# The Optimal Design of a Brushless DC Motor Using the Advanced Parallel Genetic Algorithm

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#### **Abstract**

In case of the optimization problems that have many design variables, the conventional genetic algorithms(GA) fall into a trap of local minima with high probability. This problem is called the premature convergence problem. To overcome it, the parallel genetic algorithms which adopt the migration mechanism have been suggested. But it is hard to determine the several parameters such as the migration size and the migration interval for the parallel GAs. Therefore, we propose a new method to determine the migration interval automatically in this paper. To verify its validity, it is applied to some traditional mathematical optimization problems and is compared with the conventional parallel GA. It is also applied to the optimal design of the brushless DC motor for an electric wheel chair which is a real world problem and has five design variables.

Key Words: Genetic Algorithm(GA), Migration, Optimal Design, Brushless DC Motor

#### 1. Introduction

The designer of a traction motor for an electric wheel chair should try to design a lower weight motor in order to improve the driving performance, the running distance and the low material cost. Therefore the optimal design which uses the optimization algorithm is tried to satisfy all the above condition.

The genetic algorithm(GA) is a promising search technique for finding near-optimal solutions in large space. GA is now widely recognized

as an effective search paradigm in the artificial intelligence, the image processing, the VLSI circuit layout, solving non-linear equations, the optimal design of an electric machine, and many other areas. But in the case of the optimization problems that have many design variables, such as the optimal design of a brushless DC motor(BLDCM), the conventional GAs fall into traps of local minima with high probability. This problem is called the premature convergence problem. Diversity preservation methods based on a spatial separation have been proposed in order to avoid it. The parallel genetic algorithm is most famous among them. It separates the population to several sub-populations [1].

The parallel GA is composed of several sequential algorithms operating simultaneously which are linked by the genetic operation such as

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the migration. The migration mechanism has several parameters such as the migration size, the migration interval, and the migration topology. But it is hard to determine them. Therefore, we propose a new method to determine the migration interval automatically in this paper. To verify its superiority, the proposed method is applied to some traditional mathematical problems and compared with the conventional parallel GA.

The proposed method is also applied to the optimal design of a BLDC motor which is a real world problem and has five design variables...

### 2. Parallel Genetic Algorithm

# 2.1 The Premature Convergence Problem

One common problem in traditional GAs is the premature convergence. It means that GAs find a local instead of a global optimum. Many variations on traditional GAs have been devised to address this problem. Previous researches have focused on two general approaches to avoid the premature convergence. The first approach is to lower a convergence speed so GA can do a more through search before converging, increasing the chance of finding a global optimum. These schemes affect the selection phase. The second approach focuses on keeping the diversity of a population high by modifying traditional replacement and mating operators. Of course, the parallel GA may be seen as a particularly natural and efficient version of such approaches [1]. The parallel GA maintains multiple and separate subpopulations which may be allowed to evolve independently. This allows each subpopulation to explore different parts of the each maintaining search space, its own high-fitness individuals and controlling to mix with other subpopulations. The parallel GA models the evolution of species in a way more similar to nature than a single population GA.

# 2.2 Multi-population Parallel Genetic Algorithm

There are three main types in the parallel GAs: 1) global single-population master slave GA, 2) single population fine-grained, and 3) multiple-population coarse-grained GA. In a master-slave GA there is a single population, but the evaluation of fitness is distributed among several processors. Fine-grained parallel GA's are suited for massively parallel computers and consist of one spatially structured population. Selection and recombination are restricted to a small neighborhood, in which some interaction is permitted. Multi-population parallel GA called coarse-grained GA is very popular. It uses several subpopulations that evolve independently from each other for the given number of generation and exchanges individuals occasionally. It is also effective in solving larger problems and finding better solutions in case of using single processor [2]. Therefore it is selected as the type of parallel GA in this paper. In this multi-population parallel GA, the most important topic is the parameter related to a migration. They are the migration size, the migration interval, the method of migrant selection and the migration topology. Many researches on these parameters have been proposed.

Many possibilities exist for the structure of the migration of individuals between subpopulations [3-4]. The most general migration strategy is unrestricted migration. In this method, individuals may migrate from any subpopulation to another. The selection of the individuals for migration takes place at random. And the interval of migration is predetermined. Fig. 1 is the structure of the parallel GA using migration mechinism.

# 2.3 New Strategy for Migration Interval

Generally most parallel GAs use the constant migration interval. The short migration interval (or frequent migration) retards the convergence during the final stage. On the contrary the long migration interval is prone to fall into traps of local minima [5]. So we use a new strategy for migration. If the variation of the average fitness subpopulation for each is under pre-determined level (F<sub>TH</sub>), the migration to other subpopulation started. Otherwise subpopulation continues a separate search independently.

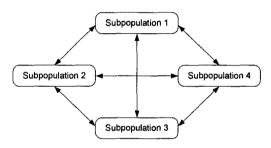


Fig. 1. The structure of parallel GA with the migration strategy

$$\left| \frac{F_j(i+1) - F_j(i)}{F_j(i)} \right| \le F_{TH} \tag{1}$$

, where  $1 \le j \le s$ ,  $1 \le i \le n$ ,

s: number of subpopulation,

n: maximum number of generation

If (1) is satisfied for any subpopulation, it means that the corresponding subpopulation is converged to a local minimum and the migration is needed. In (1), Fj(i) is the average fitness for each subpopulation where the number of generation is I. Fig. 2 is the structure of the proposed parallel GA. And some techniques for genetic operator are introduced to advance the characteristic of parallel

GA. They are scaling window and elitism. In the next section, we will show that the proposed algorithm searches a better global optimum effectively.

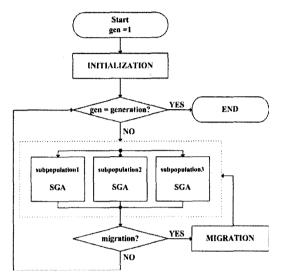


Fig. 2. The structure of parallel GA with a new migration strategy

#### 2.4 Numerical Examples

Two test functions are used to compare the performance of the proposed method with the conventional parallel GAs which use a constant migration interval [6]. The first test function, F2, is a two-dimensional function with 25 peaks. The 25 peaks are all of differing heights, ranging from 476.191 to 499.002. The global maximum occurs at (-32, -32). Fig. 3 is the graph of F2 test function. The second test function F4 is a four-dimensional function and it has a minimum value, 0, at all xi=1.

$$\begin{split} F_2(x,y) = & 500 - \frac{1}{0.002 + \sum_{i=0}^{24} \frac{1}{1 + i(x - a(i))^6 + (y - b(i))^6}}, \\ where \ a(i) = & 16[(i \operatorname{mod} 5) - 2], \ b(i) = 16[(i/5) - 2], \\ & (-65.536 \leq x, y \leq 65.536) \end{split} \tag{2}$$

$$\begin{split} F_4(x) &= 100(x_2-x_1^2)^2 + (1-x_1)^2 \\ &+ 90(x_4-x_3^2)^2 + (1-x_3)^2 + 10.5(x_2-1)^2 \\ &+ (x_4-1)^2 + 19.8(x_2-1)(x_4-1) \\ ,where &\quad -10 \leq x_1, x_2, x_3, x_4 \leq 10 \end{split}$$

We run each algorithm 100 times on each test function and average the searched result. Two criteria are introduced to evaluate the result. The first criterion (C1) is the average of searched best solutions. The second criterion (C2) is the average of best ten trails out of 100 trials. The parallel GA has three sub-populations. Therefore the size of sub-populations for PGA is one-third of simple GA. Each subpopulation has 20 individuals. The miximum number of generation is 3000. The number of migrant individuals is 10. The migrants in a given subpopulation are selected randomly.

In the case of two-dimensional function (F2), every method has same result as shown in Table 1. But in the case of four-dimensional function (F4), the result is different according to a search method as shown in Table 2. In terms of the criterion C2, the simple genetic algorithm(SGA) which uses a single population is inferior to other methods in the case of four-dimensional function. This means that the SGA is hard to converge to the global minimum. It converges to the local minima. For the conventional PGA, five migration intervals are selected. When the migration interval is short, the conventional PGA finds the better solution in terms of the criterion C1 [5]. But in terms of the criterion C2, the conventional PGA has the best result when the migration interval is 30. So the best migration interval can't be determined because the results are different according to the given criterion. But the proposed PGA searches better(lower) solutions than other methods in any criterion. Fig. 4 is the comparison of the convergence speed for the F4 test function between the simple GA and the new parallel GA. As shown in Fig. 4, the proposed PGA converges an optimum faster than the simple GA. From all the above result, we know that the proposed new PGA is more powerful than other techniques particularly in the high dimensional optimization problems.

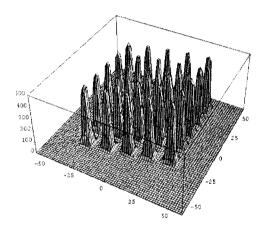


Fig. 3. The Graph of F2 test function

Table 1. Performance comparison for F2 test function

	C1	C2
PGA (I= 10)	4.990 E+2	4.990 E+2
PGA (I= 100)	4.990 E+2	4.990 E+2
SGA	4.990 E+2	4.990 E+2
New PGA	4.990 E+2	4.990 E+2

The symbol I is a migration interval in generation.

Table 2. Performance comparison for F4 test function

	C1	C2
PGA (I= 5)	2.5906E-4	5.8066E-07
PGA (I= 10)	3.0161E-4	1.1398E-11
PGA (I= 30)	3.3769E-4	2.1090E-13
PGA (I= 50)	4.1302E-4	5.0399E-12
PGA (I= 100)	6.2976E-4	8.0560E-07
SGA	3.1311E-4	6.6821E-06
New PGA	3.9450E-5	3.8992E-18

The best result on each criterion is shown in boldface.

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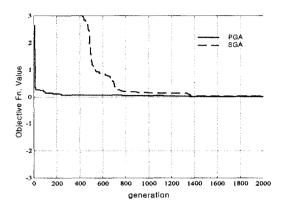


Fig. 4. The comparison of convergence characteristics for F4 test function

# Optimal design of BLDC Motor

### 3.1 Synthesis

Now, the proposed parallel method is applied to the optimal design of the BLDC motor for an electric wheel chair. It is a real world and high dimensional problem. The first step of optimization is the design synthesis of the BLDC motor. The synthesis is a procedure for producing the motor on the basis of a set of design variables, other design data, and the motor specification. We select five independent design variables consisting of the maximum flux density for a stator and a rotor punching and four geometric variables. Four geometric variables are the stator outside radius. the motor axial length, the rotor outside radius and the depth of rotor magnet. The air gap length and the stator slot opening are dictated by mechanical considerations. Fig. 5 is the cross-section of stator and rotor of BLDC motor. Topological parameters such as the number of phases, magnet poles, and slots per phase are fixed [7]. Therefore the optimization of the BLDC motor is a five-dimensional problem. So the optimal algorithm for a high-dimensional problem should be used.

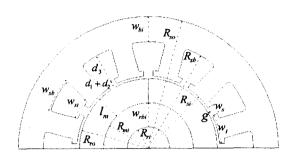


Fig. 5. The cross-section of stator and rotor

# 3.2 Objective Function and Design Constraints

The designer of a traction motor for an electric wheel chair should try to design a lower weight motor in order to improve the driving performance, the running distance and the low material cost. Therefore the weight of the BLDC motor is selected for the objective function of an optimal design. The weight of the BLDC motor is calculated by (4). And the other characteristics such as the efficiency of the BLDC motor are selected as the constraints of optimization.

$$W_{total} = W_{cu} + W_m + W_{co} + W_{sh} \tag{4}$$

$$W_{cu} = k_{co} A_s (L + \pi \tau_c/2) N_s C u_d [kg]$$
 (5)

$$W_m = A_m L N_m M_d \tag{6}$$

$$\begin{split} W_{co} &= C_d ([\pi (R_{so}^2 - R_{si}^2) - N_s A_s] L k_{st} \\ &+ \pi [(R_{ro} - l_m)^2 - R_{ri}^2] L k_{st}) [kg] \end{split} \tag{7}$$

$$W_{sh} = Sh_d \frac{\pi}{\Lambda} R_{ri}^2 L \tag{8}$$

, where

 $W_{cu}$ ,  $W_m$ ,  $W_c$ ,  $W_{sh}$ : Weight of stator winding, magnet, core and shaft [kg]  $Cu_d$ ,  $M_d$ ,  $C_d$ ,  $S_d$ : Density of stator winding, magnet, core and shaft [kg/m']

 $k_{cp}$ : Fill factor of stator slot,  $A_s$ : Area of sartor slot[ $m^2$ ]

L: Axial length of stator[m],  $\tau_c$ : Coil pitch[m]

 $N_s$ : No. of slots,  $N_m$ : No. of poles

 $A_m$ : Area of magnet [m<sup>i</sup>],  $l_m$ : Radial length of magnet[m]

 $R_{so}, R_{si}$ : Stator outside and inside radius[m]  $R_{ro}, R_{ri}$ : Rotor outside and inside radius[m]  $k_{si}$ : Stacking factor for lamination

### 3.3 Result of Optimal Design

The 210[w], 4 pole, three phase BLDC motor for an electric wheel chair is designed as a sample design. The rated speed is 2,500[rpm] and the battery voltage is 24[V]. Fig. 6 is the convergence characteristics of motor weight during the optimization process. The weight of initial model is 3.33[kg] and the weight of optimized model is 2.57[kg]. Through the optimization, the weight of motor is reduced than an initial design by 23[%].

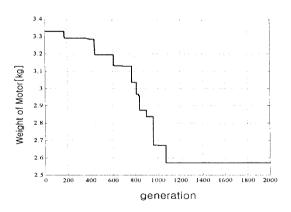


Fig. 6. The convergence characteristics of motor weight

## 4. Conclusion

To overcome the premature convergence problem, the new migration technique for a parallel GA is introduced in this paper. We showed that the proposed parallel GA found better solution than a simple GA and a conventional Parallel GA, particularly in the high-dimensional problems. The proposed parallel GA succeeded in reducing the weight of motor when it was applied to the optimal design of the BLDC motor for an electric wheel chair.

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### **Biography**

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