last column shows Black-Scholes implied volatilities (IV) obtained from the averages of the bid-ask prices.

〈Table 1〉 Call Prices and Implied Volatilities

Strike Prices	Call Prices		Bid-ask Average	IV
	Bid	Ask		
250	109.47	109.71	109.59	28.0
275	86.66	86.71	86.19	25.6
300	63.00	64.04	63.52	22.9
325	42.00	42.75	42.38	20.1
330	37.97	38.60	38.29	19.6
335	34.01	34.64	34.36	19.0
340	30.10	30.73	30.42	18.4
345	26.45	27.13	26.79	17.8
350	22.79	23.48	23.14	17.2
355	19.32	19.88	19.60	16.7
360	15.98	16.54	16.26	16.1
365	13.13	13.75	13.44	15.5
370	10.52	11.15	10.84	14.9
375	8.37	9.12	8.75	14.3
380	6.65	7.40	7.03	13.7
385	4.91	5.60	5.26	13.1

As Rubinstein emphasizes, if the Black-Scholes model is correctly specified essentially the same implied volatility should be observed across all strike prices. However, as shown in Table 1 the implied volatilities produced by the Black-Scholes formula exhibit systematic biases. Specifically, the implied volatilities decrease monotonically from 28 percent at a strike price of 250 to 13.1 percent at a strike price of 385.

3.2 Fitting alternative option models

Parameters of the Black and Scholes (BS), Heston (SV), and Corrado-Su (CS) models can be estimated from the data in Table 1 using non-linear regressions. Let Ω denote the vector of parameters to be estimated for a particular option pricing model. For example, for the BS model, $\Omega=\{\sqrt{v}\}$ for the CS model, $\Omega=\{\sqrt{v}, \mu_3, \mu_4\}$ and $\Omega=\{\sqrt{v}, \kappa, \theta, \sigma, \rho\}$ for SV model.

Parameters are estimated by minimizing the following sum of squared deviations between observed option prices and prices generated by the parameter set:

$$Min \sum_{j=1}^{N} (C_j^{obs} - C_j^{M}(\Omega))^2 \qquad (6)$$

In equation (6), C_j^{obs} represents the jth observed call price, and $C_j^{M}(\Omega)$ represents the jth call price specified by the parameter set Ω. <Table 2> presents estimated option-implied parameters obtained using equation (6) and the options data from <Table 1>.

〈Table 2〉 Option Implied Parameters

Model	Parameters	Estimates
BS	$\sqrt{v}$	0.173
	x	0.000
	θ	0.033
SV	σ	0.521
	ρ	-0.556
	$\sqrt{v}$	0.181
	μ_3	-1.253
CS	μ_4	3.188
	$\sqrt{v}$	0.162

<Table 3> contains theoretical values for the SV model, the CS model, and the BS model calculated using option implied parameters along with market option prices. In addition, fitted values from the MBS model obtained by substituting implied volatilities from the Black-Scholes model are presented. The root mean square errors (RMSE) of the pricing errors calculated from equation (7) immediately below are also provided at the bottom of <Table 3>.

$$RMSE = \sqrt{\frac{1}{N} \sum_{j=1}^{N} (C_j - C_j(\Omega))^2} \qquad (7)$$

According to the goodness of fit measured by RMSE, the SV model ranks first, the CS model second, the MBS model third, while the BS model ranks last. This ranking is not unexpected, as the SV model has the most free parameters to fit to the data, while the BS model has the least. However, differences in RMSE among the first three models are small when compared to the RMSE error of 1.61 for the BS model.

〈Table 3〉 Goodness of Fits for Option Models

Strike Prices	Call Prices	BS	SV	CS	MBS
250	109.59	109.12	109.70	109.21	109.58
275	86.19	85.16	86.39	85.87	86.23
300	63.52	61.63	63.73	63.78	63.55
325	42.38	39.89	42.25	42.51	42.14
330	38.29	35.96	38.17	38.36	38.10
335	34.36	32.22	34.19	34.29	34.17
340	30.42	28.68	30.33	30.34	30.36
345	26.79	25.37	26.60	26.53	26.69
350	23.14	22.28	23.03	22.95	23.18
355	19.60	19.44	19.65	19.54	19.86
360	16.26	16.85	16.49	16.41	16.73
365	13.44	14.50	13.58	13.57	13.83
370	10.84	12.39	10.98	11.03	11.17
375	8.75	10.51	8.71	8.79	8.79
380	7.03	8.85	6.79	6.86	6.70
385	5.26	7.40	5.23	5.23	4.91
RMSE		1.61	0.15	0.19	0.24

4. Simulation Experiments

The discussion above suggests that the SV model can largely correct for the empirical limitations of the BS model. This is consistent with the existing literature[1].

However, despite the improvements offered by a stochastic volatility option pricing model, it is quite challenging for option traders to make use of SV class models. This arises from the difficulties associated with parameter estimation. Direct estimation from observed prices is typically unreliable, though it is possible to extract option implied parameters via non-linear least squares as I did above. But even in this case, estimated parameters often exhibit unrealistic and unstable values over time[1].

In this section of the paper, a Monte Carlo simulation is used to examine the pricing performance of the BS model, the CS model, and the MBS model when the underlying asset price is assumed to follow the stochastic volatility process defined in equation (3). The objective of these simulations is to examine the extent to which theses alternative models compete with the performance of an exact Heston model. In these simulations the BS model is misspecified as it does not incorporate a stochastic volatility process, while the CS model and the MBS model represent easy-to-implement alternatives to the BS model.

4.1 Experimental simulation design

Each simulation experiment is based on a time series of security prices and variances, i.e., S_t and v_t, respectively, simultaneously generated as follows, in which z_{1t} and z_{2t} are independent standard normal variables and Δt corresponds to the passage of a single trading day, i.e., Δt = 1/252.

$$S_t = (1+\mu)S_{t-1}\triangle t + \sqrt{v_t \triangle t}S_{t-1}z_{1t}$$
$$v_t = v_{t-1} + \kappa(\theta - v_{t-1})\triangle t + \sigma\sqrt{v_{t-1}\triangle t}\,(z_{2t}\sqrt{1-\rho^2} + z_{1t}\rho) \quad (8)$$

The parameters μ, κ, σ, θ, and ρ are defined in equation (3), and are set to the values specified in Table 5. Each time series begins with an initial security price S_0 = 100 and volatility of $\sqrt{v_0}$ = 0.15, respectively, and then security prices and variances are generated randomly until option's maturity T.

<Table 4> Default Simulation Parameters

Volatility	$\sqrt{v}$	0.15
Mean Reversion	κ	1.50
Long-Tern Variance	θ	0.02
Volatility of Volatility	σ	0.40
Correlation	ρ	-0.50
Expected Return	μ	0.12
Riskless interest rate	r	0.05

For each generated series of security prices and volatilities, prices for five call options(deep out of the money, out of the money, at the money, in the money, and deep in the money options) are computed via the SV model based on the input parameters specified in <Table 4>. These five call option prices represent simulated daily market option prices. Market participants can observe these market option prices but the price generating function is assumed unknown to them. Therefore, the option models they can utilize to evaluate the market option prices confined to the BS model, the CS model and the MBS model.

The goodness of fit of the alternative option models are evaluated as follows: At day t, the parameters implied in the market option prices are estimated for CS model, BS model and MBS model. The estimated parameters are then used to calculate the fitted option prices of the alternative models across the strike prices. These theoretical prices are compared to the true option prices produced from the Heston model until option maturity to obtain the average daily percentage errors (APE) as shown in equation (9).

$$APE = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{C_t^H - C_t^{M_j}}{C_t^H}\right) \quad (9)$$

where C_t^H represents the true call price and $C_t^{M_j}$ represents jth model price at time t respectively. This simulation process is repeated for 1,000 times.

4.2 Simulation Results

<Table 5> compares the goodness of fit of the MBS model, the CS model and the BS model. Each entry of <Table 5> represents the simulation average of APE calculated for each model. The RMSE at the bottom of <Table 5> represents the average percentage pricing errors of equally weighted option portfolios across strike prices over the life of the options contracts. The RMSE measures suggest that the MBS model performs best and the BS model performs worst under stochastic volatility. With one-month maturity options, the BS model produces 0.2917% of pricing errors for a equally weighted option portfolio across strike prices. The MBS model reduces the BS model's pricing errors by 0.2796 percentage point or 95.8 percent of pricing errors. The performance of the CS model places in ranks between the MBS and the BS models, reducing the BS pricing errors by 0.1201 percentage point or 41.1 percent of pricing errors.

Similar results have been produced for longer-term options. With three-month and six month maturity options, the percentage pricing errors of the BS model

〈Table 5〉 Pricing Performance of Alternative Option Models

S/X	Maturity = 1 month			Maturity = 3 month			Maturity = 6 month		
	BS	MBS	CS	BS	MBS	CS	BS	MBS	CS
0.90	-1.4318	0.0482	0.8571	-0.5830	0.0621	0.3667	-0.3289	0.0640	0.3297
0.95	-0.2777	-0.0362	-0.0353	-0.1863	-0.0387	-0.0134	-0.1507	-0.0402	-0.0259
1.00	0.0004	-0.0001	0.0008	0.0029	-0.0061	0.0011	0.0053	-0.0059	0.0013
1.05	0.0137	0.0010	-0.0003	0.0211	0.0018	-0.0001	0.0243	0.0023	-0.0002
1.10	0.0049	-0.0001	0.0002	0.0139	-0.0003	0.0001	0.0183	-0.0004	0.0002
RMSE	0.2917	0.0121	0.1716	0.1225	0.0147	0.0734	0.0726	0.0152	0.0661

for a equally weighted option portfolio across strike prices are 0.1225% and 0.0726% respectively. However, they are 0.0147% and 0.0152% for the MBS model and 0.0734% and 0.0661% for the CS model respectively. The MBS model reduces 88.0 percent of the BS pricing errors for three month maturity options and 79.1 percent for six month maturity options. For the CS model, the reduction is 40.7 percent for three month maturity options and 8.9 percent for six month maturity options.

5. Conclusion

In this paper, the pricing performance of the BS model, the MBS model and the CS model were examined using Monte Carlo simulation experiments in a setting in which the volatility of the underlying asset was stochastic. The simulation results suggest that both the MBS and the CS models perform well in this situation substantially reducing the BS pricing errors caused by stochastic volatility across all strike prices considered. Surprisingly, however, the improvements of the MBS model over the BS model are much higher than those of the CS model. This result helps us to understand why option professionals have strong preferences to a Black-Scholes based model over more complicated theoretical models. Methods based on the Black-Scholes formula are the simplest model to implement while their performances are no worse than complicated models. This may also explain why the Black-Scholes formula is the overwhelmingly preferred choice of option professionals even though it is widely accepted that the constant volatility assumption of the Black-Scholes model is violated in real world financial market.

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