Effect of the Stagnation Temperature on the Normal Shock Wave

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Abstract

When the stagnation temperature increases, the specific heat does not remain constant and start to vary with this temperature. The gas is perfect, it's state equation remains always valid, except, it was called by gas calorically imperfect or gas at high temperatures. The purpose of this work is to develop a mathematical model for a normal shock wave normal at high temperature when the stagnation temperature is taken into account, less than the dissociation of the molecules as a generalisation model of perfect for constant heat specific. A study on the error given by the perfect gas model compared to our model is presented in order to find a limit of application of the perfect gas model. The application is for air.

Keys words : Supersonic flow, subsonic flow, high temperature, supersonic nozzle, thermodynamics ratios, normal shock wave, entropy, relative error, interpolation

Nomenclature

A cross−section area.
a sound velocity.
C_P specific heat at constant pressure.
H enthalpy.
M Mach number.
P static pressure.
R constant of gas.
S entropy
T temperature.
V gas velocity.
x_P pressure ratio through the shock.
x_T temperature ratio through the shock.
x_ρ density ration through the shock.
ε relative error (%).
γ specific heats ratio.
ρ density.
q mass flow
b_j coefficients of the polynomial $C_P(T)$.
K Number of subdivision of the interval

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Subscripts

i point number.
0 stagnation condition.
1 upstream condition of the choc
2 downstream condition of the choc
$*$ critical condition.

Introduction

The study of an ideal gas flow (*PG*) can be made under the basis of a few known cases [1], [2], [4], [7], [8] and [11]. Among these assumptions, the gas must be calorically perfect. The specific heats are constant and do not depend on temperature, which is not really the case when the temperature increases.

The purpose of this work is to develop a mathematical model by adding the effect of change of C_P with temperature, lower than the dissociation threshold of molecules, where the gas becomes calorically imperfect and thermally perfect. The development of a mathematical flow through the shock is based on the use of equations conservation of the mass, quantity of movement and energy, adding the state equation of gas perfect. The shock is characterized by the conservation of stagnation temperature [2] and [8]. In reference [14], we find, for air, a table containing some values C_P and γ depending on the temperature in the interval 55 K and 3550 K. An interpolation polynomial is made to the values of the table [10] and [11] to find an analytical function C_P (T) [3], [6], [12] and [13]. The presented mathematical relationships are valid in the general case regardless of the form of interpolation and substance, but our results are presented by the choice of an interpolation by a polynomial of the 9th degree [3], [6], [12] and [13], and substance chosen is the air. The comparison is made with the *PG* model to determine a limit for applying this model.

Mathematical Formulation

The development is based on the use of conservation equations of the mass, quantity of movement and energy including the equation of state gas perfect [1], [8] by:

$$\rho V = constante \tag{1}$$

$$dP + \rho V\, dV = 0 \tag{2}$$

$$C_P\, dT + V\, dV = 0 \tag{3}$$

$$P = \rho R T \tag{4}$$

The speed of sound is given by [3] and [6]:

$$a^2 = \gamma(T)\, R\, T \tag{5}$$

With [1], [12] and [13]

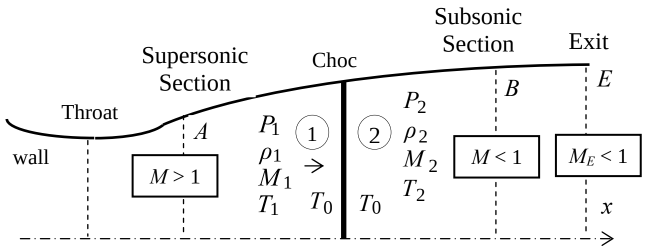


Fig. 1. Normal shock wave illustration in a nozzle

$$R = 287.1029 \text{ J/(kg K)} \tag{6}$$

and

$$\gamma(T) = \frac{C_P(T)}{(C_P(T) - R)}. \tag{7}$$

The presentation scheme of a normal shock wave is illustrated in Fig. 1:
The thickness of the shock wave is very small [8], so that one sees it as a mathematical discontinuity. The flow on both sides of the shock is isentropic.

The integration of equations (1), (2) and (3) between the upstream and downstream state of the shock gives respectively:

$$\rho_1 V_1 = \rho_2 V_2 = q \tag{8}$$

$$P_2 - P_1 + q(V_2 - V_1) = 0 \tag{9}$$

$$2H(T_1) - 2H(T_2) - (V_1^2 - V_2^2) = 0 \tag{10}$$

Where [3] and [6]

$$H(T) = \int_T^{T_0} C_P(T)\, dT \tag{11}$$

You can get an expression for V_1+V_2 and V_1-V_2 from (8) and (9) respectively. The replacement of their product in (10) then the elimination of pressure using the equation (4) and rearrangement gives the following result:

$$x_T x_\rho^2 - \left(\frac{2H(T_1) - 2H(T_2)}{R T_1} - x_T + 1 \right) x_\rho - 1 = 0 \tag{12}$$

According to (8), it gives $V_2 = \rho_1 V_1 / \rho_2$ and $q = \rho_1 V_1$. Replace those quantities in (9) and write an expression for $P_1 - P_2$. By division the result by P_1 and write an expression for V_1^2 and then eliminate P_1 and P_2 by using the relation (4) and then using the relation (5) by replacing the term RT_1 versus a_1 and $\gamma(T_1)$ and introduce the number of Mach M_1, one obtain after rearrangement, the following expression:

$$x_T x_\rho^2 - \left[1 + M_1^2 \gamma(T_1)\right] x_\rho + M_1^2 \gamma(T_1) = 0 \tag{13}$$

The equations (12) and (13) constitute a system of nonlinear equations with two unknowns T_2 and x_ρ.

Calculation procedure

The interpolation constants (b_j, j = 1, 2,⋯, 10) of the function C_P (T) are shown in [3] and [6]. The function $H(T)$ is presented in [3] and [6].

The state generator is given by (M 0, $T=T_0$) (ambient air or combustion chamber).

The integration of the relationship (3) between the stagnation state and respectively between critical condition given by (M = 1, T_*), and the upstream state before the shock given by (M_1, T_1) and introduce the number of Mach M_1, we get the following equation [3]:

$$2\,H(T_i)-M_i^2\ a^2(T_i)=0 \qquad i=*,1 \tag{14}$$

The determination of T_* and T_1 from (14) respectively when $i=*$ and $i=1$ is using the algorithm dichotomy [5] and [9]. The ratio T_*/T_0 and T_1/T_0 corresponding to T_0 and M_1 can be determined as a result.

The report ρ_1/ρ_{0_1} can be calculated using the following relationship when $i=1$, presented in [3], [12], [6] and [13], using the Simpson formulae [5] with condensation of nodes [3] by:

$$\frac{\rho_i}{\rho_{0_i}} = Exp\left(-\int_{T_i}^{T_0} F_\rho(T)\,dT\right) \qquad i=1,2 \tag{15}$$

with

$$F_\rho(T) = \frac{C_P(T)}{a^2(T)} \tag{16}$$

The isentropic pressure ratio upstream of the shock can be determined by the relationship (17) when $i=1$.

$$\frac{P_i}{P_{0_i}}=\left(\frac{\rho_i}{\rho_{0_i}}\right)\left(\frac{T_i}{T_0}\right) \qquad i=1,2 \tag{17}$$

Given that $x_T > 0$ (just above the unit), form (12) shows that one of the solutions of x_ρ is negative and the other is positive. While in the equation (13), the two solutions of x_ρ are positive. To find directly the solution, making the difference between (12) and (13), we get:

$$x_\rho = \frac{R\,T_1\left[1+M_1^2\gamma(T_1)\right]}{-2H(T_1)+2H(T_2)+R\,T_1\left[x_T+M_1^2\gamma(T_1)\right]} \tag{18}$$

The solution given by (18) is the one accepted by the physical phenomenon through the shock. If we replace the expression x_ρ given by (18) in (13), we get a non-linear algebraic equation with one unknown T_2, appointed by:

$$F(T_2)=0 \tag{19}$$

At this stage, we know only that $T_2>T_1$. To give an increase in temperature T_2, consider the discriminate of the equation (13). Then:

$$\Delta = \left[1 + M_1^2\ \gamma(T_1)\right]^2 - 4\ x_T\ M_1^2\ \gamma(T_1) \tag{20}$$

To get a solution it must $\Delta > 0$. So $T_2 < T_{Max}$ with:

$$T_{Max} = \frac{\left[1 + M_1^2\ \gamma(T_1)\right]^2}{4\ M_1^2\ \gamma(T_1)} T_1 \tag{21}$$

The relationship (21) shows that $T_{Max} > T_1$. So, $T_1 < T_2 < T_{Max}$. The obtaining the root of $F(T_2)=0$ is made by using the dichotomy algorithm [5] and [9].

The value of $F(T_1)=0$, means that it is no shock. It can demonstrate that $F(T_{Max}) > 0$, whatever the values of M_1 and T_0. One calculating the value of $F(T_M)$, with $T_M = (T_1 + T_{Max}) / 2$, mid−range of $[T1,\ T_{Max}]$. If $F(T_M) > 0$, the solution lies in $[T_1,\ T_M]$, and if $F(T_M) < 0$, the solution lies in the $[T_M,\ T_{Max}]$. This procedure will be repeated until K satisfy both the accuracy ε desired. It can show [3] and [6] that if $\varepsilon = 10^{-3}$, regardless of M_1 and T_0, the number of iteration K can not exceed 66. The report x_ρ can be measured by the ratio (18). Once the value of T_2 is found with a desired precision ε can be easily inferred the ratio of temperature x_T across the shock.

Fig. 2 illustrates the field of existence of the solution T_2 in the interval $[T_1,\ T_{Max}]$ depending on the stagnation temperature T_0 for the Mach number upstream $M_1 = 2.00$ presented by the case (a) of the figure 2 and the number of M_1=6.00 (extreme supersonic), presented by the case (b) of the Fig. 2. The physical solution still exists.

The Mach number M_2 downstream the shock can be determined using the following relationship [3], [6].

$$M_2 = \frac{\sqrt{2H(T_2)}}{a(T_2)} \tag{22}$$

The isentropic pressure ratio after the shock may be determined by the relationship (17) when i=2.

The static and total pressure ratio and the density ratio through the shock may be determined by:

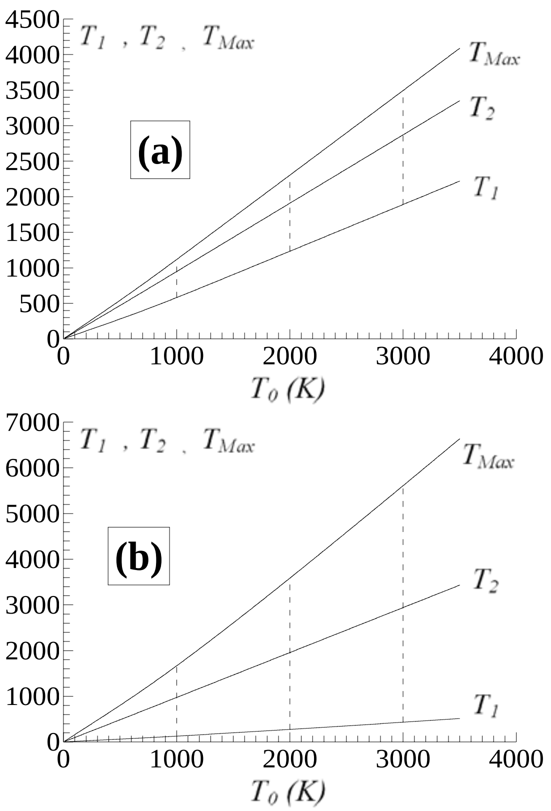


Fig. 2. Interval existence of the solution T_2 versus T_0. (a) when $M_1 = 2.00$. (b) when $M_1 = 6.00$

$$x_P = x_\rho \ x_T \tag{23}$$

$$\frac{P_{0_2}}{P_{0_1}} = \frac{\rho_{0_2}}{\rho_{0_1}} = \frac{(\rho_1/\rho_{0_1})}{(\rho_2/\rho_{0_2})} \ x_\rho \tag{24}$$

The isentropic ratio ρ_2/ρ_{0_2} after the shock can be calculated by the relationship (15) when $_i$ = 2.
The variation of the entropy through the shock for the perfect gas is given by [1], [2], [3] and [4]:

$$dS = (C_P - R)\,\frac{dT}{T} - R\,\frac{d\rho}{\rho} \tag{25}$$

Replacing the polynomial C_P (T) in relation (25) one obtained after integration between states 1 and 2, the following form:

$$S_{12} = (b_1 - R)\, Log(x_T) - R\, Log(x_\rho) + \sum_{j=2}^{j=10} \frac{b_j}{j-1}\left(T_2^{j-1} - T_1^{j-1}\right) \tag{26}$$

Considering the function $C_P(T)$ is constant when $T \le \overline{T}$, $\overline{T}$=240K , $\overline{C}_P = 1001.289$ J / (kg K) [3], then the function $S_{12}(T_1, T_2)$ took the corrected following:

If $T_2 \le \overline{T}$ then $S_{12} = (\overline{C}_P - R)\, Log\,(x_T) - R\, Log\,(x_\rho)$
If $T_1 \ge \overline{T}$ then S_{12}=relation (26)

If $T_1 < \overline{T} < T_2$ then
$$\begin{aligned} S_{12} &= \left(\overline{C}_P - R\right) Log\left(\frac{\overline{T}}{T_1}\right) - R\, Log\left(x_\rho\right) + \\ &(b_1 - R)\, Log\left(\frac{T_2}{\overline{T}}\right) + \sum_{j=2}^{j=10} \frac{b_j}{j-1}\left(T_2^{\,j-1} - \overline{T}^{\,j-1}\right) \end{aligned}$$

To make comparison with the *PG* model, these relations through a shock wave are presented in [1], [2] and [4].

The stagnation parameters corresponding to subsonic conditions after the shock can be determined:

The Mach number and the speed of sound are always equal to zero. Then:

$$M_{0_1} = M_{0_2} = M_0 = 0 \tag{27}$$

$$V_{0_1} = V_{0_2} = V_0 = 0 \tag{28}$$

Now consider the change in the total temperature corresponding to the regions before and after the shock when the specific heat CP (T) varies with temperature. Know that for the model GP, the total temperature through the shock remains constant [1].

One can write the relationship (3) in three forms.

The first form is between the total state total and the state just before the shock. The second relationship is between the total state total after the shock and the state immediately after the impact. The third relationship is between the state just before and after the shock. Yields respectively:

$$\mathrm{V}_1^2 = 2\int_{T_1}^{T_{0_1}} C_P(T)\, dT \tag{29}$$

$$\mathrm{V}_2^2 = 2\int_{T_2}^{T_{0_2}} C_P(T)\, dT \tag{30}$$

$$\left(\mathrm{V}_2^2 - \mathrm{V}_1^2\right) = 2\int_{T_2}^{T_1} C_P(T)\, dT \tag{31}$$

Let the difference between the relations (29) and (30) and compare the result with the relationship (31), we get:

$$\begin{aligned} &2\int_{T_2}^{T_{0_2}} C_P(T)\, dT - 2\int_{T_1}^{T_{0_1}} C_P(T)\, dT = 2\int_{T_2}^{T_1} C_P(T)\, dT = \\ &= 2\int_{T_2}^{T_{0_2}} C_P(T)\, dT + 2\int_{T_{0_2}}^{T_{0_1}} C_P(T)\, dT + 2\int_{T_{0_1}}^{T_1} C_P(T)\, dT \end{aligned} \tag{32}$$

After simplification one found

$$\int_{T_{0_2}}^{T_{0_1}} C_P(T)\, dT = 0 \tag{33}$$

This gives that

$$T_{0_2} = T_{0_1} = T_0 \tag{34}$$

Hence it was confirmed that the total temperature through the shock remains constant for the model HT. Then, the phenomenon of shock is through conservation of the total temperature.

One can say that the isentropic flow that was subsonic or supersonic is done with the conservation of entropy which provides that such flow is done with conservation of the total pressure.

Hence, the total speed of sound remains constant on both sides of the shock. By using relations (5), (7) and (34), we get:

$$a_{0_2} = a_{0_1} = a_0 = \sqrt{\gamma(T_0)\, R T_0} \tag{35}$$

With $\gamma(T)$ is given by the relationship (7).

The total stagnation pressure and density may be determined by the relationship (24).

The relationship (26) is given depending on conditions across the shock. We can demonstrate from the relationship (25) that the integration between the conditions total before and after the shock gives:

$$S_{12} = -R\, Log\left(\frac{\rho_{0_2}}{\rho_{0_1}}\right) = -R\, Log\left(\frac{P_{0_2}}{P_{0_1}}\right) \tag{36}$$

Error of Perfect Gas Model

The mathematically perfect gas model is developed on the basis to regarding the specific heat C_P and ratio γ as constants, which gives acceptable results for low temperature. According to this study, we can notice a difference on the given results between the perfect gas model and the model developed here. The error given by the *PG* model compared to our *HT* model can be calculated for each parameter. For each value (T_0, M) the error ε can be evaluated by the following relationship:

$$\varepsilon_y(T_0, M) = \left| 1 - \frac{y_{PG}(T_0, M)}{y_{HT}(T_0, M)} \right| x\ 100 \tag{71}$$

Resultats et commentaires

Fig. 3 to 9 show the thermodynamic ratio through the shock for T_0= 1000 *K*, 2000 *K* and 3000 *K* including the graph for the PG model for γ =1,402.

There is clearly the effect of temperature T_0 on the obtained results. We can conclude that for low-temperature, the perfect gas gives very good results closer to those of *HT* model. This case is until T_0 = 1000 *K* and M_1 = 2.00 if one accepts an accuracy of 5%.

On the Fig.3 of the temperatures ratio across the shock, we note that the *PG* model, gives superior results compared to the results given by the *HT* model, whatever the temperature T_0. We can see it on the case (b) of the Fig. 4, which means that the model of ideal gas provides a safety margin to the design of the structure that was for internal or external aerodynamics.

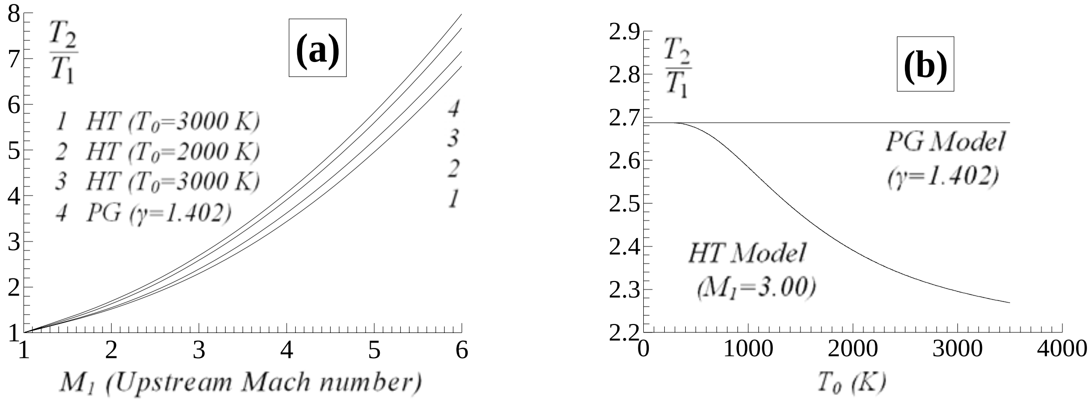


Fig. 3. Evolution of the ration T_2/T_1 (a) : versus M_1. (b) : versus T_0 when M_1=3.00

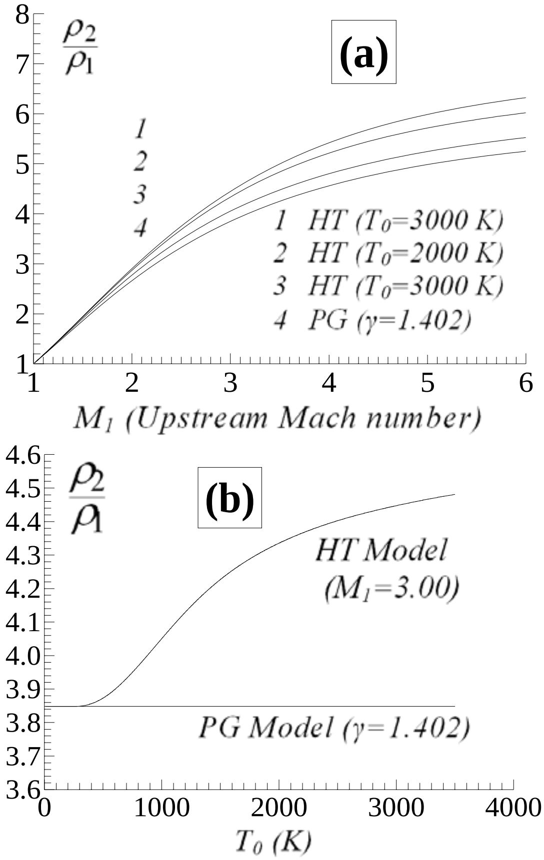


Fig. 4. Evolution of the ratio ρ_2/ρ_1 (a) : versus M_1. (b) : versus T_0 for M_1=3.00

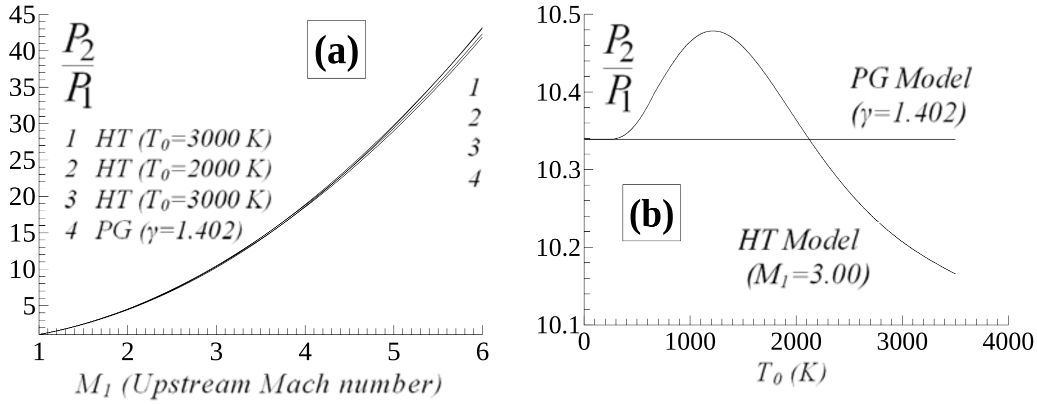


Fig. 5. Evolution of the ratio P_2/P_1 (a) : versus M_1. (b) : versus T_0 when M_1=3.00

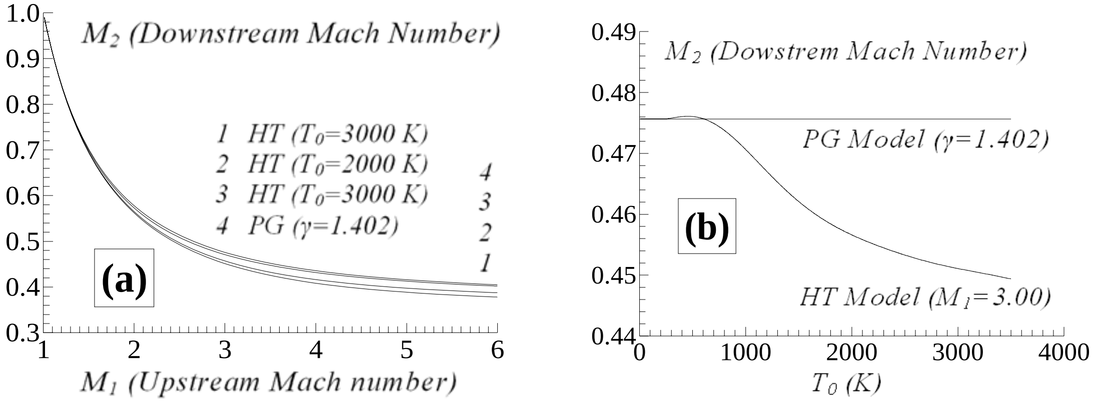


Fig. 6. Evolution of the Mach number M_2 (a) : versus M_1 (b) : versus T_0 when M_1=3.00

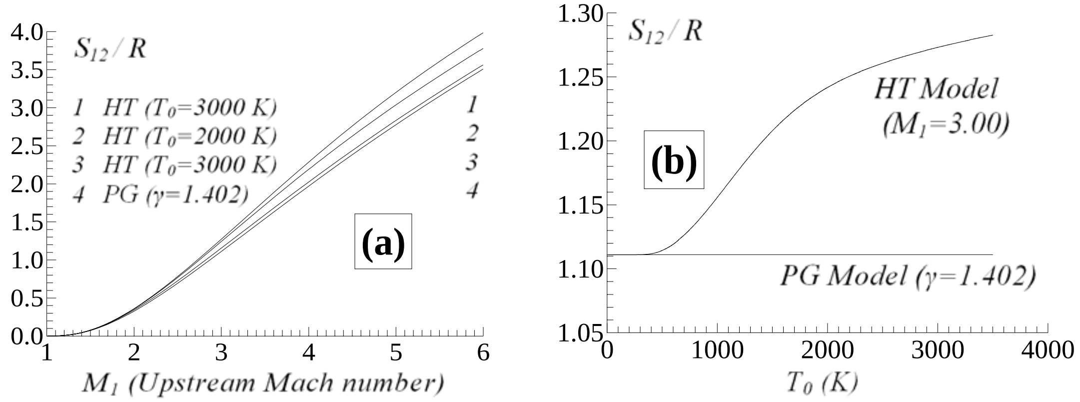


Fig. 7. Evolution of the entropy S_{12}/R (a) : versus M_1. (b) : versus T_0 for M_1=3.00

It is clear that whatever the Mach number M_1, the model of ideal gas gives good results until T_0 = 240 K. Beyond that value the two models are starting to differentiate more and more that T_0 increases.

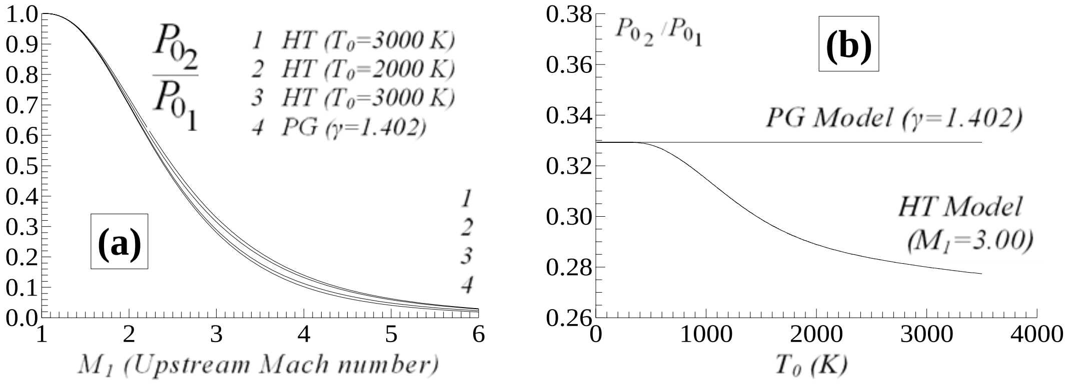


Fig. 8. Evolution of P_{0_2}/P_{0_1} (a) : versus M_1. (b) : versus T_0 when M_1=3.00

In the Fig. 6, we notice that the Mach number M_2 just after the shock given by the *PG* model is higher than the value given by the *HT* model whatever the temperature T_0, which means that if we calculate the speed V_2, The *PG* model accelerates the gas compared to the real case.

When M_1=1.00, the values of x_T, x_ρ, x_P, M_2, P_{0_2}/P_{0_1} on the part (a) of the figures 3, 4, 5, 6, 7 and 8 are equal to the unit. While the value of S_{12}/R is equal to 0. These results indicate that these parameters remain unchanged and to be physically interpreted by the absence of shock right.

Fig. 9 presents the relative error of the thermodynamics and physical parameters between the *PG* and the *HT* models for several T_0 values.

It can be seen that the error depends on the values of T_0 and M. For example, if T_0=2000 K, and M=3.00 the use of the *PG* model will give a relative error equal to ε=12.39 % for the temperature ratio, see the figure 10a, ε=11.24 % for the density ratio, ε=2.44 % for the static pressure ratio, ε=4.16% for the upstream mach number, ε=10.51 for the entropy variation and ε=13.95 %. For lower values of M and T_0, the error is weak. The curve number three in the figure 10 is under the error *5%* independently of the Mach number, which is interpreted by the use potential of the *PG* model when T_0<1000 K.

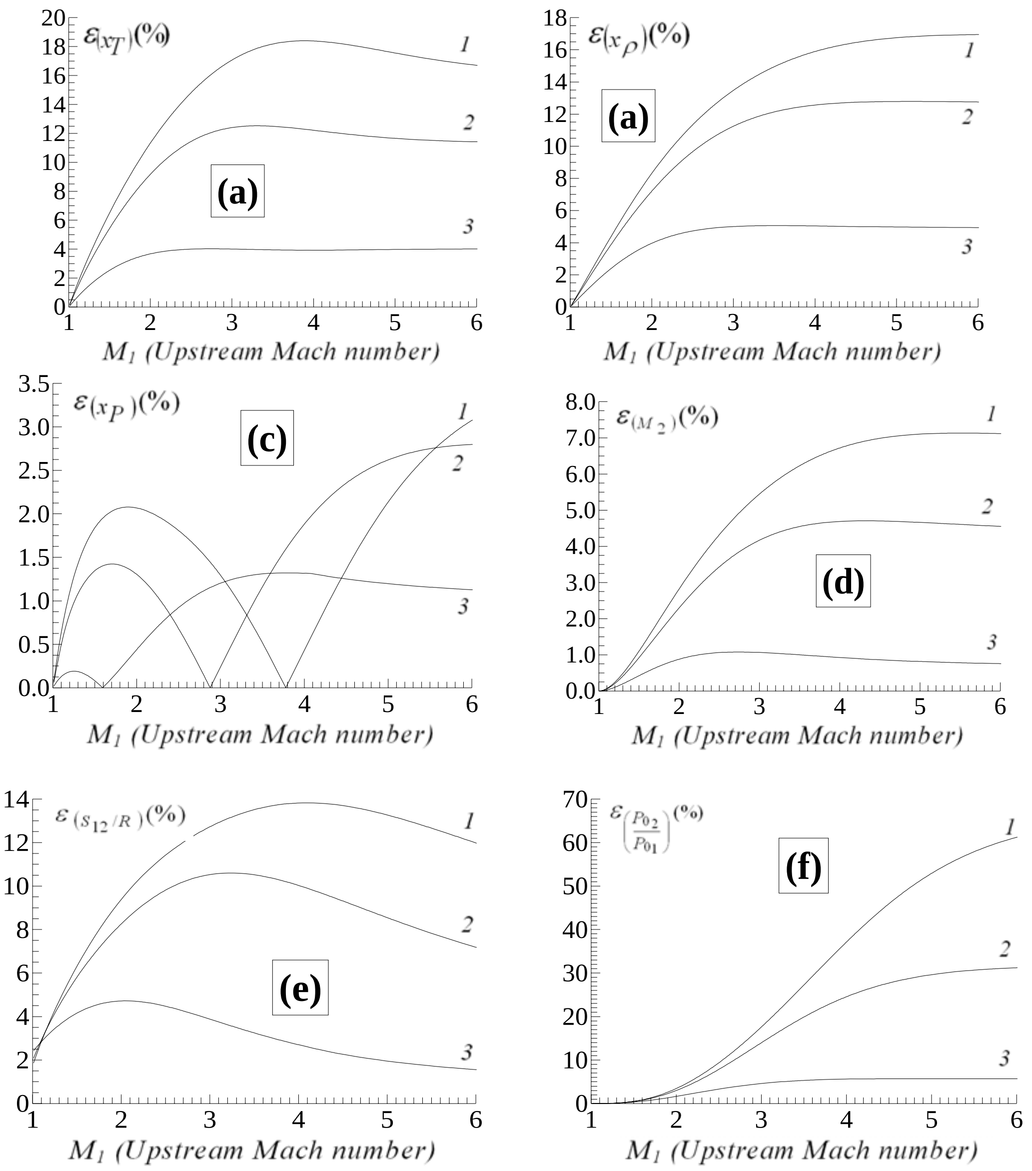
When M_1=1.00, the parameters x_T, x_ρ, x_P, M_2, P_{0_2}/P_{0_1} on the figure 9 are equal to zero. While the value of S_{12}/R is different to zero, the relationship (37) gives the following result

$$\varepsilon_{(x_T)}(T_0, M_1{=}1) = \left|1 - \frac{(x_T)_{PG}}{(x_T)_{HT}}\right| \times 100 = \left|1 - \frac{1}{1}\right| \times 100 = 0 \tag{38}$$

$$\varepsilon_{(x_\rho)}(T_0, M_1{=}1) = \left|1 - \frac{(x_\rho)_{PG}}{(x_\rho)_{HT}}\right| \times 100 = \left|1 - \frac{1}{1}\right| \times 100 = 0 \tag{40}$$

$$\varepsilon_{(x_P)}(T_0, M_1{=}1) = \left|1 - \frac{(x_P)_{PG}}{(x_P)_{HT}}\right| \times 100 = \left|1 - \frac{1}{1}\right| \times 100 = 0 \tag{41}$$

$$\varepsilon_{(M_2)}(T_0, M_1{=}1) = \left|1 - \frac{(M_2)_{PG}}{(M_2)_{HT}}\right| \times 100 = \left|1 - \frac{1}{1}\right| \times 100 = 0 \tag{42}$$



Curve 1 Error compared to *HT* model for (T_0=3000 K)
Curve 2 Error compared to *HT* model for (T_0=2000 K)
Curve 3 Error compared to *HT* model for (T_0=1000 K)

Fig. 9. Variation of the relative error given by the ratios towards the shock of PG model versus Mach number
(a): Temperature ratio. (b) : Density ratio. (c) : Pressure ratio
(d) : Mach Number. (e) : entropy. (f) : Total pressure ratio

$$\varepsilon_{(P_{0_2}/P_{0_1})}(T_0, M_1{=}1.00) = \left|1 - \frac{(P_{0_2}/P_{0_1})_{PG}}{(P_{0_2}/P_{0_1})_{HT}}\right| \times 100 = \left|\; 1-\frac{1}{1}\right| \; x\; 100 = 0\% \tag{43}$$

For the entropy we have the following result:

$$\varepsilon_{(S_{12}/R)}(T_0, M_1=1.00) = \left| 1 - \frac{(S_{12}/R)_{PG}}{(S_{12}/R)_{HT}} \right| \times 100 =$$

$$\left| 1 - \frac{0}{0} \right| x\ 100 = \begin{cases} 2.421\,\%\ \text{When}\ T_0 = 1000\,K \\ 2.076\,\%\ \text{When}\ T_0 = 2000\,K \\ 1.856\,\%\ \text{When}\ T_0 = 3000\,K \end{cases} \quad (44)$$

Conclusion

It was to prove from this study that if one accepts an error of more than 5% can be used to study the phenomenon of HT normal shock wave using the results of the ideal gas when T0 does not exceed 1000 K and the number of Mach M1 may not exceed 2.00. For Otherwise, it must take into account T0 and use the mathematical model presented.

The *PG* model is represented by explicit and simple relations and do not require much time to make calculation, unlike the proposed model which requires the resolution of nonlinear algebraic equations, and integration of two complex analytical functions. It takes more time for calculation and for data processing.

The equations presented in this study are valid for any interpolation chosen for the function $C_P(T)$. The essential one is that the selected interpolation gives small acceptable error.

We can choose another substance instead of air. The equations and relations remain always valid, except that it is necessary to have the table of variation of C_P or γ according to the temperature and to make a suitable interpolation.

We can obtain the equations of a perfect gas starting from the equations of our model by annulling all constants of interpolation except the first. In this case, the *PG* model becomes a particular case of our *HT* model.

The relations presented on the HT model can be used to study the effect of stagnation temperature on the flow in a supersonic nozzle when a shock wave will appear in the cone, Because of the diminution in the exit pressure compared with the ambient temperature.

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