General Purpose Cross-section Analysis Program for Composite Rotor Blades

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Abstract

A two-dimensional cross-section analysis program based on the finite element method has been developed for composite blades with arbitrary cross-section profiles and material distributions. The modulus weighted approach is used to take into account the non-homogeneous material characteristics of advanced blades. The CLPT (Classical Lamination Plate Theory) is applied to obtain the effective moduli of the composite laminate. The location of shear center for any given cross-sections are determined according to the Trefftz' definition while the torsion constants are obtained using the St. *Venant* torsion theory. A series of benchmark examples for beams with various cross-sections are illustrated to show the accuracy of the developed cross-section analysis program. The cross section cases include thin-walled C-channel, I-beam, single-cell box, NACA0012 airfoil, and KARI small-scale blades. Overall, a reasonable correlation is obtained in comparison with experiments or finite element analysis results.

Key words : cross-section analysis, finite element method, shear center

Introduction

Helicopter rotor blades are normally built-up of solid and/or thin-walled cross-section, and it consists of complex geometries and various topologies such as erosion shield, skin, spar and balancing weight. They are often modeled as one-dimensional beams instead of the three-dimensional beams in static and dynamic analysis. For this case, Cross-sectional properties such as tension center, shear center and section stiffness of the blades are absolutely necessary for the analysis. However, it is not easy to calculate the cross-sectional properties of the blades.

There are several cross-section analysis programs that can be applicable for beams and blades with arbitrary section geometries. The ShapeDesigner [1] can calculate the cross- sectional properties such as torsion constant, warping constant, warping and shear stresses. It is a useful tool for the analysis of steel, aluminum, and composite beams. The Precomp [2] provides span-variant cross-sectional properties for composite blades such as coupled stiffness, inertia properties, and offsets with respect to the blade pitch axis. It requires that the blade external shape and the internal layup of composite laminates be

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described for inputs. The external shape is specified in terms of the chord, twist, and airfoil geometry variation along the blade. The internal layup is specified in terms of the laminates schedule, orientation of fibers in each laminate, and the laminate properties. A modified classic laminate theory with a shear-flow approach is used to obtain the more accurate cross-sectional properties. The NuMAD [3] is a windows based pre/post-processor to produce the three-dimensional FE model of ANSYS. It is designed to enable users to quickly and easily create a three-dimensional model of a turbine blade in static and dynamic analysis. Also, it can generate the one- dimensional properties from the three-dimensional finite element model of ANSYS.

In this work, a finite element-based, two-dimensional cross-section analysis program is developed for composite blades with inhomogeneous cross-section shapes. The modulus weighted method is applied to consider the non-uniform distribution of material properties in the cross sections. In order for versatility of the cross-sectional program, MD Nastran [4] input file format is used as the pre-processor while the TECPLOT360 [5] file format is used as the post-processor. A series of validation is performed for beams and blades with various cross-section geometries as compared with commercial software, experimental data, and published data available in the literature.

Theory

In the cross-section analysis, centroidal properties such as center of gravity, tension center and shear center location should be determined and measured from the origin of the user-defined coordinate system. Fig. 1 shows the reference axis of the blade. It is assumed that X axis is located at an arbitrary position on the cross-section and Y and Z axes are perpendicular to this axis.

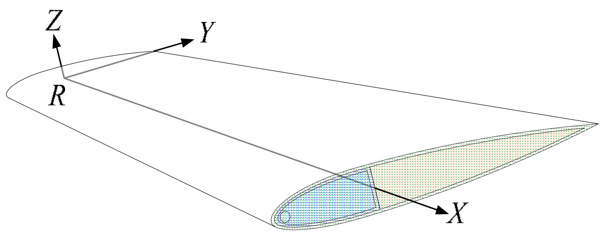


Fig. 1. Reference axis of the blade

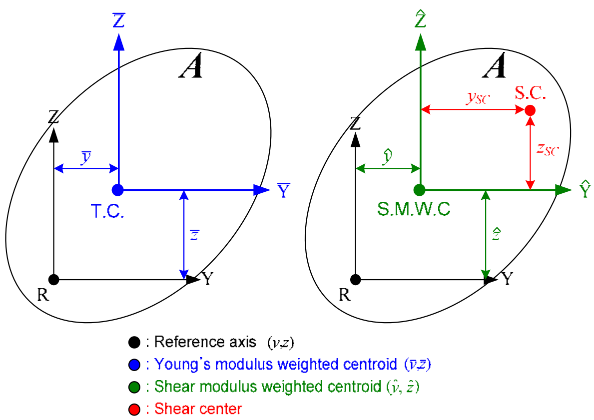


Fig. 2. The coordinate systems of a blade cross-section

Fig. 2 shows the tension center (T.C.) offset ($\bar{y}, \bar{z}$) and shear modulus weighted centroid (S.M.W.C.) measured respect to the reference axis, and shear center (S.C.) offset (y_{SC}, z_{SC}) from the shear modulus weighted centroidal position.

For isotropic material case, the tension center offset ($\bar{y}, \bar{z}$) is simply defined as:

$$\begin{aligned} A &= \iint_A dA, \\ Q_Y &= \iint_A z dA, \quad Q_Z = \iint_A y dA, \\ \bar{y} &= \frac{Q_Z}{A}, \quad \bar{z} = \frac{Q_Y}{A} \end{aligned} \tag{1}$$

where, A is the area of the cross-section, Q_Y and Q_Z are the first moments of area measured from the reference axes, respectively. In the isotropic material case, tension center is coincided with the geometric centroid of the blade cross-section. The axial (EA) and bending stiffnesses (EI_Y, EI_Z) of beams with isotropic materials are defined as

$$\begin{aligned} EA &= E\iint_A dA \\ EI_Y &= E\iint_A \bar{z}^2 dA \quad , I_Y = \iint_A \bar{z}^2 dA \\ EI_Z &= E\iint_A \bar{y}^2 dA \, , I_Z = \iint_A \bar{y}^2 dA \end{aligned} \tag{2}$$

where E is the Young' s modulus, I_Y and I_Z are the area moment of inertia of the cross-section, respectively. The torsion stiffness GJ can be expressed as a function of the shear modulus G and the torsion constant J . The torsion constant is derived from the St. *Venant* torsion theory [6].

$$J = \iint_A \left[\left(\frac{\partial \omega}{\partial z} + y \right) y - \left(\frac{\partial \omega}{\partial y} - z \right) z \right] dA \tag{3}$$

where, ω is the warping function of the cross-section and the torsion constant J can be determined from the warping function.

For beams with inhomogeneous cross-sections, each of the constituent sections with different properties can be expressed with respect to those of the reference sections having E_0 and G_0 . Hense, a given segment of the cross-sections are defined as

$$d\bar{A} = \frac{E_i}{E_0} dA, \; d\hat{A} = \frac{G_i}{G_0} dA \tag{4}$$

where E_0 and G_0 are Young' s and shear modulus of the reference section, E_i and G_i are Young' s and shear modulus of an *i-th* element.

Using Eq. (4), the previously defined section properties are modified by modulus weighted properties as is given below:

$$\begin{aligned} \bar{Q}_Y &= \iint_A z d\bar{A}, \; \bar{Q}_Z = \iint_A y d\bar{A}, \\ \bar{y} &= \bar{Q}_Z / \bar{A}, \quad \bar{z} = \bar{Q}_Y / \bar{A}, \\ \hat{Q}_Y &= \iint_A z d\hat{A}, \; \hat{Q}_Z = \iint_A y d\hat{A}, \\ \hat{y} &= \hat{Q}_Z / \hat{A}, \quad \hat{z} = \hat{Q}_Y / \hat{A} \end{aligned} \tag{5a}$$

$$\begin{aligned} \bar{I}_Y &= \iint_A \bar{z}^2 d\bar{A}, \; \bar{I}_Z = \iint_A \bar{y}^2 d\bar{A}, \\ \hat{I}_Y &= \iint_A \hat{z}^2 d\hat{A}, \; \hat{I}_Z = \iint_A \hat{y}^2 d\hat{A} \end{aligned} \tag{5b}$$

where $\overline{Q}_Y$ and $\overline{Q}_Z$ are Young's modulus weighted first moment of area, $\bar{y}$ and $\bar{z}$ are Young's modulus weighted centroid, $\hat{Q}_Y$ and $\hat{Q}_Z$ are shear modulus weighted first moment of area, and $\hat{y}$ and $\hat{z}$ are shear modulus weighted centroid, respectively. In Eq. (5b), $\bar{I}_Y$ and $\bar{I}_Z$ are Young's modulus weighted area moment of inertia, and $\hat{I}_Y$ and $\hat{I}_Z$ are shear modulus area moment of inertia, respectively. These values are measured from the reference axis. Modulus weighted second moments of area respect to the tension center can be obtained by the parallel axis theorem, and is written by

$$\bar{I}_{\bar{Y}} = \bar{I}_Y - \frac{\overline{Q}_Y^2}{\overline{A}}, \quad \bar{I}_{\bar{z}} = \bar{I}_Z - \frac{\overline{Q}_Z^2}{\overline{A}}, \\ \hat{I}_{\hat{Y}} = \hat{I}_Y - \frac{\hat{Q}_Y^2}{\hat{A}}, \quad \hat{I}_{\hat{Z}} = \hat{I}_Z - \frac{\hat{Q}_Z^2}{\hat{A}} \tag{6}$$

where, $\bar{I}_{\bar{Y}}$ and $\bar{I}_{\bar{Z}}$ are the Young's modulus weighted area moment of inertia, and $\hat{I}_Y$ and $\hat{I}_Z$ are shear modulus weighted area moment of inertia, respectively. These values are measured from Young's and shear modulus weighted centroid.

The section stiffness constants of the blade with inhomogeneous materials are now obtained by applying Eqs. (5) to (6), which are expressed as:

$$\begin{aligned} EA &= E_0 \iint_A d\overline{A} = E_0 \overline{A} \\ EI_Y &= E_0 \bar{I}_{\bar{Y}} = E_0 \iint_A \bar{z}^2 d\overline{A} \\ EI_Z &= E_0 \bar{I}_{\bar{Z}} = E_0 \iint_A \bar{y}^2 d\overline{A} \\ GJ &= G_0 \hat{J} = G_0 \left[\hat{I}_{\hat{Y}} + \hat{I}_{\hat{Z}} - \iint_A \left(\hat{z} \frac{\partial \omega}{\partial y} - \hat{y} \frac{\partial \omega}{\partial z} \right) d\hat{A} \right] \end{aligned} \tag{7}$$

The shear center is defined as a point on the cross-section where a shear force induces no twist deformation. If a cantilever beam of length L is subjected to a twisting moment at the tip, the elastic strain energy due to pure torsion is

$$U_{\text{torsion}} = \frac{L}{2G} \iint_A \left(t_{xy}^2 + t_{xz}^2 \right) dA \tag{8}$$

where, t_{xy} and t_{xz} are torsional shear stresses. The strain energy due to transverse loads V_y and V_z is given by

$$U_{\text{bending}} = \frac{1}{2E} \int_L \iint_A \sigma_x^2 dA\, dx + \frac{L}{2G} \iint_A \left(\tau_{xy}^2 + \tau_{xz}^2 \right) dA \tag{9}$$

where, τ_{xy} and τ_{xz} are the flexural shear stresses. The total strain energy due to a combined loading of torque and transverse loads should be:

$$U = \frac{1}{2E} \iint_A \int_L \sigma_x^2 dx dA + \frac{L}{2G} \iint_A \left\{ (t_{xy} + \tau_{xy})^2 + (t_{xz} + \tau_{xz})^2 \right\} dA \tag{10}$$

When the transverse loads are applied at the shear center, the bending and torsion problems are decoupled. This leads to a definition of torsion-free flexure which is called as a Trefftz condition [6]:

$$\iint_A \left(t_{xy} \tau_{xy} + t_{xz} \tau_{xz} \right) dA = 0 \tag{11}$$

The shear stresses due to pure torque are given by [6]:

$$t_{xy} = G\theta \left(\frac{\partial \omega}{\partial y} - z \right), \qquad t_{xz} = G\theta \left(\frac{\partial \omega}{\partial z} + y \right) \tag{12}$$

From Eqs. (11) to (12), the Trefftz condition can then be written as:

$$\iint_A \left(t_{xy} \tau_{xy} + t_{xz} \tau_{xz} \right) dA = \iint_A \left[\left(\frac{\partial \omega}{\partial y} - z \right) \tau_{xy} + \left(\frac{\partial \omega}{\partial z} + y \right) \tau_{xz} \right] dA = 0 \tag{13}$$

Assuming that V_y or V_z is applied at the shear center, the above equation results in:

$$\begin{aligned} y_{sc}V_z &= \iint_A \left(\tau_{xz}y - \tau_{xy}z\right)dA = -\iint_A \left(\frac{\partial \omega}{\partial y}\tau_{xy} + \frac{\partial \omega}{\partial z}\tau_{xz}\right)dA \\ &= -\iint_A \omega \frac{\partial \sigma_x}{\partial x}dA = \iint_A \omega \frac{V_z}{I_y I_z - I_{yz}^2}(I_{yz}y - I_z z)dA \\ -z_{sc}V_y &= \iint_A \omega \frac{V_y}{I_y I_z - I_{yz}^2}(I_{yz}z - I_y y)dA \end{aligned} \tag{14}$$

The above derivation can readily be extended to inhomogeneous material cases by using the modulus-weighted approach. The location of shear center are derived as

$$y_{SC} = \frac{\hat{I}_{\hat{Z}}\hat{I}_{\hat{Z}\omega} - \hat{I}_{\hat{Y}\hat{Z}}\hat{I}_{\hat{Y}\omega}}{\hat{I}_{\hat{Y}}\hat{I}_{\hat{Z}} - \hat{I}_{\hat{Y}\hat{Z}}^2}, \quad z_{SC} = \frac{\hat{I}_{\hat{Y}\hat{Z}}\hat{I}_{\hat{Z}\omega} - \hat{I}_{\hat{Y}}\hat{I}_{\hat{Y}\omega}}{\hat{I}_{\hat{Y}}\hat{I}_{\hat{Z}} - \hat{I}_{\hat{Y}\hat{Z}}^2} \tag{15}$$

where the sectorial products of area and they are defined as:

$$\begin{aligned} \hat{I}_{\hat{Y}\omega} &= \iint_A \hat{y}\omega d\hat{A} \\ \hat{I}_{\hat{Z}\omega} &= \iint_A \hat{z}\omega d\hat{A} \\ \hat{I}_{\hat{Y}\hat{Z}} &= \iint_A \hat{y}\hat{z} d\hat{A} \end{aligned} \tag{16}$$

In order for the cross-section analysis, the finite element methods are employed in this study. As is shown in Fig. 3, a three node triangular element is introduced due to its convenience of modeling complex geometries such as helicopter blades and wind turbine blades.

The cross section coordinates are transformed into y and z coordinates using the shape function N.

$$\mathbf{y} = \mathbf{N}(\eta,\zeta)y, \quad \mathbf{z} = \mathbf{N}(\eta,\zeta)z \tag{17}$$

where the shape functions are defined in natural coordinates as given below:

$$\begin{aligned} N &= \lfloor \psi_1 \quad \psi_2 \quad \psi_3 \rfloor \\ \psi_1 &= 1-\zeta-\eta, \quad \psi_2 = \zeta, \quad \psi_3 = \eta \end{aligned} \tag{18}$$

In general, the numerical integration method is used to deal with a function $f(y,z)$ for cross section.

$$\iint_A f(y,z)dA = \sum_{i=1}^{n} \int_0^1 \int_0^{1-\zeta} f(\eta,\zeta)\left|J_i\right| d\eta d\zeta \tag{19}$$

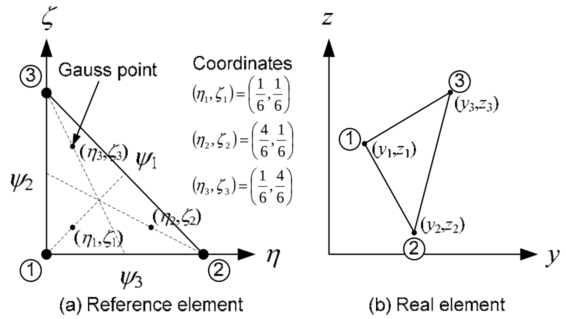


Fig. 3. Order of Gauss integration and location of integration points for 3-node triangular element

where $|J_i|$ denotes the determinant of Jacobian matrix *J*.

The governing equations of torsion warpings are obtained through the principle of virtual work using the St. *Venant* principle. The variational statement results in the following form for the warpings [6]:

$$\iint_A \left[\left(\frac{\partial}{\partial y}\delta\omega\frac{\partial\omega}{\partial y} + \frac{\partial}{\partial z}\delta\omega\frac{\partial\omega}{\partial z} \right) - \left(\frac{\partial}{\partial y}\delta\omega z - \frac{\partial}{\partial z}\delta\omega y \right) \right] dA = 0 \tag{20}$$

where ω is the the warping function in the cross section. The discretized form for the warping variables are denoted as in Eq. (21) by use of Eq. (17).

$$\boldsymbol{\omega} = \mathbf{N}\boldsymbol{\omega}_e, \ \delta\boldsymbol{\omega} = \mathbf{N}\delta\boldsymbol{\omega}_e \tag{21}$$

where $\boldsymbol{\omega}_e$ is the warping vector of an element.
Substituting of Eq. (21) into Eq. (20) gives the warping equation.

$$K_e\omega_e - F_e = 0 \tag{22}$$

where the element stiffness matrix K_e and loading vector F_e for an element are defined respectively as:

$$K_e = \iint \left(\frac{\partial N^T}{\partial\eta}\frac{\partial N}{\partial\eta} + \frac{\partial N^T}{\partial\zeta}\frac{\partial N}{\partial\zeta} \right)|J| d\eta d\zeta, F_e = \iint \left(\zeta\frac{\partial N^T}{\partial\eta} - \eta\frac{\partial N^T}{\partial\zeta} \right)|J| d\eta d\zeta \tag{23}$$

Discussions

A series of benchmark tests are performed to validate the developed cross-sectional analysis program which is called KSec2D. The first example considered is a C-channel section. The geometry and coordinate axes of a C-channel section are given in Fig. 4. Table 1 shows the comparison between KSec2D and analytical results for the geometric properties such as area, centroid, area moment of inertia, torsion constant and shear center location. The cross-section results are obtained using a total of 636 3-node triangular elements. As is seen Table 1, the current cross-section analysis results in an excellent agreement with those of closed form solution.

The geometry FE mesh and warping distribution of the I-beam section are shown in Fig. 5. KSec2D results were obtained using 342 3-node linear elements. The FE meshes are generated with CTRIA3 element of MD Patran [8] and plotted the warping distributions using TECPLOT 360.

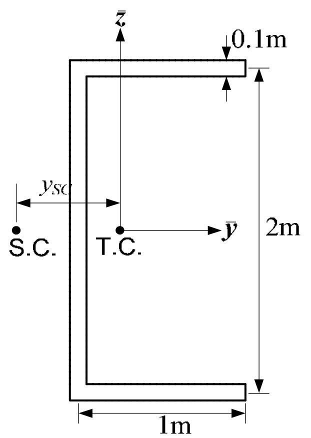


Fig. 4. The geometric properties of a C-channel beam

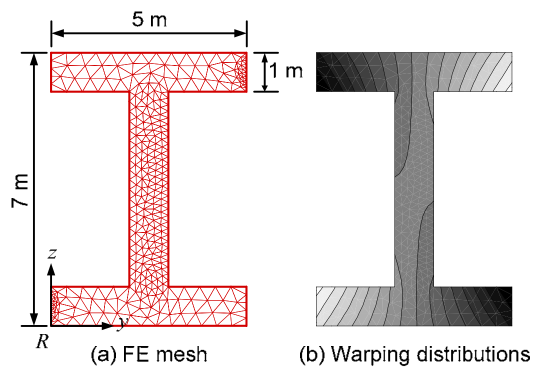


Fig. 5. FE mesh and warping distributions of I beam

Table 1. Comparison results of a channel section beam

Properties		Analytical Solution [7]	KSec2D	% error
Area (m^2)		0.4	0.4	0.0
Centroid (m)	y−dir	2.9938E−01	2.9938E−01	0.0
	z−dir	1.0500E+00	1.0500E+00	0.0
Area moment of inertia (m^4)	y−dir	2.6733E−01	2.6733E−01	0.0
	z−dir	4.1958E−02	4.1958E−02	0.0
Torsion Constant (m^4)		1.4933E−03	1.5001E−03	0.46
Shear center (m)	y−dir	−6.2438E−01	−6.2010E−01	0.69
	z−dir	1.0500E+00	1.0500E+00	0.0

Table 2. Cross section properties for I beam

Properties		KSec2D	MD Patran	% error
Area (m^2)		15.0	15.0	0.0
Centroid (m)	y−dir	2.50	2.50	0.0
	z−dir	3.50	3.50	0.0
Area moment of inertia (m^4)	y−dir	101.25	101.25	0.0
	z−dir	21.25	21.25	0.0
Torsion constant (m^4)		5.5451	5.3333	3.97
Shear center (m)	y−dir	2.4988	2.5000	−0.05
	z−dir	3.4992	3.5000	−0.02

Table 2 shows comparison of the cross section properties obtained using KSec2D and MD Patran. It is observed that the predictions of the KSec2D are in good agreement with MD Patran.

The next example considered is a single−cell box section, as is shown in Fig. 6. The KSec2D results are obtained using 1,022 3−node triangular elements. The comparison of KSec2D results with those of MD Patran is presented in Table 3. The maximum error between the two set of results is about 8.9% for the torsion constant. Overall a good correlation is clearly seen.

Numerical simulations are also carried out for elastically−coupled composite blades with two−cell airfoil section [9]. The beam has D−type spar having $[0/15]_2$ layups, whereas the skin consists of [15/−15]. Fig. 7 shows the finite element model for the two−celled beam. The geometry and the material properties of the blade are given in Table 4.

For KSec2D results, a total of 2,185 3−node triangular elements are used to discretize the section. Inertial properties, stiffness constants and centroidal offset properties are compared with other experimental results [9]. A good correlation is obtained in comparison with the experimental results.

The last example considered is the KARI small−scaled blade. The FE mesh of KARI small−scaled blade is presented in Fig. 8. The results of KARI are calculated using the CORDAS software which was developed by Russia−Korea technology transfer program for composite rotor blade cross−section analysis. The CORDAS program is developed and operated in the windows 98 platform. MD Patran [8] is used to model the KARI small−scaled blade for Ksec2D input file.

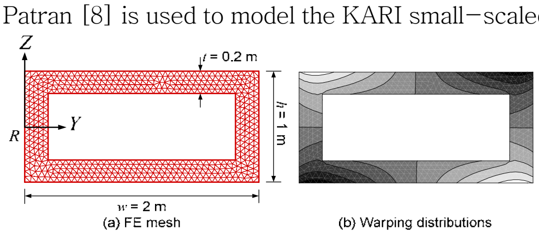


Fig. 6. FE mesh and warping distributions of single cell box beam

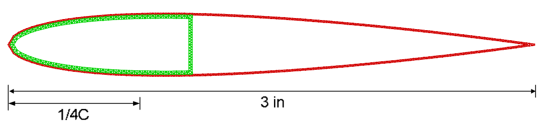
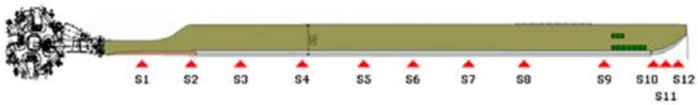
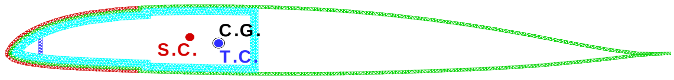


Fig. 7. FE meshes used for two−cell airfoil section



(a) KARI small-scaled blade with variable cross-section



(b) FE meshes of cross-section at S2 location

Fig. 8. FE meshes of KARI small-scaled blade

Table 3. Cross section properties for box beam

Properties		KSec2D	MD Patran	% error
Area (m^2)		1.4	1.4	0.0
Centroid (m)	y-dir	1.0	1.0	0.0
	z-dir	0.0	0.0	0.0
Area moment of inertia (m^4)	y-dir	0.13787	0.13787	0.0
	z-dir	0.46187	0.46187	0.0
Torsion constant (m^4)		0.3503	0.3191	8.91
Shear center (m)	y-dir	0.9999	1.0000	-0.01
	z-dir	0.0001	0.0000	0.01

Table 4. Geometry and material properties of composite blades

Properties	Values
E_1	131 GPa
E_2	9.3 GPa
G_{12}	5.86 GPa
ν_{12}	0.40
Ply thickness	0.127 mm
Airfoil	NACA 0012
Length	641.4 mm
Chord	76.2 mm
Airfoil thickness	9.144 mm

Table 5. Cross-section properties of two-cell airfoil section

$\theta = 15^{\circ}$			KSec2D	Experiment [9]	% error
Section Stiffness	EA (N)		7.3840E+06	-	-
	EI_Y (N-m^2)		7.9408E+01	7.7141E+01	2.94
	EI_Z (N-m^2)		3.2085E+03	-	-
	GJ (N-m^2)		2.5662E+01	2.5427E+01	0.93
Offset	Tension center (m)	y-dir	2.7661E-02	-	-
		z-dir	0.0000E+00	-	-
	Shear center (m)	y-dir	1.6825E-02	-	-
		z-dir	0.0000E+00	-	-

Table 6. Cross-sectional properties of KARI small-scaled blade

Properties		CORDAS (KARI)	KSec2D	% error
Tension center offset (m)	y-dir	0.2023	0.2179	0.0199C
	z-dir	0.0013	0.0022	0.0011C
Shear center offset (m)	y-dir	0.1927	0.2250	0.0411C
	z-dir	0.0024	0.0021	0.0004C
Section stiffness	EA (N)	5.07E+06	5.29E+06	4.34
	EI_Y (N-m^2)	5.45E+01	5.70E+01	4.59
	EI_Z (N-m^2)	2.22E+03	2.33E+03	4.95
	GJ (N-m^2)	6.74E+01	8.42E+01	24.93

Table 6 shows the comparison of the cross-sectional properties between KSec2D and CORDAS for the locations of tension center, shear center and cross-section stiffnesses. The KSec2D results are obtained using 2,061 3-node triangular elements. Generally, a good correlation is obtained between the two codes, except the torsion rigidity.

Conclusion

In this work, a general purpose, finite element-based, two-dimensional cross-sectional analysis program (code name: KSec2D) was developed. In order for versatility of the program, the KSec2D uses MD Nastran input file format as pre-processor and TECPLOT360 input file format as post-processor. The modulus-weighted method was used to consider inhomogeneous cross-sectional properties of composites blades. The classical laminate plate theory is applied to obtain the effective moduli of the walls of composite blades. The St. *Venant* torsion theory is used to calculate the torsion constant of blade cross section. The location of shear center for any given cross- sections are determined according to the Trefftz' definition. A series of benchmark tests have been conducted to ensure adequate accuracy solution of cross-section properties for various cross-section blades. It is indicated that fair to good correlation is obtained for various cross-section beams and blades considered in comparison with experimental data and/or commercial finite element sectional analysis results. It is believed that KSec2D can be used as a useful tool to model and analyze arbitrary cross-sections in the preliminary design stage of rotary wing vehicles.

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