THE JOURNAL OF THE ACOUSTICAL SOCIETY OF KOREA Vol.28, No.1E 2009. 3. pp. 16~22

Robust Active Noise Controller with Hybrid Adaptive Nonlinear Compensator

Oh-Sang Kwon*
*Dept. of Computer and Information Communication, Korea Digital University
(Received November 10 2008; revised December 31 2008; accepted January 28 2009)

Abstract

In this paper, the new robust active noise controller was proposed to be applied for attenuating the noises when the nonlinear distortion in the secondary path exists. Through computer simulations as well as the analytical analysis, it could be shown that it is possible for both conventional LMS controller and proposed controller, to be applied for actively controlling the noises and linearizing the nonlinear distortion in the secondary path. Also, the simulations results demonstrated that the proposed controller may have faster convergence speed and better capability of controlling the noises and compensating the nonlinear distortion than the conventional LMS controller.

Keywords: *Active noise controller, Nonlinear distortion, Convergence speed*

I. Introduction

The active noise control (ANC) of sound and vibration involves the introduction of a number of controlled secondary sources driven such that the field generated by these sources interferes destructively with the field caused by the original primary source. The field of active control of sound and vibration has been growing very rapidly in the last three decades, and many applications have been developed. Most of this development has been driven by advances in control theory and the availability of low cost electronics for digital signal processing.

A number of different techniques and control algorithms have received considerable attention until now for active control of sound and vibration fields [1-3]. With regards to the active control of sound fields, there have been significant gains made in a theoretical understanding of the mechanisms of active control, and the interaction between the primary excitation sources and the secondary control sources. Along with the theoretical developments, there has also been significant progress made in implementing practical active control systems. A number of different control algorithms have been developed for active control. However, the active control systems reported in the literature have relied most prominently only on the feedforward or the feedback structures.

The feedforward control algorithm for active noise control exhibits high stability and performance robustness. But it has a slow convergence speed and a correlated reference signal must be available [1]. Broadband active control systems typically use feedback control in order to increase the convertgence speed and to avoid the problems associated with obtaining and decoupling a suitable reference signal for use in a feedforward least-mean-square (LMS) configuration [3]. However, it is well known

Corresponding author: Oh-Sang Kwon (narcis@kdu.edu)
1-21, Gye-dong, Jongno-gu, Seoul 110-800, Korea

that conventional feedback control systems have a gain-bandwidth limitation. Thus, a given level of noise reduction can only be achieved over a limited bandwidth. And modelling uncertainty can degrade closed-loop performance and lead to stability problems for the closed-loop system. In real applications, the presence of a transfer function in the auxiliary-path following the adaptive controller and/or in the error-path, has been shown to generally degrade the performance of the LMS algorithm. Thus, the convergence rate is lowered, the residual power is increased, and the algorithm can become unstable.

In this paper, I present a robust active noise controller based on the combination of feedforward and feedback control systems with very small complexity increment and large performance increment. In this controller, the secondary path is modeled with hybrid adaptive Volterra filter (HAVF) [4], so overall performance of active noise controller is increased.

II. Analysis of Proposed Controller

2.1. Derivation

A robust active noise controller is proposed which is based on the combination of both feedforward and feedback control system in order to increase the convergence speed and the performance robustness with very small complexity increment. In the proposed controller, a new generated reference signal comprised of weighted sum of an original reference signal and an error signal, is used as an input to the controller. This new signal may depend on the properties of both an original reference signal and a feedback error signal according to the weighting factors(α,β). Thus, the proposed controller can have faster convergence speed and better performance than either feedforward or feedback system using the general LMS algorithm as almost equivalent complexity of computation as it. The structure of Fig. 1 The block diagram representing the combination of both feedback and feedforward active noise controller the proposed controller can be produced by combination of both feedback control system and feedforward control system, as illustrated in Fig. 1. In the figure, the original reference signal and the error signal via a negative feedback may be summed. A physical illustration of the proposed controller and its equivalent block diagram can be represented as Fig. 2 and Fig. 3, respectively. In these figures, w describes the adaptive controller and an original reference signal $x(n)$ is weighted by a parameter α and an error signal $e(n)$ is weighted by a parameter β and the difference between $\alpha x(n)$ and $\beta e(n)$, $r(n)$ is used as an input to the adaptive controlle. In general, $x(n)$ and $e(n)$ may be uncorrelated each other. The controller uses both a reference sensor and an error sensor, and can attenuate broadband noise as well as narrow-band noise. The reference

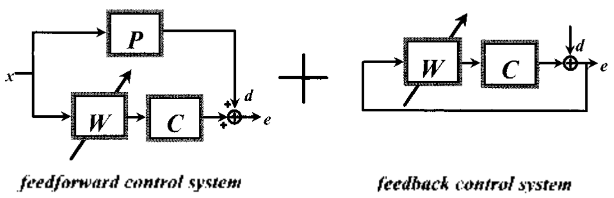


Fig. 1. The block diagram representing the combination of both feedback and feedforward active noise controller.

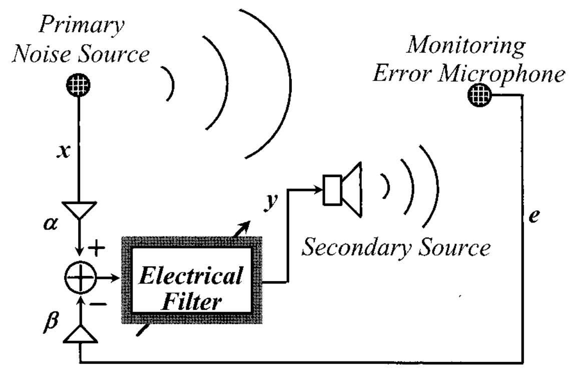


Fig. 2. The physical representation of the proposed active noise controller.

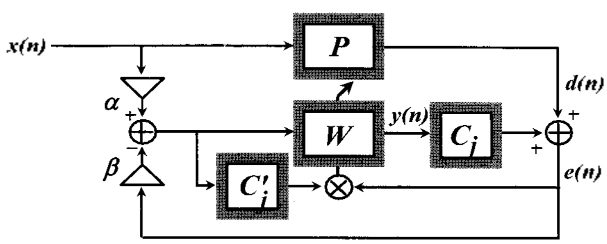


Fig. 3. The equivalent block diagram of the proposed active noise controller.

sensor measures the primary noise to be canceled while the error sensor supplies advanced informations of the primary noise as well as monitors performances of the active noise controller. The proposed controller can use both the feedforward control strategy that it can cancel the primary noise components that are correlated with the reference signal, and the feedback control strategy that it can cancel the noise components by the use of feedback error signal in the closed loop. Also, the proposed controller can be implemented as various types of adaptive filters such as finite impulse response (FIR), infinite impulse response (IIR), lattice structures, *etc.*

2.2. Analysis

In the Fig. 3 let the sampled input $y(n)$ to the actuator be obtained by filtering the weighted reference signal and the weighted error signal using an adaptive FIR controller whose i-th coefficient at the n-th sample is $w_i(n)$, so that [5]

$$y(n)=\sum_{i=0}^{I-1} w_i(n)[\alpha x(n-i)-\beta e(n-i)] \quad (1)$$

where I is the coefficient number of the adaptive FIR filter, and α and β are the weighting factors of the original reference signal and the error signal, respectively. Let the sampled output of the error sensor be $e(n)$, which is equal to the sum of the desired signal $d(n)$ due to the primary source alone, and an output due to an actuator. And let the transfer function from the input of the actuator to the output of the sensor be modeled as a J-th order FIR filter, whose j-th coefficient is C_j, so that

$$e(n)=d(n)+\sum_{j=0}^{J-1} C_j y(n-j) \quad (2a)$$

$$=d(n)+\sum_{i=0}^{I-1}\sum_{j=0}^{J-1} C_j w_i(n-j)[\alpha x(n-i-j)-\beta e(n-i-j)] \quad (2b)$$

$$=d(n)+\sum_{i=0}^{I-1} w_i \sum_{j=0}^{J-1} C_j\,[\alpha x(n-i-j)-\beta e(n-i-j)] \quad (2c)$$

Let the cost function or the total error be defined as

$$J = E[e^2(n)] \fallingdotseq e^2(n) \quad (3)$$

where $E[\cdot]$ denotes an expectation value. If the weighted reference signal is at least partly correlated with the desired signal $d(n)$, it is possible to reduce the value of J by deriving the secondary source. The total error may be a quadratic function of this filter coefficient and the optimal set of this coefficient required to minimize J may thus be evaluated adaptively using gradient descent method.

The gradient vector is evaluated as follows.

$$\nabla_i(n)=\frac{\partial J}{\partial w_i}=2e(n)\frac{\partial e(n)}{\partial w_i} \quad (4)$$

where $\frac{\partial e(n)}{\partial w_i}=r(n-i)$

$$=\sum_{j=0}^{J-1} C_j\,[\alpha x(n-i-j)-e(n-i-j)]$$

Here, $r(n-i)$ is the weighted filtered reference signal and this sequence is the same as the one which would be obtained at the sensor if the weighted reference signal and the weighted error signal, delayed by i samples, were applied to the actuator.

If the coefficient is now adjusted at every sample time by an amount proportional to the negative instantaneous value of the gradient, the filter coefficient adaptation mechanism is produced.

$$\therefore w_i(n+1)$$

$$=w_i(n)+\mu(-\nabla_i(n)) \quad (5a)$$

$$=w_i(n)-2\mu e(n)r(n-i) \quad (5b)$$

$$=w_i(n)-\alpha' e(n)s(n-i)+\beta' e(n)t(n-i) \quad (5c)$$

where μ is the convergence parameter that determines both adaptation speed and stability, and $\alpha'=2\mu\alpha$ and $\beta'=2\mu\beta$ are the parameters that determine the weighting of the original reference signal and the feedback error signal, respectively. Here, $s(n-i)$ and $t(n-i)$ represent a feedforward filtered version signal and a feedback filtered version signal, respectively and are expressed as follows.

$$s(n-i) = \sum_{j=0}^{J-1} C_j x(n-i-j),\ t(n-i) = \sum_{j=0}^{J-1} C_j e(n-i-j)$$

For zero weighting factor of the original reference signal, this corresponds exactly to the "filtered-error LMS" algorithm [3] and for zero weighting factor of the error signal, this corresponds exactly to the "filtered-x LMS" algorithm [5].

In order to analytically demonstrate the shape of the error surface, and so determine the optimum Wiener set of filter coefficients, it is convenient to assume that the filter coefficients are exactly time-invariant. The assumption of time invariance in the filter coefficients is equivalent, in practice, to assuming that the filter coefficients change only slowly compared to the time-scale of the response of the system to be controlled. This time-scale is defined by the values of the coefficients.

The sampled output of the error sensor may be written as

$$e(n) = d(n) + \sum_{i=0}^{I-1} w_i\, r(n-i) \quad (6a)$$

$$= d(n) + \boldsymbol{r}^T \boldsymbol{W} \quad (6b)$$

where $\boldsymbol{r}^T = [r(n), r(n-1), \cdots, r(n-I+1)]$
and $\boldsymbol{W}^T = [w_0,\ w_1,\ w_2,\ \cdots,\ w_{I-1}]$.

The error criterion can now be written as

$$J = E[e^T(n)e(n)] \quad (7a)$$

$$= E[d^T(n)d(n)] + 2E[d(n)]\boldsymbol{W} + \boldsymbol{W}^T E[\boldsymbol{r}^T]\boldsymbol{W} \quad (7b)$$

$$= E[d^T(n)d(n)] + 2\boldsymbol{P}^T\boldsymbol{W} + \boldsymbol{W}^T\boldsymbol{R}\,\boldsymbol{W} \quad (7c)$$

where $\boldsymbol{P}^T = E[rd(n)]$ represents the cross-correlation matrix between a weighted filtered reference input vector and a desired signal, and $\boldsymbol{R} = E[\boldsymbol{r}^T r]$ represents the auto-correlation matrix of a weighted filtered reference input vector.

The quadratic nature of the error surface can now be clearly identified, and it can be confirmed that the surface has a unique global minimum by examining the positive definiteness of $\boldsymbol{R}$. By setting the differential of this expression with respect to $\boldsymbol{W}$ to zero, the optimum Wiener set of coefficients may be obtained as

$$\therefore\ \boldsymbol{W}_{opt} = (E[\boldsymbol{r}^T r])^{-1} E[\boldsymbol{r}^T d(n)] = \boldsymbol{R}^{-1}\boldsymbol{P}. \quad (8)$$

This set of filter coefficients gives a minimum error criterion equal to

$$J_{min} = E[d^T(n)d(n)] - E[d^T(n)](E[\boldsymbol{r}^T])^{-1}E[\boldsymbol{r}^T d(n)] \quad (9a)$$

$$= E[d^T(n)d(n)] + \boldsymbol{P}^T\boldsymbol{W}_{opt}. \quad (9b)$$

The error criterion can now be rewritten as

$$J = J_{min} + (\boldsymbol{W} - \boldsymbol{W}_{opt})^T \boldsymbol{R} (\boldsymbol{W} - \boldsymbol{W}_{opt}) \quad (10)$$

III. Computer Simulation and Results

The overall active control systems may be nonlinear due to some components of the systems. The most significant cause of nonlinearity present in an active noise control system is usually due to the loudspeaker acting as the secondary source, including amplifiers, converters, and microphones acting as the sensor, *etc.* In this environment, the performance of active noise control may be decreased by the extent of nonlinearities, and an active control system compensating nonlinear distortions is needed.

For computer simulations, the overall active noise control scheme shown as Fig. 4 is used in real car environments. Fig. 5 and Fig. 6 show the block diagram of active noise control to attenuate the car noises and compensate the nonlinear distortions in the secondary path simultaneously, for the general LMS controller and the proposed controller, respectively. In the both figures, the transfer function from the input of the actuator to the output of the sensor is modeled as a J-th order FIR filter, whose j-th coefficient is C_j. And it is assumed that the nonlinear distortion exist and represented as a non-linear filter H. In general, the nonlinear filter to represent the nonlinear system, may be based on the functional series. The characteristics of the non-linear filter is similar to that of linear systems because the filter output is composed of linear combination of filter coefficients. Also, the adaptive algorithms in linear systems to adapt the filter coefficients, can be applied to the nonlinear system directly, and the analysis is similar to that of the linear system. As the adaptive algorithms to adapt the filter coefficients in the nonlinear system, the least mean square (LMS) and least square (LS) algorithms *etc.* can be used. However, the nonlinear filter may have more com-putation complexities and slower convergence speed than those of the linear system because it may use more coefficients than the nonlinear system. There-fore, the nonlinear control system with less com-putation complexities and faster convergence speed is needed.

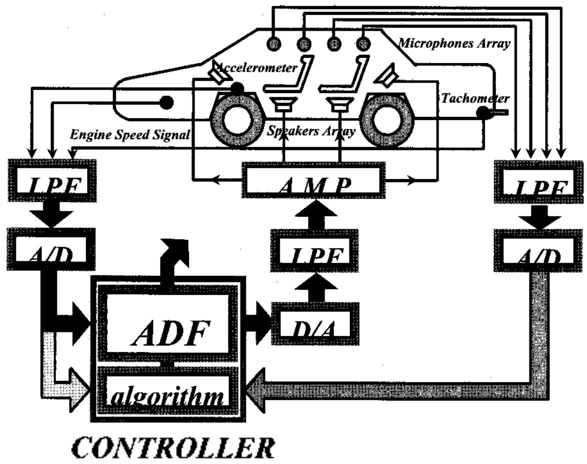


Fig. 4. The overall active noise control scheme in real car environments.

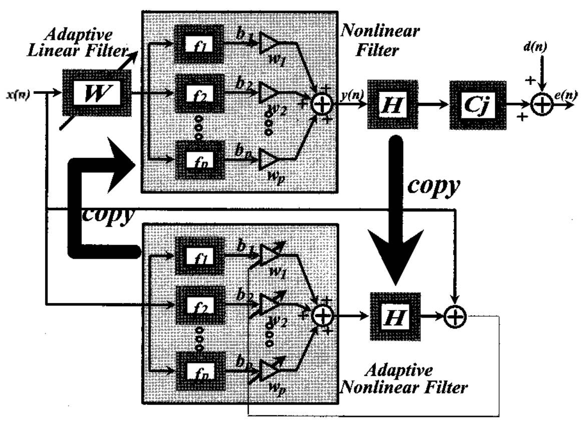


Fig. 5. The block diagram of the active noise control system with a compensating nonlinear filter using the general filtered-x LMS algorithm.

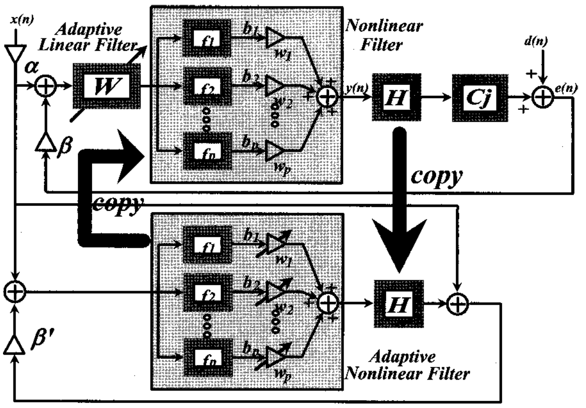


Fig. 6. The block diagram of the active noise control system with a compensating nonlinear filter using the pro-posed algorithm.

In the simulations, the sigmoid function is used as a nonlinear filter. In the figure, the adaptive nonlinear filter is a pre-distorter and functions as a inverse filter to estimate the transfer function of secondary path C_j [4]. Also, the filtered-x LMS algorithm is used to update the adaptive linear filter W that controls noises in a car.

In the simulations, in order to estimate the impulse response of the secondary path, the maximum length sequence [6] is used as an input to a loudspeaker. The maximum-length sequence is used mainly to estimate the impulse response of the time-invariant system and is composed of two values. And the sequence has merits that the fast computation is possible due to the fast Hadamad transform and in the presence of the noise, it increases the signal to noise ratio (SNR). In environments of Fig. 4, the estimated impulse responses using the maximum length sequence are shown in the Fig. 7 From the figure, it is known that the coefficient number is 256 and the responses in a car are almost a lightly -damped acoustic systems.

Fig. 8 and Fig. 9 represent the results for the active control of engine noises, for the general LMS

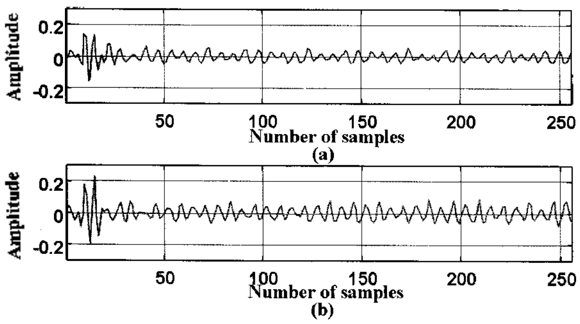
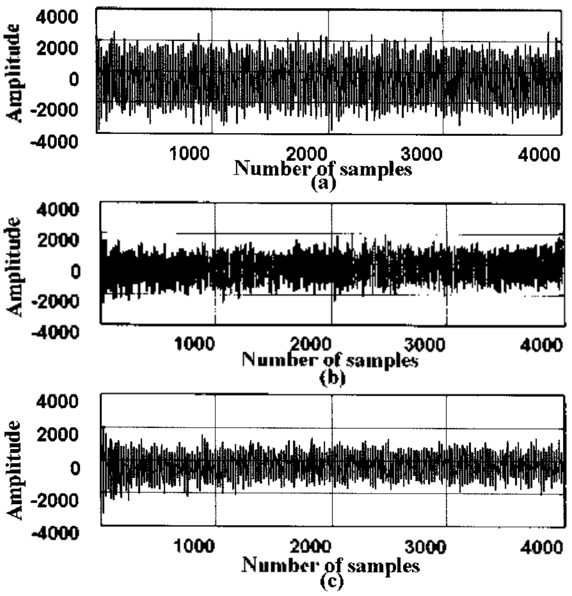
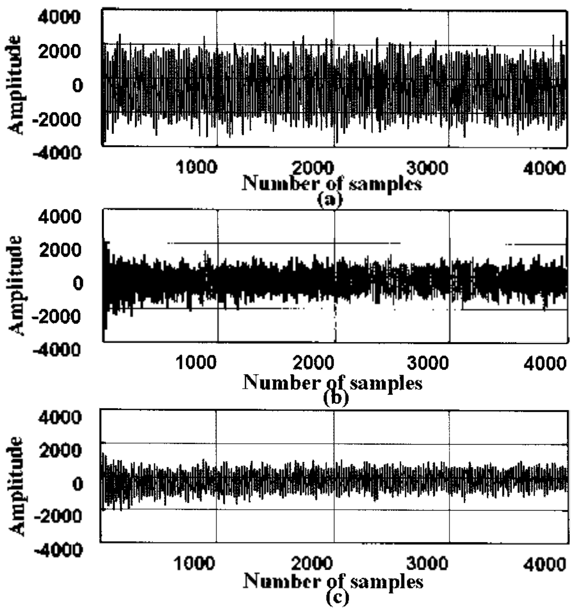


Fig. 7. The impulse responses of the primary path and the secondary paths (a) primary path (b) secondary path.



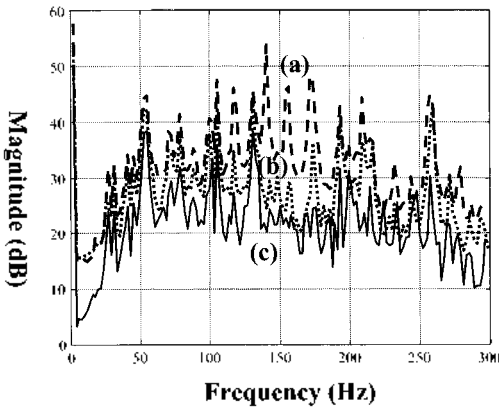
(A) without the adaptive nonlinear compensator



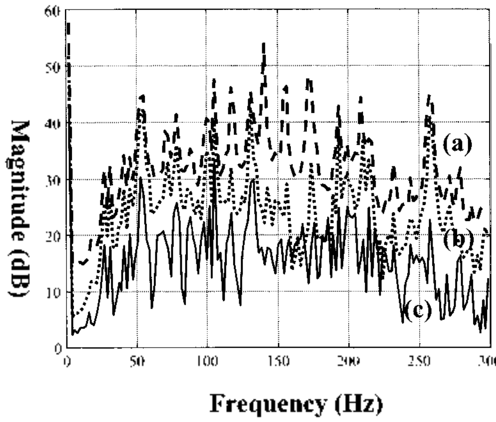
(B) with the adaptive nonlinear compensator

Fig. 8. The time sampled primary noise signal before control (a) and the time sampled error signals of the general LMS control (b) and the proposed control (c).

controller and the proposed controller in the time domain and the frequency domain, respectively. The real engine noises are picked up directly in a car, under the structure of Fig. 4 The time length of data samples is four seconds by the sampling rate of 1000 Hz. And the number of adaptive linear filter coefficients and the impulse response coefficients used are 64 and 64, respectively. The engine rotations per minutes (RPM) are ranged from 3000 to 5000 rpm and their dominant noise components (C2) are distributed between 100 Hz to 166.7 Hz. For simulations, the engine ignition signal is used as a reference signal and an error sensor output in a cabin is used as a primary signal, and both reference signal and primary signal are band-limited to 300 Hz using four-pole Butterworth low pass filter (LPF). In the figures, the active noise controller with the adaptive



(A) without the adaptive nonlinear compensator



(B) with the adaptive nonlinear compensator

Fig. 9. The power density spectrum of (a) the primary noise before control (dashed line) (b) the general LMS control (dotted line) (c) the Proposed control (solid line).

nonlinear compensator has significant advantages over the active control system without the adaptive nonlinear compensator in convergence speed and noise attenuation performance in time domain and frequency domain, respectively. Especially at low frequencies, the performance difference of both algorithms is distinct. That is because the former system cannot control well due to the distortions of nonlinear elements such as speakers, whereas the latter system can control the primary noise, compensating for the nonlinear distortions at the same time. Also the proposed system has significant advantages over the general LMS control system in convergence speed and noise attenuation performance in time domain and frequency domain, respectively. In the figures, the results represent the control performances without an adaptive nonlinear filter for compensation of nonlinearity in *H* and with the adaptive nonlinear filter, respectively, in which the coefficients of the adaptive nonlinear filters are used to be copied to the nonlinear filters when both adaptive nonlinear filters converge to their optimum solutions sufficiently. From the figures, it is verified that the performance of noise control with nonlinear compensator is better than without nonlinear compensator. In the figures, (a), (b), and (c) represent the signals of primary noise before control, of the general LMS control, and of the proposed control, respectively. From the figures, it is verified that in both case of (A) and (B), the proposed controller can control the engine noise better than the general LMS algorithm, and both controllers can control the noises with the nonlinear compensator better than without the compensator. This is because the compensators may linearize the secondary path, so the adaptive linear filter can control better.

IV. Conclusions

In this paper, I present a robust active noise controller based on the combination of feedforward and feedback control systems. Through computer simulations as well as the analytical analysis, it could be shown that it is possible for both conventional LMS controller and proposed controller, to be applied for actively controlling the noises and linearizing the nonlinear distortion in the secondary path. Also, the simulations results demonstrated that the proposed controller may have faster convergence speed and better capability of controlling the noises and compensating the nonlinear distortion than the conventional LMS controller.

References

1. L.J.Erikkson, M.C.Allie and R.A.Greiner, "The Selection and Application of an IIR Adaptive Filter for Use in Active Sound Attenuation," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. ASSP-35, no. 4, pp. 433-437, 1987.
2. A.M.McDonald, S.J.Elliott and M.A.Stokes, "Active Noise and Vibration Control within the Automobile," *Proc. ASJ Symposium'91*, pp. 147-156, April 1991.
3. O.S.Kwon, I.W.Cha, "The Optional Summed Algorithm for Active Noise Control," *J. of the Acoustical Society of Korea*, vol. 16, no. 5, pp. 18-25, 1997.
4. Oh Sang Kwon, "Hybrid Adaptive Volterra Filter Robust to Nonlinear Distortion," *J. of the Acoustical Society of Korea*, vol. 27, no. 3E, pp. 95-103, 2008.
5. B.Widrow and S.D.Stearns, *Adaptive Signal Processing*, Englewood Cliffs, NJ : Prentice-Hall, 1985.
6. D.D.Rife, "Modulation Transfer Function Measurement with Maximum-Length Sequences," *J. Audio Eng. Soc.*, vol. 40, no. 10, pp. 779-789, 1992.

[Profile]

• **Oh-Sang Kwon**

1988.03~1992.02 연세대학교 전자공학 공학사
1992.03~1994.02 연세대학교 전자공학 공학석사
1994.03~1999.02 연세대학교 전자공학 공학박사
1999.03~2003.10 삼성전자 책임연구원
2005.03~현재 한국디지털대학교 컴퓨터정보통신학과 전임교수