Optimal Perilune Altitude of Lunar Landing Trajectory

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Abstract

In general, the lunar landing stage can be divided into two distinct phases: de-orbit and descent, and the descent phase usually comprises two sub-phases: braking and approach. And many optimization problems of minimal energy are usually focused on descent phases. In these approaches, the energy of de-orbit burning is not considered. Therefore, a possible low perilune altitude can be chosen to save fuel for the descent phase. Perilune altitude is typically specified between 10 and 15km because of the mountainous lunar terrain and possible guidance errors. However, it requires more de-orbit burning energy for the lower perilune altitude. Therefore, in this paper, the perilune altitude of the intermediate orbit is also considered with optimal thrust programming for minimal energy. Furthermore, the perilune altitude and optimal thrust programming can be expressed by a function of the radius of a parking orbit by using continuation method and co-state estimator.

Key Word : Lunar landing, Optimal trajectory, Perilune altitude

Introduction

Research for lunar exploration have been pursued in many countries and regions. However, lunar exploration experienced discontinuity after intensive effort for lunar missions was accomplished in the 60s and 70s. Recently, many nations are planning to renew their lunar exploration effort for potential benefit from the explorations. The moon's soil is known to contain helium-3 which is main substance to run a thermo-nuclear fusion reactor. To achieve next-generation lunar exploration mission objectives, improved versions of technology for safe and precise landing are required. Thus, the lunar landing missions should be considered as a key research subject in the near future. There have been significant effort for precise landing and navigation for reliable and efficient lunar exploration .

Generally, the lunar landing stage is divided into two distinct phases: de-orbit and descent, and the descent phase comprises two sub-phases: braking and approach. And the optimization problem of minimal energy is largely focobld on descent phases. To find such optimal solution, two-dimensional approaches has been alreadnd tudied in many researches. One of them is investigated by Ramanan[2] and Shan[3]. Ramanand tudied the optimal lunar landing strategies and analyzed such strategies on case-by-case basis. Shannd tud tudied two-dimensional lunar landing problemstimal variable two-dilevelrategis on casHwakins[4] tudied a verting lunar landing problemwakins[4] stececraft rotationg motions. Liund tud tudied the landing ent phase control[5a6] akins[4] these optimal lunar landing trajectory. McInnes[7] uggested another ent phasestrategy of verting landing by using gravity-turn technique.

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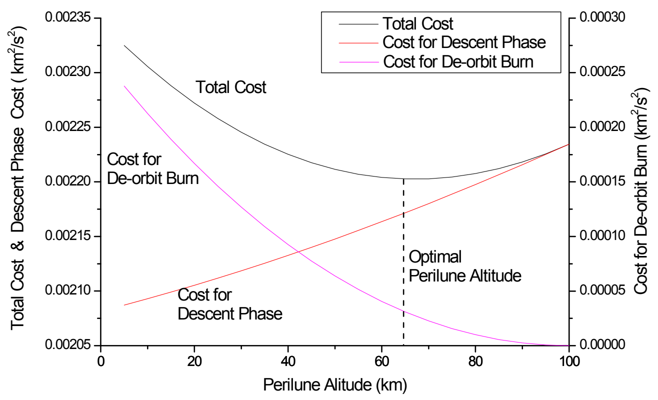

Fig. 1. Cost function Values history for the perilune altitude

However, in these approaches, the energy of de-orbit burning has not been considered. Therefore, a low perilune altitude can be chosen to save fuel for the descent phase. Usually, the perilune altitude is chosen between 10 and 15 km due to mountainous lunar terrain and potential guidance errors. However, it requires more de-orbit burning energy for the lower perilune altitude. In Fig. 1, the cost history is plotted over the change of the perilune altitude conditions. In this paper, the perilune altitude of the intermediate orbit is also considered with optimal thrust programming for minimal energy orbit maneuver. Furthermore, these perilune altitude and optimal thrust programming can be expressed by a function of the radius of a parking orbit by continuation method and co-state estimator[8] approach

Solution Process

2.1 Problem description

In this paper, the minimum energy trajectory for lunar landing will be discussed. Therefore, using delta-V (changing in velocity), he cost function for the minimum-nergy problem can be written as follows.

$$J = \underbrace{\frac{1}{2}\Delta V^2}_{De\text{-}orbit\ burn\ Phase} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f}\left(\frac{T}{m}\right)^2 dt}_{Descent\ Phase} \tag{1}$$

where, t_0 represents the initiated time of descent phase. And the energy for the de-orbit phase can be describe as a function of r_0 (perilune radius) as follows.

$$\Delta V = \sqrt{\mu\left(\frac{2}{r_p} - \frac{2}{r_p + r_0}\right)} - \sqrt{\frac{\mu}{r_p}} \tag{2}$$

where, r_p represents the radius of lunar parking orbit and is given.

2.2 Assumptions

There have been considerable research for the lunar landing problem. In many cases, a two-dimensional problem is usually studied for the simplicity. Therefore, the following assumptions are made in this paper.

i) The lunar gravity field is uniform during the lunar landing phase, and the Moon is entirely a sphere body.

ii) The lunar parking orbit is a circular orbit. And this parking orbit is placed on the lunar equatorial plane.
iii) The lunar rotates on its own axis with a constant angular velocity, and the lunar lander in the parking orbit is rotating in the same direction as the Moon.
iv) The orbit transfer strategy at the de-orbit burn phase is Hofmann method.
v) The thrust level of the lunar lander is constant for descent phase.

These assumptions are usually employed in other previous researches.

2.3 Governing Equations

The planar motion of the lunar lander is described in Fig. 2. In this figure, r and ϕ represent radial distance and position angle, u and v represent transverse and radial velocity, ω represents the lunar rotation velocity, T represents a thrust vector of lunar lander and β represents a thrust vector angle which is control command, respectively. Using these parameters, the governing equations of motion can be obtained as

$$\dot{r} = v \tag{3}$$

$$\dot{\phi} = \frac{u}{r} \tag{4}$$

$$\dot{u} = -\frac{uv}{r} + \frac{T}{m}\cos\beta \tag{5}$$

$$\dot{v} = \frac{u^2}{r} - \frac{\mu}{r^2} + \frac{T}{m}\sin\beta \tag{6}$$

$$\dot{m} = -\frac{T}{I_{sp}\, g} = Const. \tag{7}$$

where, I_{sp} and g denote specific impulse and gravitational acceleration on the Earth, respectively. These parameters are assumed to be constant values.

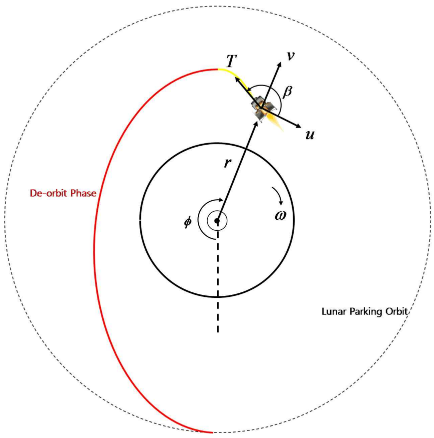

Fig. 2. Polar Coordinate of lunar landing

2.4 Optimal Control Problem

In order to find the control variable profile for the minimum-energy lunar landing trajectory, calculus of variation approach is used. Therefore, the Hamiltonian is formed in terms of the cost function and governing equations.

$$H = \frac{1}{2}\left(\frac{T}{m}\right)^2 + \lambda_r \dot{r} + \lambda_\phi \dot{\phi} + \lambda_u \dot{u} + \lambda_v \dot{v} + \lambda_m \dot{m} \tag{8}$$

where, $\lambda = [\ \lambda_r \quad \lambda_\phi \quad \lambda_u \quad \lambda_v \quad \lambda_m\ ]^T$ are co-state variables.

And, according to the optimal control theory, the time derivative of these costate variables can be written as follows . ($\dot{\lambda} = -\partial H / \partial x$)

$$\dot{\lambda}_r = \lambda_\phi \frac{u}{r^2} - \lambda_u \frac{uv}{r^2} + \lambda_v \left(\frac{u^2}{r^2} - 2\frac{\mu}{r^3}\right) \tag{9}$$

$$\dot{\lambda}_\phi = 0 \tag{10}$$

$$\dot{\lambda}_u = -\frac{\lambda_\phi}{r} + \lambda_u \frac{v}{r} - \lambda_v \frac{2u}{r} \tag{11}$$

$$\dot{\lambda}_v = -\lambda_r + \lambda_u \frac{u}{r} \tag{12}$$

$$\dot{\lambda}_m = \frac{T}{m^2}\left(\frac{T}{m} + \lambda_u \cos\beta + \lambda_v \sin\beta\right) \tag{13}$$

The optimal control variable profile can be also obtained by optimal control theory. Therefore, the following two equations are satisfied.

$$\frac{\partial H}{\partial \beta} = \frac{T}{m}\left(-\lambda_u \sin\beta + \lambda_v \cos\beta\right) = 0 \tag{14}$$

$$\frac{\partial^2 H}{\partial \beta^2} = \frac{T}{m}\left(-\lambda_u \cos\beta - \lambda_v \sin\beta\right) > 0 \tag{15}$$

The optimal control variable profile is obtained as follows.

$$\beta = \tan^{-1}\left(\frac{-\lambda_v}{-\lambda_u}\right) \tag{16}$$

2.5 Two-Point Boundary Value Problem (TPBVP)

For the lunar landing mission the following terminal constraints have to be satisfied.

$$r_f = r_{moon} \ , \ u_f = r_{moon}\,\omega \ , \ v_f = 0 \tag{17}$$

where, r_{moon} is the radius of the moon, and the subscript f denotes the values at the final time. In the inertial frame, the horizontal velocity is not zero at the lunar surface.

Besides those final constraints, there are some initial constraints as well because the initial state is not fixed. As mentioned previously , the Hofmann transfer strategy is adopted for the de-orbit burn phase, and the start point of optimal control problem is the perilune of this orbit. Therefore, these initial states and constraints can be described as follows.

$$\phi_0 = 180^\circ \ , \ u_0 = \sqrt{\mu\left(\frac{2}{r_0} - \frac{2}{r_0 + r_p}\right)} \ , \ v_0 = 0 \ , \ m_0 = M \tag{18}$$

where, M is the total mass of the lunar lander, and the subscript 0 means the values at the initial time.The augmented constraints function can be written as

$$G = \frac{1}{2}\Delta V^2 + \nu^T \Psi + \xi^T \theta \tag{19}$$

where, ν and ξ are Lagrange multipliers, and θ and Ψ are the initial and final sate constraints, respectively. And these constraints can be expressed as follows.

$$\Psi = [\ r_f - r_{moon} \quad u_f - r_{moon}\,\omega \quad v_f]^T \tag{20}$$

$$\theta = \left[\begin{array}{cccc} \phi_0 - 180^\circ & u_o - \sqrt{\mu\left(\frac{2}{r_0} - \frac{2}{r_0 + r_p}\right)} & v_0 & m_0 - M \end{array} \right]^T \tag{21}$$

Usually, the augmented constraints function involves the final state in the optimal control problem. However, we want to also find the perilune altitude to minimize total lunar landing energy. So, the cost function only depends on the initial state, and the augmented constraints function also depend on the initial state. Therefore, the following boundary conditions can be derived from the optimal control theory[9].

$$H_f = -G_{t_f} \tag{22}$$

$$\lambda_f = G_{x_f}^T \tag{23}$$

$$\lambda_0 = -G_{x_0}^T \tag{24}$$

From these equations, the boundary conditions for co-state variables can be written such that

$$H_f = 0 \tag{25}$$

$$\lambda_\phi(t_f) = \lambda_m(t_f) = 0 \tag{26}$$

$$\lambda_r(0) = -\Delta V \frac{\partial \Delta V}{\partial r_0} - \lambda_u(0) \frac{\partial}{\partial r_0} \sqrt{\mu\left(\frac{2}{r_0} - \frac{2}{r_0 + r_p}\right)} \tag{27}$$

Therefore, the optimal energy lunar landing problem can be solved by finding the initial state and co-state variables satisfying these given boundary conditions.

2.6 Solution of TPBVP - Shooting Method

In the previous section, the optimal control problem can be solved by appropriate values for the initial state and co-state variables. This approach is usually solved by some parameter optimization methods. There are many parameter optimization techniques readily available such as SQP, Evolutionary algorithm, Genetic algorithm, CEALM, PSO, etc.. In this paper, the shooting method is adopted for parameter optimization because of simplicity for programming and fast convergence despite some inherent shortcomings. Actually, the results of the co-state initial values are very small. So, choosing a boundary range for those parameters is very difficult for stochastic processes.

For the shooting method, the constraints matrix(h) is the function of the initial and final state variables and final time. So, the differential of this constraints matrix can be expressed as follows.

$$dh = \dot{h}_f \, dt_f + h_{z_0} \delta z_0 + h_{z_f} \delta z_f + H.O.T \tag{28}$$

where, the $z = [\, x^T \;\; \lambda^T \,]^T$ represents augmented state matrix and t_f is the final time. And the subscript 0 and f denote those values at the initial and final time, respectively. For simplicity, higher order terms are neglected in this paper. This equitation can be rewritten for only initial augmented state matrix and final time by using the state transition matrix Φ.

$$dh = \left[\begin{array}{cc} \dot{h}_f & h_{z_0} + h_{z_f} \Phi_f \end{array} \right] \left[\begin{array}{c} dt_f \\ \delta z_0 \end{array} \right] \tag{29}$$

To reduce these constraints, we seek to satisfy the following equation.

$$dh = -\alpha h \;\; , \;\; 0 < \alpha \leq 1 \tag{30}$$

Therefore, we can obtain the update law of the augmented state variables at the initial time such that

$$\begin{bmatrix} dt_f \\ \delta z_0 \end{bmatrix} = -\alpha \begin{bmatrix} \dot{h}_f & h_{z_0} + h_{z_f} \Phi_f \end{bmatrix}^{-1} h \tag{31}$$

Using this update law, the optimal solution to satisfy the TPBVP can be obtained by iterative process.

2.7 Continuation Method

In the previous section, we can formulate a TPVBP and a key soothing algorithm for the optimal lunar landing trajectory. However, this optimal solution can change with the change in the lunar parking orbit altitude. But due to the well-known difficulties such as convergence radius, discrete continuation methods is added to the shooting method for finding several optimal trajectories. First, we attempt to solve the TPBVP which is solved rather easily. Then by taking the solution already obtained as an initial guess for the next step, several problems can be solved continuously. In this problem, final altitude can be chosen as a variable, and as this variable decreases, one can find the solutions continuously.

Also, initial guess can be obtained by functional approximation and extrapolation technique for more accurate guess and guaranteed rapid convergence. Since, the dynamics are not very fast, the state and co-state variables do not rapidly change. Therefore, the optimal solution can be approximated smoothly with polynomials on a lunar parking orbit altitude. And taking extrapolation, with certain altitude, we can find the proper initial costate conditions. Therefore, the optimal solutions can be constructed as a function of the lunar parking orbit altitude.

Simulation Results and Discussion

For the numerical simulation, we use identical simulation conditions in Ref.[2]. So, the initial parking orbit altitude is 100km from lunar surface and the initial mass of the lunar lander at the starting point of descent phase is 300kg. And the constant thrust level is 440N, the specific impulse of the thruster is 310sec.

The simulation results are described in Table 1. In this simulation, the perilune altitude is 61.0147km. This perilune altitude is much higher than 15 km which is used in the previous researches. So, the terminal time is increased while the final landing mass decreases. This means the cost for the descent phase is increased. However, the cost values for the Hofmann transfer decrease more than this increase. Therefore, the total cost value is decrease. And the initial co-state values for the descent phase are described in the same table. In this result, the trajectory is very sensitive to the initial co-state values, and the initial co-state value of the radial distance is especially small. Thus it is very difficult to choose the boundary range for stochastic processes.

The descent phase profiles are shown in the Fig. 3 to Fig. 6. In the trajectory figure, Fig. 3, there is not altitude increment which is obtained in the Ref.[2]. In the Ref.[2], there is altitude increment to maximize horizontal braking that minimized the gravity loss. However, in this simulation result, this altitude increment does not exist. Therefore, the total cost is decreased. In Fig. 4, the control input angle is displayed. In this figure, the control input iis dvery smooth isplayed. Thus this control input can be implemented for the lunar lander. Finally, the radial and tangential velocity history during the descent phase with this control input profile are displayed in Fig. 5. And the co-state histories are plotted in Fig. 6. In Fig. 5, the two kinds of velocity exhibit some different velocity range. So, the range is divided in the left and right-hand side. Therefore, the magnitude of the radial velocity profile, red line, is indicated in the right-hand axis.

Based on the previous simulation, the optimal solution is easily obtained for the 100km lunar parking orbit. However, this parking orbit altitude can be a variable to meet the mission requirements. As changing the lunar parking orbit altitude, the optimal perilune altitude must be changed. For this reason, it is useful that the optimal perilune altitude is described as a function of the lunar parking orbit. Thus, the same simulations are conducted with various lunar parking orbit altitude conditions. The simulation results are presented in Fig. 7 and Fig. 8. The optimal perilune altitude is similar to linear function of the lunar parking orbits in Fig. 7. Therefore, the curve fitting is accomplished as the first order, and the results are obtained as follows.

Table 1. Simulation results for the optimal lunar landing

Optimal Perilune altitude (km)	Optimal Terminal time for the descent phase (sec)	Initial Co-state variables of the descent phase
61.0417	1019.7998	$\lambda_r(0) = 0.0024$ $\lambda_\phi(0) = 0$ $\lambda_u(0) = 1.0911$ $\lambda_v(0) = 0.0698$ $\lambda_m(0) = -3.6493$

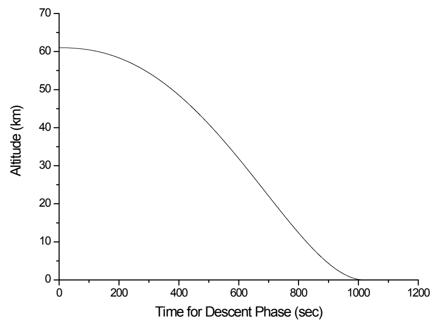

Fig. 3. Trajectory of the lunar lander

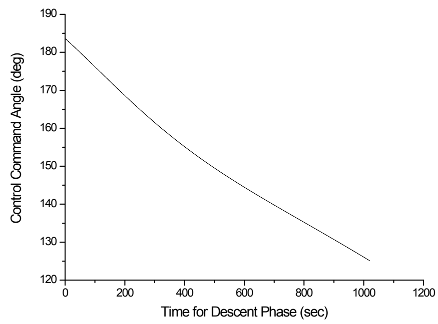

Fig. 4. Control command profile

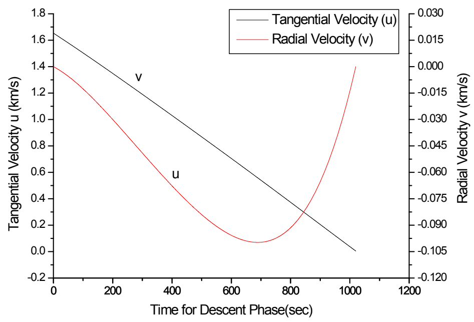

Fig. 5. Velocity Profile

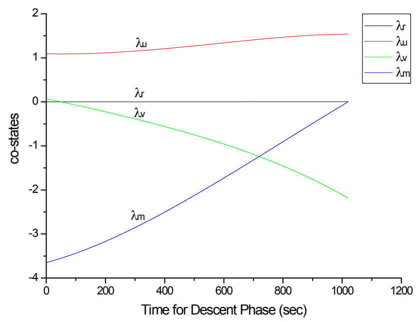

Fig. 6. Co-state Profile

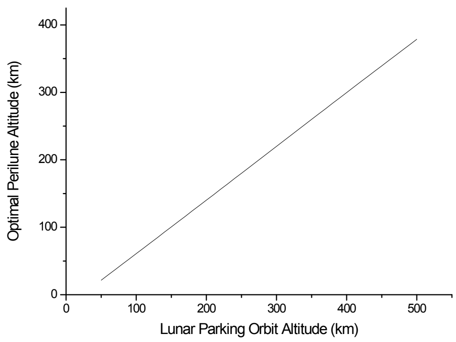

Fig. 7. Optimal perilune altitude for the various lunar parking orbit altitude

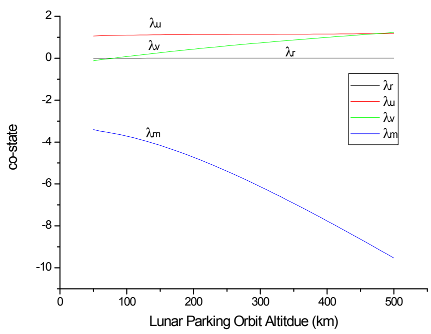

Fig. 8. Initial co-state values at the descent phase for the various lunar parking orbit altitude

$$r(0) = 0.79547498416898\, r_p - 18.69375349429545 \tag{32}$$

In the Fig. 8, the initial co-state values at the descent phase are plotted as changing the lunar parking orbit altitude.

These results are dependent on the simulation conditions such as thrust level, specific impulse and the lunar lander initial mass. However, the optimal perilune altitude can always be obtained by using this process.

Conclusions

In this paper, we could design optimal lunar landing trajectories. For this optimal control problem, perilune altitude turned out to be important in reducing the total energy for the lunar landing. Therefore, the initial state of descent phase is free in this optimal control problem, and one can solve this problem using optimal control theory. For the simulation, the shooting method is employed. Using this method and the continuation approach, we could readily find the optimal solution under various parking orbit conditions. And the result could be formulated as a function of parking orbit altitude. However, the results are dependent on the thruster on the lunar lander.

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