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대칭 토플리츠 시스템의 선행조건에 대한 특정성질 연구

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요 약

Tyrtshnikov[9]의 연구에서는 토플리츠 선형시스템에서 토플리츠 선행조건으로 일반해를 구하는 방법들을 제시하고 있다. 또한 대칭 토플리츠 행렬에서의 선행조건 행렬을 선택하는 방법도 소개 하였다. 본 연구는 토플리츠 시스템에서 새롭게 선행조건 찾는 방법을 소개하고 있으며, 선행조건행렬들의 분석을 통해 대칭 토플리츠 행렬 T의 고유값들과 대칭 토플리츠행렬로 부터 생성된 선행조건행렬의 고유값들이 매우 근접하다는 결과를 나타내고 있다. 즉, 선행조건시스템 $C_0^{-1}T$ 의 고유값들은 1에 모두 접근하게되면, 선행조건 시스템의 수렴속도는 superlinear이다. 본 연구에서 생성된 선행조건행렬 C_0은 선행조건시스템의 superlinear의 수렴속도로 계산하게 된다. 또한 토플리츠 행렬은 이미지 프로세싱이나 시그널 프로세싱에서 많이 응용되고 있으므로 본 연구에서 개발한 선행조건행렬로부터 다양한 응용성을 높일 수 있다. 본연구의 또 다른 특징은 토플리츠 행렬의 중요한 성질을 보존하면서 선행조건행렬을 생성하였다.

A Study for Spectral Properties of Preconditioner of Symmetric Toeplitz Systems

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Abstract

In [9], Tyrtshnikov proposed a preconditioned approach to derive a general solution from a Toeplitz linear system. Furthermore, the process of selecting a preconditioner matrix from symmetric Toeplitz matrix, which has been used in previous studies, is introduced. This research introduces a new method for finding the preconditioner in a Toeplitz system. Also, through analyzing these preconditioners, it is derived that eigenvalues of a symmetric Toeplitz T are very close to eigenvalues of a new preconditioner for T. It is shown that if the spectrum of the preconditioned system $C_0^{-1}T$ is clustered around 1, then the convergence rate of the preconditioned system is superlinear. From these results, it is determined to get the superliner at the convergence rate by our good preconditioner C_0. Moreover, an advantage is driven by increasing various applications i. e. image processing, signal processing, etc. in this study from the proposed preconditioners for Toeplitz matrices. Another characteristic, which this research holds, is that the preconditioner retains the properties of the Toeplitz matrix.

Keywords : **Toeplitz, Circulant, Preconditioner, Hermitian, Preconditioned Conjugate Gradient(PCG)**

1. Introduction

In this paper we investigate a new preconditioner for preconditioned Teoplitz System. The studies on the preconditioning symmetric positive definite (SPD) Toeplitz matrices with circulant matrices have been [1], [3], [4], [5]. Toeplitz systems arise in a variety of practical applications in engineering fields. For instance, in signal processing, solutions of Toeplitz systems are

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required in order to obtain the filter coefficients in the design of recursive digital filters[Chui, Chan] and time-series analysis also involves solutions of Toeplitz systems for unknown par ameters of stationary autoregressive models [3]. An iterative method for solving the SPD system $Ax=b$ can be derived by minimizing the quadratic functional $\frac{1}{2}x^TAx-b^Tx$ with the conjugate gradient (CG) method, and the unique minimum gives the desired solution.

The convergence rate of the CG method dep ends on the spectrum of A. In general, the CG method converges faster if A has a small condition number of clustered eigenvalues.

To accelerate its convergence rate, a pre-conditioning step is often introduced at each CG iteration, which leads to the PCG method.

A good choice of preconditioners for A is a matrix P that approximates A well (in the sense that the spectrum of the preconditioned matrix $P^{-1}A$ is clustered around 1 or has a small condition number), and for which the matrix-vector product $P^{-1}v$ can be computed efficiently for a given vector v. With such a preconditioner, one solves in principle the preconditioned system $\tilde{A}\tilde{x}=\hat{b}$, where $\tilde{A}=P^{1/2}AP^{-1/2}$, $\tilde{x}=P^{-1/2}x$ and $\hat{b}=P^{-1/2}x$, by the CG method[3]. The idea of preconditioning is a simple one but is now recognized as critical to the effectiveness of the PCG method.

A Toeplitz preconditioner also has been proposed by Strang, and analyzed by Chan and strang[3]. Strang's Preconditioner S is obtained by preserving the central half diagonals of A and using them to form a circulant matrix. Since S is a circulant, the matrix-vector product $S^{-1}v$ can be conveniently computed via fast Fourier transformation (FFT) with $O(n\log n)$ operations. It has been shown [1] - [5] that a eigenvalue class of $S^{-1}A$ is clustered around 1 except a finite number of outlier. The converg ence rate of preconditioned iterative methods depends on the singular value or eigen-value distribution of the preconditioned matrices.

In our research, we propose a new type of preconditioners C_0 for Toeplitz matrices as the following. We discuss about spectral properties of a circulant preconditioner C_o in Section 2 and how to develop a circulant preconditioner C_o for Toeplitz T_n in Section 3. It describes the eigenvalues distributions of $C_0^{-1}T_n$, C_o, S and T_n in section 4. The concluding remarks are given in Sections 5.

2. Spectral properties of a new circulant preconditioner

we denote T_n by the set of n by n matrices and Toeplitz matrices, respectively:

$$T_n \equiv \left\{ \begin{bmatrix} t_0 & t_1 & \cdots & t_n-1 \\ t_{-1} & \ddots & \ddots & \vdots \\ \cdots & \cdots & \ddots & \vdots \\ t_{-1(n-1)} \cdots & & t_{-1} & t_0 \end{bmatrix} \right\} \subseteq M_n.$$

We denote by T_n^R the set of all real symmetric Toeplitz matrices:

$$T_n^R \equiv \left\{ \begin{bmatrix} t_0 & t_1 & \cdots & t_n-1 \\ t_{-1} & \ddots & \ddots & \vdots \\ \cdots & \cdots & \ddots & \vdots \\ t_{-1(n-1)} \cdots & & t_{-1} & t_0 \end{bmatrix} \right\} \subseteq T_n(R) \quad (1)$$

and by T_n^H the set of all hermitian Toeplitz matrices:

$$T_n^H \equiv \left\{ \begin{bmatrix} \underline{t_0} & t_1 & \cdots & t_n-1 \\ \overline{t_1} & \ddots & \ddots & \vdots \\ \cdots & \cdots & \ddots & \vdots \\ t_{-1(n-1)} \cdots & & t_{-1} & t_0 \end{bmatrix} \right\} \subseteq T_n(C) \quad (2)$$

A Toeplitz matrix $C=[c_{ij}]\in T_n$ is called a c irculant if $c_{ij}=c_{(n+j-i)\bmod n}, 0\le i,j\le N-1$, i. e.

$$C= \begin{bmatrix} c_0 & c_1 & \cdots & c_{n-1} \\ c_{n-1} & c_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_1 \\ c_1 & \cdots & c_{n-1} & c_0 \end{bmatrix} \in T_n.$$

From the structure of the circulant matrix, we see that $C = c_0 I + c_1 J + \cdots + c_{n-1} J^{n-1}$ where

$$J = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

Therefore, every circulant matrix is generated by a simple permutation matrix J.

Thus, $C = F(c_0 I + c_1 + \cdots + c_{n-1} W^{n-1}) F^*$
$= F diag(\sum_{j=0}^{n-1} c_j, \sum_{j=0}^{n-1} c_j w^j, \cdots, \sum_{j=0}^{n-1} c_j w^{j(n-1)}) F^*$

where $\sum_{j=0}^{n-1} c_j w^{j(k-1)}$ is the kth eigen-value, $\frac{1}{\sqrt{n}}(1 \quad w^{(k-1)} \cdots \mathrm{w}^{(\mathrm{k}-1)(\mathrm{n}-1)})^{\mathrm{T}} \in \mathbb{C}^{\mathrm{n}}$ and $w = e^{(2\pi i)/n} = \cos\frac{2\pi}{n} + i sin \frac{2\pi}{n}$ is the correspond -ing kth eigen-vector of C for $k = 1, 2, \ldots, n$.

We denote by C_n the set of all circulant matrices. Because all circulant matrices are generated by J, C_n forms a commutative ring over real or complex field. Also it should be noted that C_n is completely characterized by the diagonalizability under F-unitary similarity. Thus, C_n is a very special commutative subclass of the normal Toeplitz matrices. We denote by C_n^R the set of real symmetric circulant matrices and T_n^R the set of real symmetric Toeplitz matrices:

$$C_n^R \equiv \begin{cases} T_n^R(c_0, c_1, \cdots, c_k, c_{k-1}, \cdots, c_1) \\ T_n^R(c_0, c_1, \cdots, c_{k-1}, c_k, c_k, \cdots, c_1) \end{cases} \qquad (3)$$

where $k = \left[\frac{n}{2}\right]$.

Let $T \in T_n^R$. Choose $C_o \in C_n^R$ such that $\|T - C_0\|_F = \min \lim_{C \in C_n^R} \|T - C\|_F$. It is known that if $T = T_n^R(t_0, t_1, \cdots t_{n-1}) \in T_n^R$ then $C_0 = T_n^R(c_0, c_1, \cdots, c_k, \cdots, c_1) \in C_n^R$ such that

$$c_j = \frac{j t_{(n-j)} + (n-j) t_j}{n}, \; j = 0, \cdots, k. \qquad (4)$$

Thus, (4) gives the formula for the initial symmetric circulant matrix choice for this case. Note that .

$$|\lambda_j - \alpha_j|^2 \le \sum_{j=0}^{n} (\lambda_j - \alpha_j)^2 \le \|T - C_0\|_F^2$$

where λ_j and α_j are the eigenvalues of the matrices T and C_0, respectively. Thus, if $\|T - C_0\|_F$ is small then the eigenvalues of C_0 are close to the corresponding eigenvalues of T and we have a good choice of the precondit ioner.

Let C_n^R be the set of real symmetric circula nt matrices in (3). We denote by C_n^H the set of hermitian circulant matrices:

$$C_n^H \equiv \begin{cases} T_n^H(c_0, c_1, \cdots, c_{k-1}, c_k, c_{k-1}, \cdots, \overline{c_1}) \\ T_n^H(c_0, c_1, \cdots, c_{k-1}, c_k, c_k, \cdots, \overline{c_1}) \end{cases} \qquad (5)$$

where $k = \left[\frac{n}{2}\right]$ and T_n^H in (2).

Note that C_n^H is the F-real diagonalizable class of matrices: $C \in C_n^H$ if and only if $C = F diag(\alpha_1, \cdots, \alpha_n) F^*$ where $\alpha_i \in R$ for $i = 1, \cdots, n$. It can be verified easily that $C_n^R \subseteq C_n^H$ has the following finer spectral characteristic: $C \in C_n^R$ if and only if $C = F diag(\alpha_1, \cdots, \alpha_n) F^*$ where $\alpha_i \in R$ for $i = 1, \cdots, n$ such that the algebraic multiplicity of each α_i must be greater than or equal to 2, that is, there is no simple eigen value fo r real symmetric circulant matrices. A Toeplitz matrix $K = [k_{ij}] \in T_n$ is called a skew circulant ma trix if

$$k_{ij} = \begin{cases} c_{j-i} & \text{for } 0 \le i \le j \le n-1 \\ -c_{-(i-j)} & \text{for } 0 \le j \le i \le n-1. \end{cases}$$

Thus, $K = k_{0I} + k_{1L} + k_{2L}^2 + \cdots + k_{n-1} L^{n-1}$ where

$$L = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & 1 & \vdots & \vdots \\ 0 & \ddots & \ddots & \ddots & 1 \\ -1 & 0 & \cdots & \cdots & 0 \end{bmatrix} \in M_n.$$

Note that $P^*LP = \theta J$ where $P = diag(1,\theta,\theta^2,\cdots,\theta^{n-1})$, $\theta = e^{(\pi i)/n}$ and $\theta^n = -1$.

Therefore, $P^*KP = k_0 I + \theta k_1 J + \theta^2 k_2 J^2 + \cdots + \theta^{n-1} k_{n-1} J^{n-1} \in C_n$

and hence F^*P^*KPE is a diagonal matrix. It is easy to identify that $P^{s^*}LP^s = \theta^s J$ for any odd number $s = 1,3,\cdots$ and K is diagonalizable under P^sF-unitary similarity for $s = 1,3,\cdots$.

Note that $J = FWF^*$ and $J^T = F^*WF$ where $P^2 = W = diag(1,\omega,\omega^2,\cdots,\omega^{n-1})$, $\omega = e^{(2\pi i)/n}$,

hence F^*PF must be a square root of J^T..

Note that we need only to consider for $s = 1,\cdots,2n-1$, because of $P^{(2n+1)} = W^nP = P$. We denote by $K_n \subseteq T_n$ the set of all skew circulant matrices, and by $K_n^R \subseteq K_n$ the set of all real symmetric skew circulant matrices.

Lemma 1: The following are equivalent.

(i) $K \in K_n^R$. (ii) K is P^sF-real diagonalizable for $s = 1,\ 3,\ 5,\ \cdots,\ 2n-1$

i.e., $K = F^*P^{s^*}diag(\alpha_1,\cdots\alpha_n)P^sF$, $\alpha_i \in R$

(iii) $P^{s^*}KP^s \in C_n^H$ for $s = 1,\ 3,\ 5,\ \cdots,\ 2n-1$

Proof) (i) $\Rightarrow$ (iii) Suppose $K \in K_n^R$ is a given real symmetric skew circulant matrix. Consider the set

$$G = \left\{P^{s^*}KP^s\right\}_{s=1,3,\cdots,2n-1} = \left\{W^{i^*}P^*KPW^i\right\}_{i=0,\cdots,n-1} = \left\{W^{i^*}C^{(0)}W^i\right\}_{i=0,\cdots,n-1}$$

where $C^{(0)}$ is a hermitian circulant matrix, $P = diag(1,\theta,\cdots,\theta^{n-1})$ and $W = diag(1,\omega,\cdots,\omega^{n-1})$, $\theta = e^{i\pi/n}$.

Since $FWF^* = J$,

$$G = \left\{F^*(FW^{i^*}F^*FC^{(0)}F^*FW^iF^*)F\right\}_{i=0,\cdots,n-1} = \left\{F^*(J^{i^*}diag(\alpha_1,\cdots,\alpha_n)J^i)F\right\}_{i=0,\cdots,n-1} = \left\{C^{(i)}\right\}_{i=0,\cdots,n-1} \subseteq C_n^H,$$

where $\{\alpha_1,\cdots,\alpha_n\}$ are the eigenvalues of K.

(iii) $\Rightarrow$ (ii)

$P^{s^*}KP^s \in C_n^H$, $F^{*P^{s^*}}KP^sF$ is a diagonal. Thus, K is P^sF- real diagonalizable for $s = 1,3,5,\cdots,\ 2n-1$.

(ii) $\Rightarrow$ (i)

Let $C_n^K \equiv C_0(G)$ be the convex hull of G, *i.e.*, the set of all convex combination of the matrices from G.

We present the following general result about hermitian matrices. We denote by H_n the set of all n by n hermitian matrices. Suppose

$$A \in H_n,\ A = V\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}V^*, \lambda_i \in R$$

and $V \in M_n$ is unitary.

By D, we denote the set of all real diagonal matrices, $D = \{diag(\alpha_{1,\ldots,}\alpha_n)/\alpha_i \in R\}$.

For $a,b \in R$, $a \geq b$, we let $D(a,b) = \{diag(\alpha_1,\cdots,\alpha_n)/b \leq \alpha_i \leq a \text{ for } all\ i = 1,\cdots,n\} \subseteq D$.

A subset $D^s(a,b) = \{diag(\alpha_1,\cdots,\alpha_n)/\sum_{i=1}^{n}\alpha_i = s$ and $b \leq \alpha_i \leq a\}$.

Theorem 2: Suppose

$$A = V\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}V^* \in H_n.$$

Let $s = \sum_{i=1}^{n}\lambda_i$. Then $\min_{\Sigma \in D}\|A-\Sigma\|_2 = \min_{\Sigma \in D(\lambda_{\max},\lambda_{\min})}\|A-\Sigma\|_2 = \min_{\Sigma \in D^s(\lambda_{\max},\lambda_{\min})}\|A-\Sigma\|_2$.

As a simple consequence of Theorem 2, we have the following.

Corollary 3 : Let $K_0 \in K_n^R$ be given. Then $\min_{C \in C_n^H} \|K_0 - C\|_2 = \min_{C \in C_0(G)} \|K_0 - C\|_2$

where $C_0(G)$ is the convex hull of

$$G = \{W^{i*} P^* K_0 P W^i P\}_{i=0,\cdots,n-1}$$
$$= F^*(J^{i*} diag(\alpha_{1,\cdots,}\alpha_n) j^i) F\}_{i=0,\cdots,n-1}$$
$$= \{C^{(i)} \in C_n^H\}_{i=1,\cdots,n.}$$

3. A generation of circulant approximations

Now suppose $T \in T_n^R$. We define T^C, the complement of T by $T^C \in T_n^R$.

Then notice that $T = \dfrac{T+T^C}{2} + \dfrac{T-T^C}{2}$ where $\dfrac{T+T^C}{2} \in C_n^R$ and $\dfrac{T-T^C}{2} \in K_n^R$.

Thus we have $T = C_T + K_T$ (6)

where $C_T \in C_n^R$ and $K_T \in K_n^R$, and it is easy to verify that the decomposition is unique. Note that

$$\min_{C_0 \in C_n^H} \|T - C_0\|_2 = \min_{C_0 \in C_n^H} \|C_T + K_T - C_0\|_2$$
$$= \min_{C_0 \in C_n^H} \|C_T + K_T - (C_T + C)\|_2$$
$$\leq \min_{C_0 \in C_n^H} \|K_T - C\|_2$$

where the last equality is from the Corollary 3.

Since the object of this section is to obtain an initial hermitian circulant matrix, it is clear from above equality that we need to only consider a hermitian circulant matrix from the convex hull of $\{P^* K_T P, \cdots, P^*(W^{n-1^*} K_T W^{n-1}) P\}$. We use the following steps to choose the initial hermitian circulant matrix for $T \in T_n^R$.

Set $S(\alpha,\beta) = \alpha P^* K_T P + \beta P^* W^T K_T W P$, $\alpha, \beta > 0$, $\alpha + \beta = 1$.

Compute $\gamma(\alpha,\beta) = \dfrac{\|K_T - S(\alpha,\beta)\|_2}{\|C_T + S(\alpha,\beta)\|_2}$ for $\alpha = 0, 0.1, \cdots, 0.9, 1$.

Then the initial circulant matrix S of given T is the hermitian circulant matrix $C_0 = C_T + S(\alpha,\beta)$ with the minimum values of $\gamma(\alpha,\beta)$ where $T = C_T + K_T$. Some examples of the distribution of the eigenvalues of T and $C_0 \in C_n^H$ are given in Section 4.

Example: Choice of the scalars α and β. We choose α and β, $\alpha + \beta = 1$ such that $\gamma(\alpha,\beta) = \dfrac{\|K_T - S(\alpha,\beta)\|_2}{\|C_T + S(\alpha,\beta)\|_2}$ is the minimum.

Suppose $\gamma(\alpha_0,\beta_0) = \min_{0 \leq \alpha,\beta \leq 1} \gamma(\alpha,\beta)$, $\alpha + \beta = 1$. Then $C_0 = C_T + S(\alpha_0, \beta_0)$.

We provide the following two examples.

(a) $T \in T_n^R(t_1,\cdots,t_n)$ such that $t_i = \dfrac{1}{(i+1)^2}$ for $i = 0, \cdots, 49.$ $\alpha_0 = 0.7$, $\beta_0 = 0.3$

(b) $T \in T_n^R(t_1,\cdots,t_n)$ such that $t_i = \dfrac{(-1)^i}{i+1}$ for $i = 0, \cdots, 49.$ $\alpha_0 = 1.0, \beta_0 = 0.3$

Find α_0 and β_0 and β_0 such that $\gamma(\alpha,\beta)$ is minimized. Then

$C_0 = C_T + \alpha_0 P^* K_T P + \beta_0 P^* W^* K_T W P.$

4. Numerical experiments

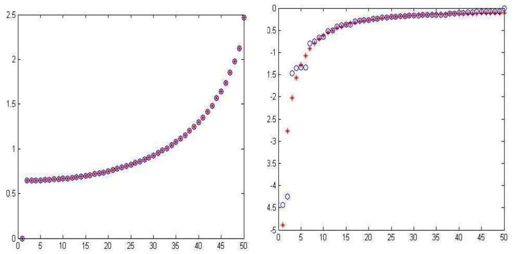
In this section, we compare our hermitian circulant preconditioner C_0 with Strang's circulant preconditioner S_n.

$$S_n = \begin{bmatrix} t_0 & t_{-1} & \cdots & t_2 & t_1 \\ t_1 & t_0 & t_{N-M} & \bullet & t_2 \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & t_{2-M} & t_{1-M} & t_{N-M} \\ t_{N-M} & \bullet & \bullet & t_{2-M} & t_{1-M} \\ t_{1-M} & t_{N-M} & t_{-1} & \bullet & t_{2-M} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ t_{-2} & \bullet & t_1 & t_0 & t_{-1} \\ t_{-1} & t_{-2} & \bullet & t_1 & t_0 \end{bmatrix}.$$

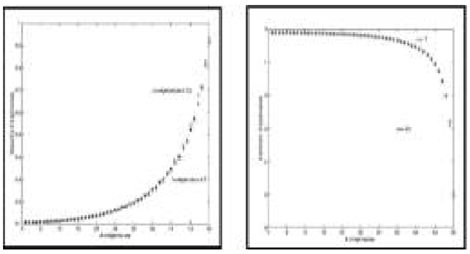
According to Strang's proposal shown above in the given matrix, we construct preconditioner S_n by preserving n consecutive diagonal in T

and bring them around to form a circulant matrix.

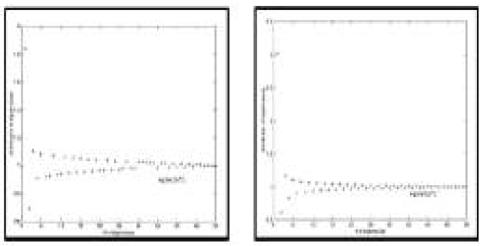
We compare with Strang preconditioners, S_n and our preconditioner C_0 from the distribution of eigenvalues based on the objective matrix T with example (a), (b) (Figure 1, 2). We also have a experiment for eigenvalue distributions of $C_0^{-1}T$ with example (a), (b) (Figure 3).



(Figure 1) Eigenvalue Distribution of S_n and T for given example (a), (b)



(Figure 2) Eigenvalue Distribution of C_0 and T for given example (a), (b)



(Figure 3) Eigenvalue Distribution of $C_0^{-1}T$ for given example (a), (b)

5. Conclusions

From (Figure 1, 2) our new preconditioner C_0 is closer than Strang preconditioners S_n to Toeplitz T. We can apply a new and better preconditioner C_0 instead of the given matrix T by the iterative method for Toeplitz systems. We expect all eigenvalues of the given Toeplitz matrices to be close to all eigenvalues of new circulant preconditioners (Figure 2).

Also the distributions of eigenvalues of $C_0^{-1}T$ are between 0.6 and 1.2 and are between 0.5 and 1.3 (Figure 3) as shown in section 3. We have the property that all eigenvalues of $C_0^{-1}T$ are very close to 1 excluding the extreme eigenvalues. It supports to reduce the iterations for the iterative method on the Toeplitz system.

The objective matrix for case (a) is a sym metric positive matrix and $\|I-C_0^{-1}T\|_F^2$ ≈ 1.1591 and (b) is a symmetric matrix and $\|I-C_0^{-1}T\|_F^2 \approx 8.3689$.

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