

Analysis and Design of Surface Plasmon Waveguide

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ABSTRACT

In this paper, we developed and presented a design result for optimizing the geometry of Ag circular SPP waveguide for subwavelength waveguide applications. We investigated the effect of the design parameters on the light propagation and find the optimum design for small modal size, high coupling coefficient, and low sensitivity. The results show that the globally optimal design locates optimal waveguide geometries more efficiently than individual optimal points for multivalued objective function.

Key Words : Surface Plasmon Polariton (SPP), circular waveguide, optimal design

1. Introduction

Recent development in nanoscale fabrication and characterization techniques has given rise to interest in optical devices for guiding and confining electromagnetic energy to scales within diffraction limit of light, which are based on surface plasmon polaritons (SPPs) [1]. SPPs are charge oscillations of the conduction electrons of metallic media localized at the interface with a dielectric at the optical and near infrared frequency spectrum. In this frequency range, the real part of the dielectric function of metal is a negative dielectric function. The electromagnetic field associated with SPPs is sharply bounded along the interface and decreases exponentially in the direction perpendicular to the interface resulting in a subwavelength confinement of the electromagnetic wave [2].

In this paper, we present comprehensive analysis of fundamental SPP waveguide structures and a novel design result considering narrow confinement and coupling efficiency with conventional optical waveguide for circular SPP waveguides.

2. Thick Ag Films

The free electron gas model was first introduced by

Drude to express electron transport in metals. In the limit of negligible collision frequency, the dielectric function takes the form

$$\epsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \quad (1)$$

where ω_p defines the bulk plasma frequency of the material. While this form of the dielectric function is commonly quoted for metallic films [3-4], its validity can be limited only to near infrared frequencies. A more accurate description of surface plasmon behavior requires use of empirically determined optical constants. Bulk Ag is most often described by the refractive index ($n+ik$) data sets from the Palik *Handbook of Optical Constants of Solids* [5].

Figure 1 shows the relative dielectric functions of Ag both from Drude model Eq. (1) and Palik' data. In this paper, we choose the dielectric constant from Palik' data to obtain realistic results.

Since the real part of the dielectric constant $\epsilon(\omega)$ of metallic media is negative up to the plasma frequency ω_p , and thus of opposite sign as the dielectric constant ϵ_2 of the adjacent dielectric,

The dispersion relation $w(\beta)$ of SPPs at a single, flat interface between a metal and a dielectric can be obtained by solving a suitably defined boundary condition on Maxwell's equations as follows

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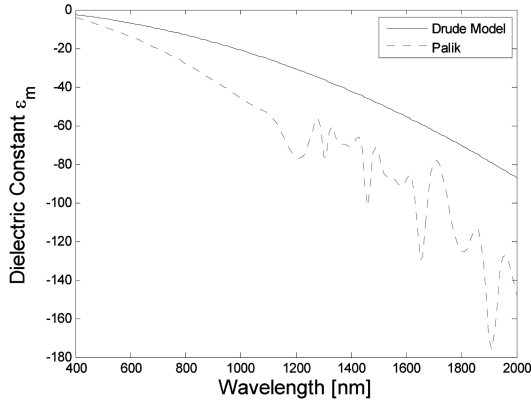


Fig. 1. The Dielectric Constant of Ag using a free-electron model (solid line) and data from Palik (dotted line).

$$\beta = \frac{\omega}{c} \sqrt{\frac{\epsilon(\omega)\epsilon_2}{\epsilon(\omega) + \epsilon_2}}. \quad (2)$$

Figure 2 illustrates the dispersion characteristics using the dielectric function of a free electron gas with $\omega_p = 5.833$ eV. The dielectric constant ϵ_2 of a dielectric is taken as 4 and we consider dielectric constant of Ag as real. For energies below $\omega_{sp} = 3.3$ eV, the typical bound SPP mode is observed. Above ω_p , the dielectric constant of Ag becomes positive and thus, bound mode cannot be sustained by two media with both positive dielectric constant, resulting in creating radiative plasmon polariton (RPP)

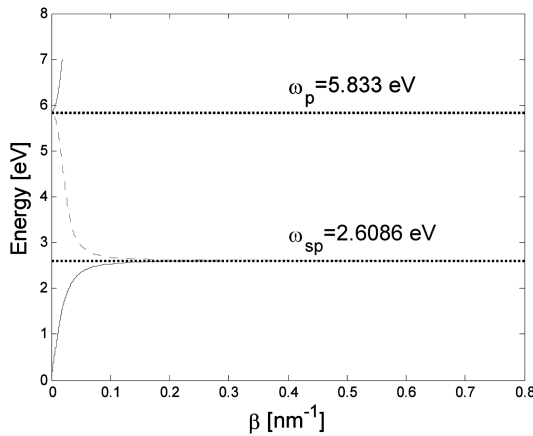


Fig. 2. Surface plasmon dispersion relation at a single interface of Ag and a dielectric ($\epsilon_2 = 4$).

mode. For energies between the SPP and RPP modes, that is, in the range between ω_{sp} and ω_p the plasmon wave vector is purely imaginary, indicating that modes in this regime are forbidden, which is shown by dotted line.

3. Analysis of Ag Circular Waveguide

The structure considered in this paper is shown in Fig. 3. It consists of a metal cylinder of radius a and negative dielectric constant surrounded by an infinite homogeneous dielectric of permittivity $\epsilon_2 = 4$.

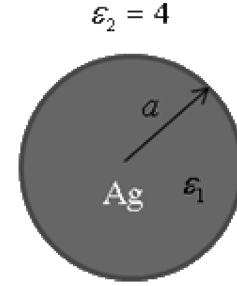


Fig. 3. The cross section of Ag circular waveguide embedded by dielectric material ($\epsilon_2 = 4$). The shaded area shows Ag.

We calculated the phase constant and the beam diameter of an optical waveguide by solving Maxwell's equations analytically. We consider the lowest- (0th-) order TM mode. The electromagnetic field in this structure is assumed to be propagating in the z direction in a sinusoidal form. It is easily found that the longitudinal electric field is given as

$$E_z(r) = \begin{cases} AI_0(hr), & r \leq a \\ BK_0(qr), & r \geq a \end{cases} \quad (3)$$

where A and B are any constants, r is the radius direction of the cylindrical coordinates, a is the radius of the core, I_n and K_n are the n th-order modified Bessel functions, and h and q are defined as

$$h = \sqrt{\beta^2 - n_1^2 k_0^2}, \quad q = \sqrt{\beta^2 - n_2^2 k_0^2} \quad (4)$$

where $n_1^2 = \epsilon_1$, $n_2^2 = \epsilon_2$.

From the boundary conditions for E_z and H_f at $r=a$, we obtain the following constant relation

$$B = A \frac{I_0(ha)}{K_0(qa)} \quad (5)$$

and the characteristic equation of the 0th-order TM mode as

$$\frac{n_1^2 I_1(ha)}{ha I_0(ha)} = -\frac{n_2^2 K_1(qa)}{qa K_0(qa)} \quad (6)$$

Transverse magnetic field are calculated as following

$$H_\phi = \begin{cases} AI_1(hr), & r \leq a \\ -A \frac{I_0(ha) h n_2^2}{K_0(qa) q n_1^2} K_1(qr), & r > a \end{cases} \quad (7)$$

The effective beam radius can be calculated numerically by using Eq. (6) defined as

$$K_1(hr_H) = \frac{1}{e} K_1(qa) \quad (8)$$

Figure 4 shows calculated effective beam radius with respect to the core radius. From Fig. 4, it can be seen that the effective beam radius increases with the core radius. Therefore, to confine electromagnetic wave sharply using this waveguide, it is required to select small radius value.

4. Design of Ag Circular Waveguide

To design Ag circular waveguide, we must under-

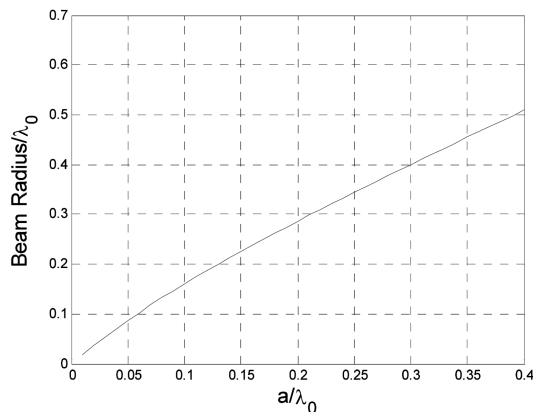


Fig. 4. Effective beam radius in Ag circular waveguide with respect to the core radius.

stand the relationship between the geometry and important design considerations such as modal size, coupling efficiency with conventional optical device, and its sensitivity. Figure 5 depicts the magnetic field distribution of the SPP mode (H_ϕ) and fundamental mode of optical fiber with core radius 100 nm, $\Delta n=2.8\%$, and wavelength 633 nm. From this figure, it is known that the coupling efficiency become small at the both end of small and large core radius, and has the largest value at a certain point.

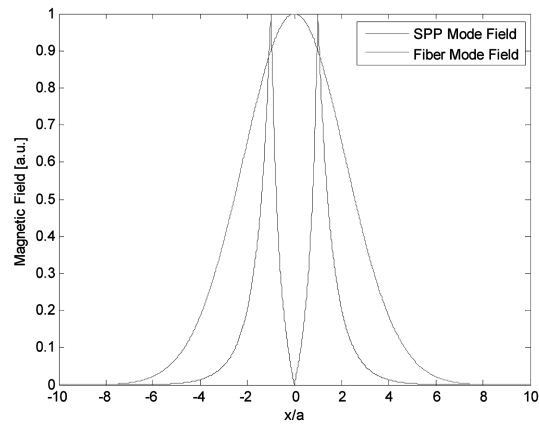


Fig. 5. the magnetic field distribution of the SPP mode (H_ϕ) and fundamental mode of optical fiber with core radius 100 nm, $\Delta n=2.8\%$, and wavelength 633 nm.

Figure 6 shows the calculated launching efficiency from optical fiber into SPP waveguide with Ag cylinder defined as

$$\eta = \frac{\int_0^\infty H_{SPP} H_{fiber} r dr}{\left[\int_0^\infty H_{SPP}^2 r dr \int_0^\infty H_{fiber}^2 r dr \right]^{1/2}} \quad (9)$$

To obtain large coupling efficiency, the optimal point must be taken. But it becomes clear from Fig. 4 and Fig. 6 that individual optima may not match so the “total” optima is required for optimization of a given objective function including all design parameters.

Figure 7 shows the calculated sensitivity of coupling efficiency defined as

$$S_i = \frac{\eta_{i+1} - \eta_i}{\eta_i} \quad (10)$$

Figure 8 shows each optimal point corresponding to design parameter, launching efficiency, sensitivity, and effective beam radius.

To obtain a optimal result, we have developed a figure of merit to evaluate the tradeoff between modal size, coupling efficiency and its sensitivity. Reducing the modal size of the waveguide is essential for miniaturization of this photonic device. The figure of merit is defined as

$$OF = \frac{\eta}{M_s} e^{-|S|} \quad (11)$$

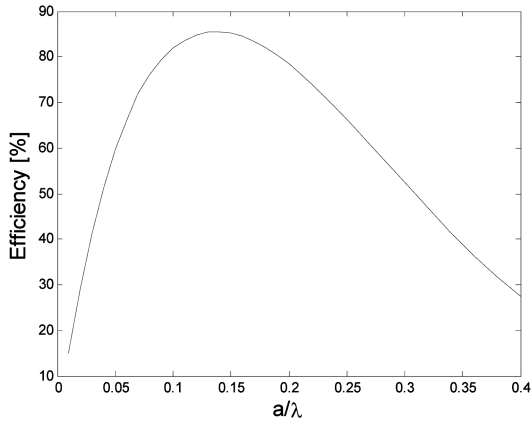


Fig. 6. The calculated launching efficiency from optical fiber into SPP waveguide with Ag cylinder.

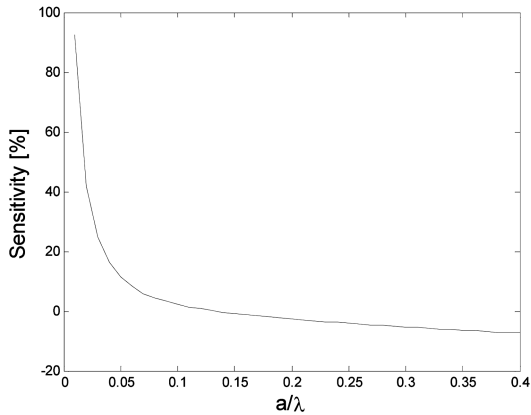


Fig. 7. The calculated sensitivity of coupling efficiency.

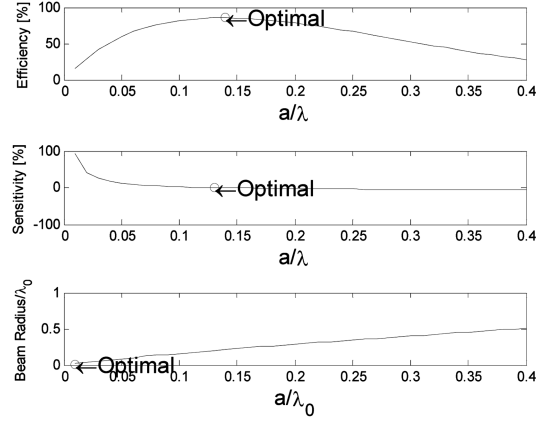


Fig. 8. Each optimal point corresponding to design parameter, launching efficiency, sensitivity, and effective beam radius.

where $M_s = r_H^2$, and S is the sensitivity of the coupling efficiency.

Figure 9 shows the optimal point for our figure of merit including all design parameters, launching efficiency, sensitivity, and effective beam radius. Although each design parameter has its optimal point, the optimization procedure locates the globally optimal geometry for the Ag circular waveguide.

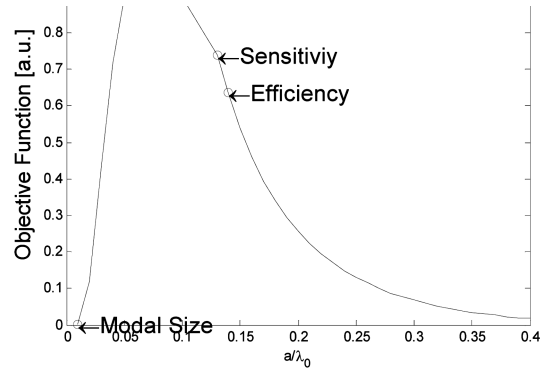


Fig. 9. The optimal point for our figure of merit including all design parameters, launching efficiency, sensitivity, and effective beam radius.

5. Conclusions

In this paper, we developed and presented a design result for optimizing the geometry of Ag circular SPP waveguide while simultaneously achieving small modal size, high coupling coefficient, and low sensitivity. The

results indicate that the globally optimal design locates optimal waveguide geometries more efficiently than individual optimal points, which is essential for the case when various design parameters are required simultaneously.

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