

Validation of a Rate-Sensitive Model for Clayey Soils

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점성토에서 전단속도 의존 모델의 검증

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Abstract In this study, the rate-sensitive constitutive model, which was developed in the previous paper of this journal, was validated using the experimental results obtained from the well-calibrated triaxial compression test conducted with the Boston blue clay. The validation was performed for the various cases of the strain rate of 0.05%/hr, 0.5%/hr, 5.0%/hr and OCR of 1, 2, 4, 8. The developed model was validated for the normally and slightly overconsolidated cases; however, the cases of heavily overconsolidation needs further research.

요약 본 논문에서는 전 논문에서 개발되었으나 검증을 필요로 한 간단한 전단속도 의존적 구성모델을 유명한 보스톤 블루 점토를 대상으로 엄격하게 수행된 비등방 삼축압축시험 결과를 활용하여 검증하였다. 검증은 전단속도 0.05%/hr, 0.5%/hr, 5.0%/hr 및 OCR=1, 2, 4, 8 경우에 있어서 수행되었다. 연구결과, 개발된 모델은 OCR=4, 8 경우 등 심한 과압밀 경우에는 변형을 완화, 비배수강도 등 응력-변형을 예측에 있어 개선이 필요한 것으로 보였으나, OCR=1, 2 등 정규압밀 또는 약간 과압밀 경우에는 비교적 신뢰도 높게 응력-변형을 관계를 예측하여, 그 활용도를 기대할 수 있겠다.

Key Words : constitutive, rate-sensitive, model, strain rate, stress-strain

1. Introduction

The strain rate affects significantly the stress-strain behavior of clayey soils(Richardson and Whitman 1963, Hight 1983)[1,2]. The strain rate plays an important role in determining if the ground condition is the drained or undrained condition, that might be one of the most essential issues in overall soil mechanics. For example, such standards as the shearing rate of 1%/hr in conventional triaxial test and the penetration rate of 2cm/sec in the piezocone testing have been regulated for more reliable analyses and applications. The strain rate effects have been extensively studied(Vaid and Campanella 1977, Sheahan 1995)[3,4]. The various efforts also in the category of the constitutive modeling have been made for the precise prediction of the rate-sensitive stress-strain behavior of clayey soils.

The relatively simple rate-sensitive constitutive model was developed in the previous paper(Kim 2009a 2009b)[5,6]. The model was proposed in the scheme of the classical elastic-plastic-viscous relation together with the Adachi's model(Adachi and Oka 1982), and verified [7]. The model was further simplified to reduce the number of the model parameters but the verification was not sufficient. The concept of the model and the details on the validation with the approved experimental results are presented in the following sections in this paper.

2. Concept of the Model

The total strain rate in the proposed model in the previous papers was obtained, as in the classical scheme, by just superposition of the elastic, plastic, and viscous

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strain rates. The generalized Hooke's law and the anisotropic modified Cam-clay model(Dafalias 1987) were adopted for the elastic and plastic simulations respectively[8].

The anisotropic modified Cam-clay model presents the distorted elliptical yield surface in the triaxial space with the associated flow rule, and the rotational and distortional hardenings in p' (mean effective principal stress= $\frac{\sigma_1 + 2\sigma_3}{3}$) - q (principal stress difference= $\sigma_1 - \sigma_3$) space, namely, kinematic and distortional hardenings in principal stress space as in eqs. (1) and (2)[8].

$$f = p^2 - pp_o + \frac{1}{2M^2} \{ (s_{ij} - p\alpha_{ij}) (s_{ij} - p\alpha_{ij}) + (p_o - p)p\alpha_{ij}\} = 0 \quad (1)$$

$$\dot{\alpha} = \langle \bar{\lambda} \rangle \left\{ \frac{1+e_o}{\lambda - \kappa} \left[r \frac{\partial f}{\partial \sigma_{mn}} \right] \frac{c}{p_o} (s_{ij} - xp\alpha_{ij}) \right\} \quad (2)$$

where $\langle \rangle$ denotes the Macauley bracket, $\bar{\lambda}$ is the loading index, e_o is the initial void ratio, s_{ij} is the deviatoric stress tensor, and p_o is the p' value at the intersection point between the p' axis and the distorted and rotated yield surface f . The M , λ , κ , c , and x are the material parameters described in Table 1. The plastic modulus can be obtained through the consistency condition as other conventional models.

The viscous and rate-sensitive relation was devised using the Perzyna's generalized viscous theory(Perzyna 1966) and Adachi's rate equation(Adachi and Oka 1982) as in eq. (3)[7,9]. The constituted model was, in the previous papers(Kim 2009a 2009b), further modified in the assumption that the initial yield surface and the dynamic loading surface are identical in the generalized viscous theory: the yield surface has the exactly same mathematical form and hardening rules with the plastic yield surface(eq. 4)[5,6]. These modifications were made, with keeping the simulation capability of the model, to reduce the number of material parameters which might produce rather complexity than higher reliability. The model has the advantage that the parameter values determined at a strain rate could be used at different strain rates.

[Table 1] Material Parameters

No.	Parameter	Identification
1	M	slope of CSL(critical state line) in p' - q space
2	λ	slope of NCL(normally consolidation line) in e - $\ln p'$ space
3	κ	slope of swelling line in e - $\ln p'$ space
4	c	anisotropic hardening parameter (controlling the anisotropy level developing under a constant p'/q)
5	x	anisotropic hardening parameter (controlling the pace at which such anisotropy develops)
6	\hat{V}	overstress function parameter (assumed a parameter instead of a form in the prior papers)
7	m'	rate-dependent parameter. (determining the slope of the line connecting the maximum deviatoric stresses to the corresponding strain rates)

$$\dot{\epsilon}^v = \langle \Phi \rangle \frac{\partial f}{\partial \sigma_{ij}} \quad (3)$$

$$\Phi = \frac{1}{V} \exp \left(\frac{\frac{1}{2} \left[\frac{N \times \frac{1}{3} I_1 \epsilon_{11}^{(1)}}{m'} \ln \frac{\epsilon_{11}^{(1)}}{\epsilon_{11}^{(2)}} - \sqrt{2J_2^{(1)}} \right]^2}{N I_1}} \right) \times \dot{\sigma}_{ij} \quad (4)$$

where Φ is called the overstress function, I_1 is the first invariant of the stress tensor, J_2 is the second invariant of the deviatoric stress tensor, and N is the slope of the critical state line in I_1 - $\sqrt{J_2}$ space($N=M/3\sqrt{3}$). The \hat{V} and m' are the material parameters(Table 1). The superscripts (1) and (2) in eq. (4) respectively denote a rate (1) and a different rate (2). For example, the $\epsilon_{11}^{(1)}$ indicates the axial strain rate at rate (1).

3. Validation of the Model

Sheahan et al.(1994) conducted a series of one-dimensionally(K_o conditions) consolidated undrained triaxial compression(CK_o ,UC) tests for the well-known Boston blue clay specimens[10]. The testings were reliably performed under the combined conditions of four overconsolidation ratio(OCR=1, 2, 4, and 8) and four strain rates(0.05, 0.5, and 5.0%/hr).

The specimens were of water content 39.9%, liquid limit 45.4%, and plastic index 23.7%. Tables 2 and 3 respectively denote the material parameter values obtained from the reference and input values for the model simulations. The e_o indicates the initial void ratio and r means $\frac{\epsilon_{11}^{(1)}}{\epsilon_{11}^{(2)}}$ in eq. (4).

[Table 2] Values of the Material Parameters

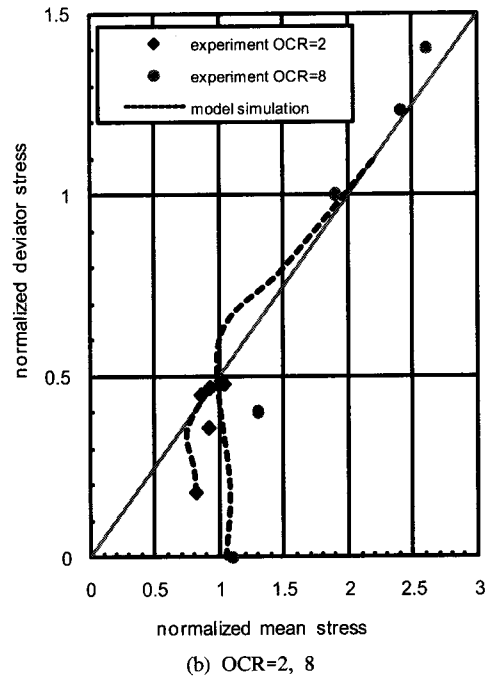
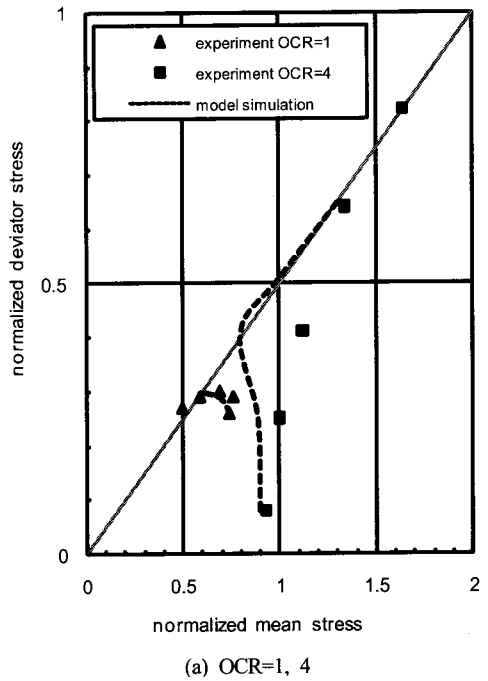
No.	Parameter	Value
1	M	0.5
2	λ	0.31
3	κ	0.34
4	c	0.15
5	x	0.07
6	\hat{V}	2×10^7
7	m'	35

[Table 3] Input Values

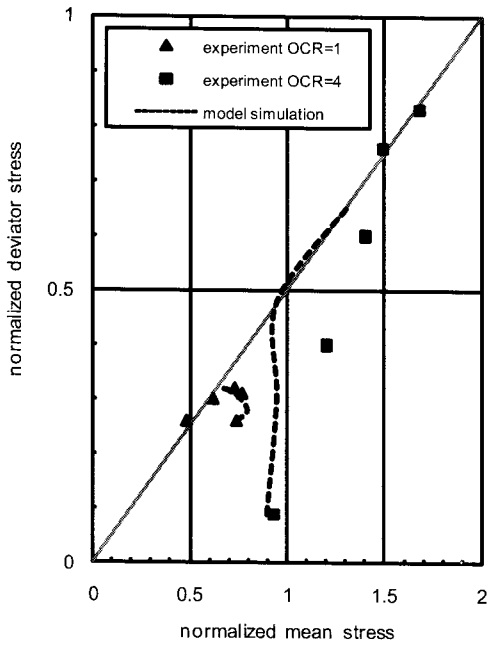
Input	Value
e_o	1.8
$r(0.05\%/hr)$	10
$r(0.5\%/hr)$	1.0
$r(5.0\%/hr)$	0.1

The strain rate of 0.5 to 1.0%/hr was typically recommended for standard practice so 0.5%/hr was taken as $\epsilon_{11}^{(1)}$ (Sheahan et al. 1994)[10]. The parameter values determined from the testing results at OCR=1 and 0.5%/hr were commonly used for the simulations at all the three rates and all the OCR.

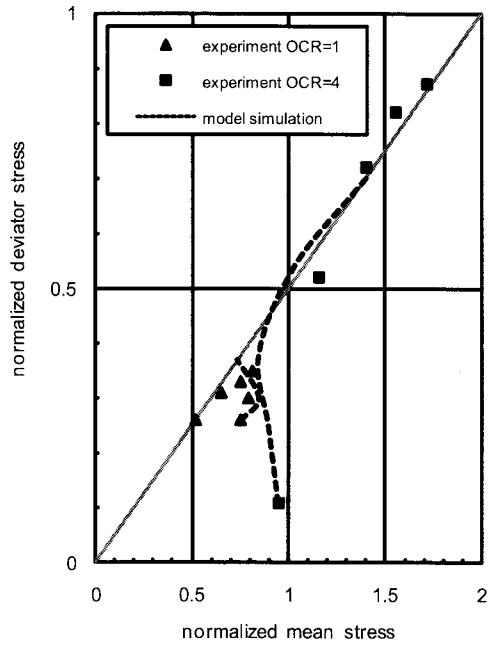
The comparisons of the model simulations and the experimental results, at various strain rates and the overconsolidation ratios, were presented in Figures. 1 to 3. The results of OCR=1 and 4 were shown on a plot and the results of OCR=2 and 8 were separately presented on the other plot to avoid complexity.



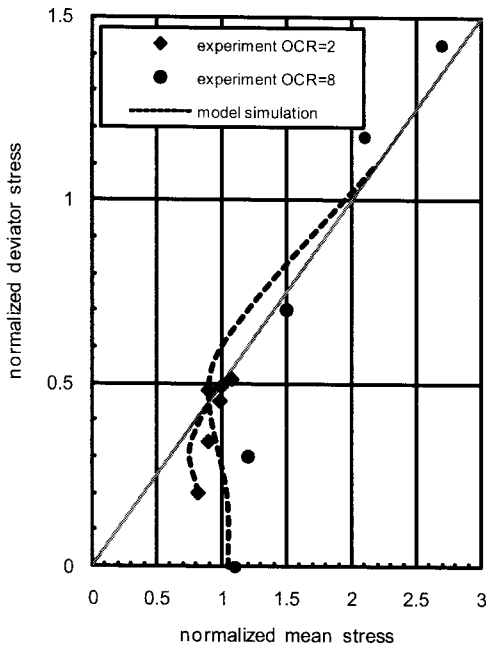
[Fig. 1] Strain Rate=0.05%/hr



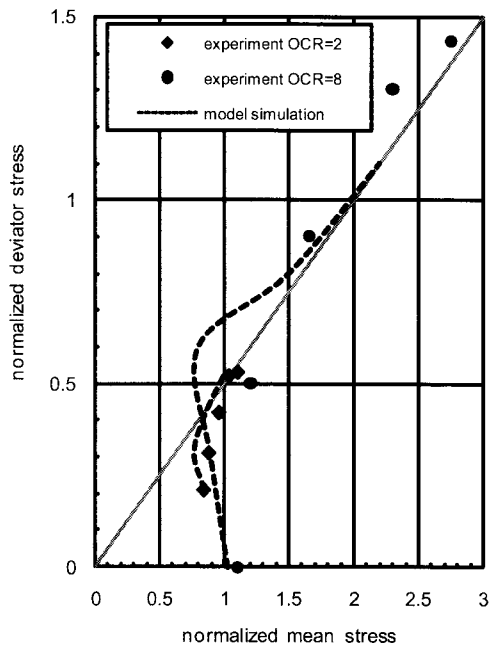
(a) OCR=1, 4



(a) OCR=1, 4



(b) OCR=2, 8



(b) OCR=2, 8

[Fig. 2] Strain Rate=0.5%/hr

[Fig. 3] Strain Rate=5.0%/hr

The model simulations for OCR=1 shows so good agreement with the experimental results at all the three strain rates. This could be expected since the normal modified Cam-clay model from which the developed model was modified is intrinsically based on the normally consolidation state. The model simulations for OCR=2 also gives relatively reliable results even though they are not as accurate as those for OCR=1.

The larger the OCR is, the larger the discrepancy between the model simulations and the experimental data is. For the heavily overconsolidated cases of OCR=4 and 8, the simulation results do not so good in both the peak undrained strength and the stress path. Actually the strain softening for the heavily overconsolidated clays has been still one of the major topics to overcome in constitutive modeling. In this area, one point is to use as small number of material parameters as possible. The simulations in Figures 2 to 4 showed the possibility of the strain softening but seemed to predicate some problems in undrained strength.

In the sense of the strain rate, the model simulation at 0.5%/hr shows generally better agreement with the experimental results than that at the other rates. This is, as expected, because all the parameters used for the simulations at all the three strain rates were obtained from the experimental results at 0.5%/hr. One reason of the discrepancy may go to the engineering characteristics of the specimen, that is the very low value of M , and the high values of λ and κ reflecting the softness of the clay specimen. The results of modeling at 0.05%/hr gave not less discrepancy than those at 5.0%/hr as the larger r value was inputted in 0.05%/hr than in 5.0%/hr.

The Boston blue clay is one of the representatives of soft clayey soils but is known to have various characteristics even in the category of softness. Specially the specimen utilized in this study is estimated to be extremely soft, which is judged to be a reason of discrepancy, considering the following: the value of M , a very fundamental parameter in the models based on the critical state soil mechanics, is so low that it is half to two-thirds of that in normal soft clay; the large void ratio of 1.8; the smaller value of c and the larger value of x than the respective normal values derive the great change of the anisotropic factor a in eqs. (1) and (2) which makes the simulation of anisotropy possible and

remarkable; the values of \hat{V} and m' are so low that the overstress function ϕ becomes large then the viscous deformation tends to be great following the eqs. (3) and (4). The \hat{V} was considered as a parameter and the m' was in multiplication form in eq. (4) so they resulted in large effect on the simulation results.

Though the clayey specimen could be thought to be too soft for the validation of the developed model, the use of it in this study is sufficiently meaningful since most soils offshore could be, in practice, as soft as the specimen.

The function and sensitivity of the parameter M regarding the constitutive models in the critical state theory needs to be further studied.

4. Conclusions

The rate-sensitive constitutive model developed in the prior papers was validated through the comparisons of the model simulation with the experimental results from the well-calibrated triaxial testings for the Boston blue clay. The model simulation gave constantly successful results regarding the various strain rates, specially good results for the OCR=1 and 2 specimens.

The use of the developed model is practically convenient since the material parameter values determined at a strain rate could be used at different strain rates. The modification, development, and validation of the model made in the prior and this papers is estimated successful even though further study is required for the heavily consolidated soils.

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<Research Interests>

Soil Mechanics&Foundation, Geotechnical Engineering, Ground Exploration&Testing, Numerical Analysis, Constitutive Relation