Detection Techniques for MIMO Multiplexing: A Comparative Review

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Abstract

Multiple-input multiple-output (MIMO) multiplexing is an attractive technology that increases the channel capacity without requiring additional spectral resources. The design of low complexity and high performance detection algorithms capable of accurately demultiplexing the transmitted signals is challenging. In this technical survey, we introduce the state-of-the-art MIMO detection techniques. These techniques are divided into three categories, viz. linear detection (LD), decision-feedback detection (DFD), and tree-search detection (TSD). Also, we introduce the lattice basis reduction techniques that obtain a more orthogonal channel matrix over which the detection is done. Detailed discussions on the advantages and drawbacks of each detection algorithm are also introduced. Furthermore, several recent author contributions related to MIMO detection are revisited throughout this survey.

Keywords: MIMO spatial multiplexing, MIMO detection, maximum-likelihood, tree search, lattice reduction

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1. Introduction

Multiple-input multiple-output (MIMO) techniques have gained much attention due to their capabilities to improve the transmission reliability and/or increase the channel capacity. Depending on their objective, MIMO techniques are grouped into two main categories: (*i*) MIMO diversity techniques are aiming for increased transmission reliability. Examples of these transmit diversity technqiues include the Alamouti code [1] and orthogonal codes proposed by Taroukh *et al.* [2]. Receive diversity can be achieved by receiving redundant forms of the same signal. These forms can be combined using maximal-ratio combiner (MRC), equal-gain combiner (EGC), or selection combiner (SC), to improve the receive signal-to-noise ratio (SNR) [3][4]. (*ii*) MIMO spatial multiplexing (MIMO-SM) techinques linearly increase the channel capacity without requiring additional spectral resources [5]. In MIMO-SM, independent data symbols are sent via sufficiently-separated antennas, leading to a linear increase in the channel capacity that is proportional to the number of transmit antennas. In this technical survey, we only consider the receiver structure for the MIMO-SM techniques.

The capacity achieved by the MIMO-SM techniques, and the bit error rate (BER), is highly dependent on the detection algorithms employed at the receiver side to demultiplex the transmitted signals [6]. Thus, a variety of detection algorithms were proposed in the literature. The maximum-likelihhod detector (MLD) is the optimum receiver for the MIMO-SM systems [7]. Although MLD achieves the optimum performance in terms of capacity and BER, the high complexity of its *brute-force strategy* makes it inapplicable for computational complexity and latency limited communication systems. Hence, suboptimal detection algorithms were proposed to achieve a tradeoff between performance and complexity. In this paper, we categorize the detection algorithms into three main categories based on their employed demultiplexing strategies. These categories are linear detection (LD), decisision feedback detection (DFD), and tree-search detection (TSD). Also, we introduce lattice basis reduction-aided detection schemes.

LD algorithms linearly treat the received vector using a filtering matrix constructed from performance-based criteria. These criteria are zero-forcing (ZF) and minimum-mean square error (MMSE), which are used in the linear ZF (LZF) and the linear MMSE (LMMSE) detector, respectively [8]. The idea behind DFD techniques is that an already-detected signal is cancelled out from the received vector, resulting in a system with fewer interferers [9]. The error performance and diversity order of this category depends on the accuracy of the order in which signals are detected.

Since the error perfomances of the LD and DFD techniques depend on the orthogonality of the channel matrix, several techniques were proposed in the literature to obtain a more orthogonal channel matrix over which the detection is carried out. Such techniques are known as "lattice basis reduction" [10][11][12][13]. The optimum diversity order was achieved when the lattice reduction techniques were combined with simple detection.

TSD algorithms achieve a quasi-ML performance while tremendously reducing the computational complexity of the MLD [14][15][16]. The Sphere decoder (SD) has a random complexity with a low average value and a high worst-case value. Fixed-complexity SD (FSD) limits the search of the SD to a number of hypotheses so that the complexity becomes fixed. The FSD employs a specific ordering scheme to attain the quasi-ML performance. Also, QR-decomposition with M-algorithms (QRD-M) was introduced to overcome the random complexity of the SD by retaining a fixed number of candidates per detection level. The conventional QRD-M algorithm was the subject of several studies and improvements that outlined its merits and overcame several drawbacks (see [17] and [18] and references therein).

The rest of this technical survey is as follows. In Section 2 we present the system model and state the detection problem. In section 3 and Section 4, LD and DFD techniques are presented. Lattice reduction aided detection is presented in Section 5, and TSD algorithms are introduced in Section 6. In Section 7 we summarize the paper by introducing a general comparison between the investigated detection schemes. Finally, conclusions are drawn in Section 8.

2. MIMO-SM System Model and Problem Statement

2.1 System Model

We consider an open-loop single-user MIMO-SM system, where a base station equipped with n_T transmit antennas communicates with a single user equipped with n_R receive antennas [9]. The received complex vector $\mathbf{r} \in \mathbb{C}^{n_R}$ is given by:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{n_T}$ is the transmitted vector with elements independently drawn from a quadrature amplitude modulation set with size C, and $\mathbf{n} \in \mathbb{C}^{n_R}$ is the additive white Gaussian noise vector whose independent and identically distributed (i.i.d.) elements have zero mean and σ_n^2 variance. In (1), $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix whose elements are i.i.d. circular-symmetric Gaussian random variables with zero mean and unit variance. The (i, j)-th element $h_{i,j}$ is the channel coefficient coupling the i-th receive antenna to the j-th transmit antenna.

2.2 Problem Statement

Working on the vector **x**, the channel matrix **H** generates the complex lattice [19]

$$\begin{aligned} \Delta(\mathbf{H}) &= \{\mathbf{H}\mathbf{x} : \mathbf{x} \in \Omega^{n_T}\}, \\ &= \{x_1\mathbf{h}_1 + x_2\mathbf{h}_2 + \cdots + x_{n_T}\mathbf{h}_{n_T} \,|\, x_i \in \Omega\}, \end{aligned} \tag{2}$$

where Ω is the modulation set and $\mathbf{h}_i$ is the i-th column of matrix **H**. In light of that, the MLD is defined as finding the lattice point $\mathbf{H}\hat{\mathbf{x}}$ such that $\|\mathbf{r} - \mathbf{H}\hat{\mathbf{x}}\|^2$ is minimized. That is

$$\mathbf{x}_{\mathrm{ML}} = \arg\min_{x \in \Omega^{n_T}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2. \tag{3}$$

Note that the relationship between the basis vector of the resulting lattice , i.e., the columns of matrix **H**, directly affect the performance of the detection process. **Fig. 1(a)** shows an example of a 2-dimensional real lattice with basis vectors $\mathbf{h}_1 = [0.39 \;\; 0.59]^T$ and $\mathbf{h}_2 = [-0.59 \;\; 0.39]^T$, and **Fig. 1(b)** shows an example with basis vectors $\mathbf{h}_1 = [0.39 \;\; 0.6]^T$ and $\mathbf{h}_2 = [0.5 \;\; 0.3]^T$, where $\Omega = \{-3, -1, +1, +3\}$. In the first example, the columns of the channel matrix are perfectly orthogonal with equal norms, which indicates that the decision regions have square forms. On the other hand, **Fig. 1(b)** shows an example where the lattice basis vectors are correlated, resulting in decision regions which have parallelogram shapes. Clearly, the effect of lattice perturbation by additional noise on the detection algorithm of the example given in **Fig. 1(b)** is more severe. This is because the shortest diagonal of the parallelgoram becomes small, where an additive noise with small variance may lead to error in the detection process.

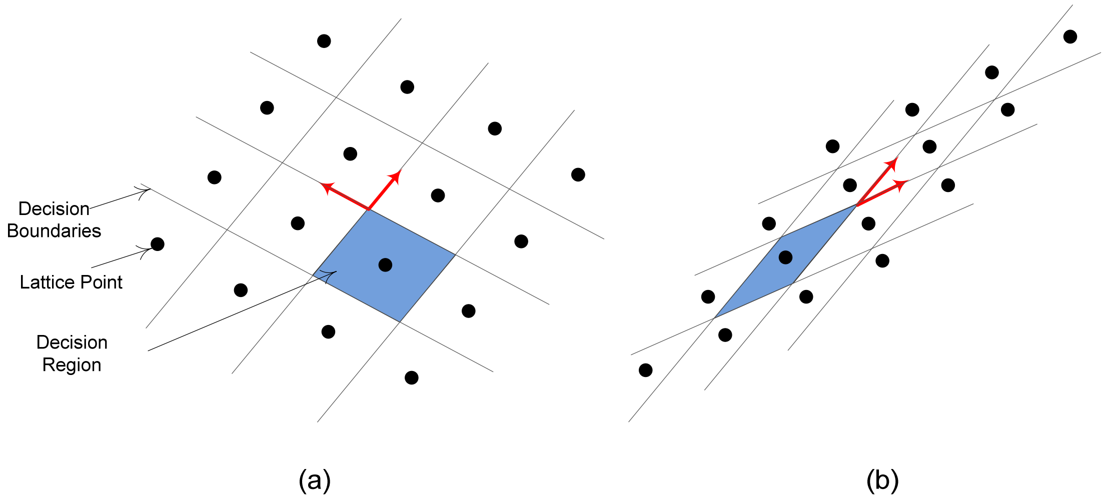

Fig. 1. Examples of 2-dimensional real lattice, with (a) orthogonal basis vectors and (b) correlated basis vectors.

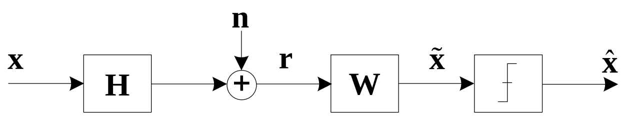

Fig. 2. Block diagram of MIMO-SM with linear receiver.

3. Linear Detection

The idea behind linear detection schemes is to treat the received vector by a filtering matrix **W**, constructed using a performance-based criterion, as depicted in **Fig. 2**. The well known ZF and MMSE performance-based criteria are used in the LZF and LMMSE detector, respectively.

LZF treats the received vector by the pseudo-inverse of the channel matrix, resulting in full cancellation of the interference with colored additive noise. The detector in matrix form is given as follows:

$$\mathbf{W}_{\mathrm{ZF}} = \left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H, \tag{4}$$

where $(\mathbf{A})^H$ and $(\mathbf{A})^{-1}$ denote the Hermitian transpose and inverse of matrix **A**, respectively.

When the channel matrix is ill-conditioned, $\mathbf{W}_{\mathrm{ZF}}$ has high power, which leads to noise enhancement. To alleviate the noise enhancement, the LMMSE detector was introduced, where the noise is considered in constructing the filtering matrix. To accomplish that, the filtering matrix is given by:

$$\begin{aligned} \mathbf{W}_{\mathrm{MMSE}} &= \arg\min_{\mathbf{G}} \left(\mathrm{E}\left[\left\| \mathbf{G}\mathbf{r} - \mathbf{x} \right\|^2 \right] \right), \\ &= \left(\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T} \right)^{-1} \mathbf{H}^H, \end{aligned} \tag{5}$$

where σ_n^2 is the noise variance, with $\mathrm{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}$. It has been shown in [20] that the improvement by the LMMSE detector over the LZF one does not depend only on the plain value of σ_n^2, but on how close σ_n^2 is to the minimum singular values of the channel matrix **H**.

It was shown in [20] that the ratio between the condition numbers of the filtering matrices of the LMMSE and LZF detectors is approximated by:

$$\frac{\operatorname{cond}(\mathbf{W}_{\mathrm{MMSE}})}{\operatorname{cond}(\mathbf{W}_{\mathrm{ZF}})} \approx \frac{1+\sigma_n^2/\sigma_1^2(\mathbf{H})}{1+\sigma_n^2/\sigma_{n_T}^2(\mathbf{H})}, \tag{6}$$

where $\sigma_1^2(\mathbf{H}) \geq \sigma_2^2(\mathbf{H}) \geq \cdots \geq \sigma_{n_T}^2(\mathbf{H})$ is the power of the singular values of **H**. Also, because the minimum singular value of **H** vanishes as the matrix dimension increases [21][22], the power of $\mathbf{W}_{\mathrm{ZF}}$ increases, leading to severe noise enhancement.

Fig. 3 shows the BER performance of the linear detection schemes in 4×4 MIMO-SM system using 4-QAM. Although σ_n^2 equals 0.008 at E_b/N_0 of 30dB, this small regularization of the channel matrix leads to about 4.5dB of gain in the E_b/N_0. This emphasizes that the improvement by the LMMSE detector is rather dependent on the closeness of the noise variance to the small singular values of the channel matrix **H**. This corrects the misconception which assumes that the improvement by the LMMSE detector is only dependent on the noise variance.

Although the linear detection schemes are favorable in terms of computational complexity, their BER performance is severely degraded due to the noise enhancement in the ZF detector case, and when the channel matrix is ill-conditioned.

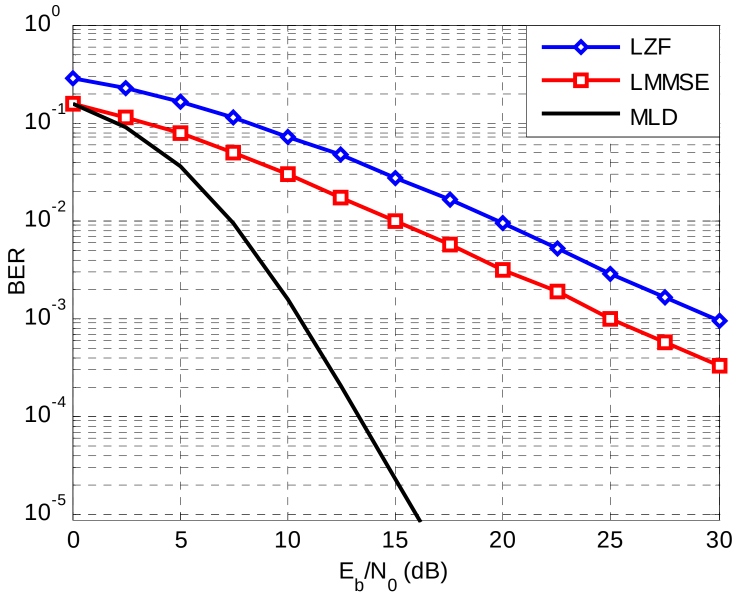

Fig. 3. BER performance of the linear detection schemes in 4×4 MIMO-SM system using 4-QAM.

4. Decision-Feedback Detection

Although linear detection approaches are attractive in terms of computational complexity, they lead to degradation in the BER performance. This is because the components of **x** are detected independently. Superior performance can be obtained if non-linear approaches are employed, as in the decision-feedback detection (DFD) algorithms. In DFD approaches, symbols are detected successively, where already-detected components of **x** are subtracted out from the received vector. This leads to a system with fewer interferers. In the following two subsections, we introduce two categories of DFD algorithms.

4.1 The V-BLAST Detection Algorithm

In vertical Bell Laboratories layered space time (V-BLAST) detection schemes, symbols are detected successively using the thus far mentioned linear detection approaches. At the end of each iteration, the already-detected component of **x** is subtracted out from the received vector. Also, the corresponding column of matrix **H** is removed. When decision-feedback approach is used, error propagation becomes a challenging issue. Therefore, the order in which symbols are detected has a great impact on the system performance.

The idea behind the zero-forcing V-BLAST algorithm (ZF-VB) is to detect the components of **x** that suffer the least noise amplification first. For the first decision, the pseudo-inverse, i.e., **G** equals $\mathbf{H}^{\dagger}$, of matrix **H** is obtained. Assume that the noise components are i.i.d. and that noise is independent of **x**, then, the row of **G** with the least Euclidean norm corresponds to the required component of **x**. That is,

$$k_1 = \arg\min_j \left(\left\| \underline{\mathbf{g}}_j \right\|^2 \right), \tag{7}$$

$$\tilde{x}_{k_1} = \underline{\mathbf{g}}_{k_1} \mathbf{r}^{(1)}, \tag{8}$$

and

$$\hat{x}_{k_1} = Q(\tilde{x}_{k_1}), \tag{9}$$

where $\underline{\mathbf{g}}_j$ is the *j*-th row of the filtering matrix **G**, *Q*(.) is the demodulation function, and the superscript in (8) is the iteration index. At the first iteration, $\mathbf{r}^{(1)} = \mathbf{r}$ and $\mathbf{G}^{(1)} = \mathbf{H}^{\dagger}$. At the end of the first iteration, the interference due to the k_1-th component of **x** is cancelled out as follows:

$$\mathbf{r}^{(2)} = \mathbf{r}^{(1)} - \hat{x}_{k_1} \mathbf{h}_{k_1}, \tag{10}$$

$$\mathbf{H}^{(2)} = \mathbf{H}^{(1)^{-k_1}} = \left[\cdots, \mathbf{h}_{k_1-1}, \mathbf{h}_{k_1+1}, \cdots \right]. \tag{11}$$

This strategy is repeated iteratively until detecting the last component of **x**. We refer to the ZF-VB algorithm without the sorting stage as the assorted ZF-VB algorithm. It is implemented by skipping the 6-th line of the pseudocode in Table 1 and setting $k_i = i$ at the *i*-th iteration. The assorted ZF-VB algorithm is introduced herein to show the effect of signal ordering on the bit error performance.

The MMSE filtering strategy can be used where the resulting detector is referred to as the MMSE-VB detector. Also, we refer to the MMSE-VB algorithm as the assorted MMSE-VB algorithm when the sorting stage is skipped. In this case, the components of **x** are detected in an ascending order.

Table 1 shows the pseudocode for the ZF-VB detection algorithm. The MMSE-VB detection algorithm can be obtained by using the MMSE criterion in constructing the filtering matrix.

Fig. 4 shows the BER performance of the VB detection schemes as well as that of the MLD. In the case of the VZF-VB scheme, a gain of about 4dB is achieved at a target BER of 10^{-3} when signal ordering is imployed. On the other hand, the sorted MMSE-VB scheme outperforms the assorted MMSE-VB one by more than 7dB. In all cases, the MMSE-VB scheme outperforms the ZF-VB one. At a target BER of 10^{-4}, MMSE-VB lags the optimum performance by about 6.7 dB.

Although the performance of the VB detection techniques are superior to those of the linear schemes, their complexity is high due to the matrix inversion required at each iteration. The complexity of the VB detection schemes is $O(n_T^4)$ [23]. Despite that several reduced complexity implementations of the VB scheme were introduced in the literature [23][24], its complexity is still high as compared to the QR-decomposition-based detection schemes.

Table 1. Pseudocode for the ZF-VB detection algorithm.

Input : $\mathbf{H}$, $\mathbf{x}$, n_T, $\mathbf{U} = \mathbf{I}_{n_T}$
$\mathbf{H}^{(1)} = \mathbf{H}$
$\mathbf{r}^{(1)} = \mathbf{r}$
for $i = 1$ *to* n_T **do**
    $\mathbf{G}^{(i)} = \mathbf{H}^{(i)\dagger}$
    $k_i = \arg\min_j \left(\left\| \underline{\mathbf{g}}_j \right\|^2 \right)$
    $\underline{\mathbf{w}}_i = \underline{\mathbf{g}}_{k_i}$
    Exchange columns k_i and i in $\mathbf{U}$
    $\tilde{x}_i = \underline{\mathbf{w}}_i \mathbf{r}$
    $\hat{x}_i = \mathcal{Q}(\tilde{x}_i)$
    $\mathbf{r}^{(i+1)} = \mathbf{r}^{(i)} - \hat{x}_i \mathbf{h}_{k_i}$
    $\mathbf{H}^{(i+1)} = \mathbf{H}^{(i)-k_i}$
end
Output: $\mathbf{U}\hat{\mathbf{x}}$

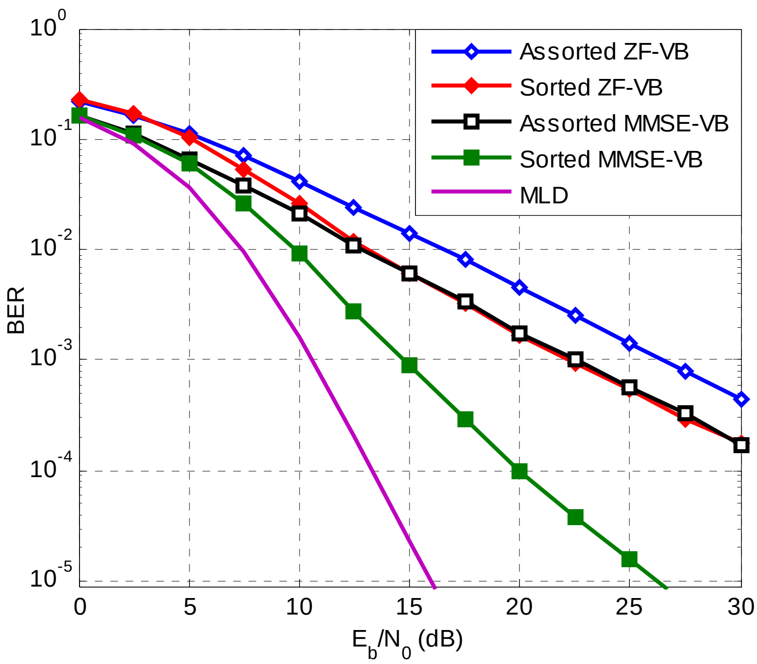

Fig. 4. BER performance of the VB detection schemes in 4×4 system using 4-QAM.

4.2 QRD-based Detection

DFD based on the QR decomposition (QRD) of the MIMO channel requires only a fraction of the computational efforts required by the V-BLAST detection algorithm [25][26]. This is why QRD-DFD is preferable for power and latency limited wireless communication systems. **Fig.**

5 depicts the block diagram of the DFD using the QRD.

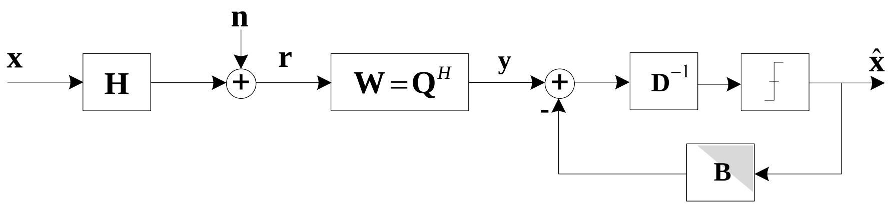

Fig. 5. Block diagram of the decision-feedback detection (DFD) based on the QR decomposition.

In the ZF-SQRD (sorted QRD) detection scheme, the channel matrix is decomposed into the multiplication of a unitary matrix $\mathbf{Q}\in\mathbb{C}^{n_R\times n_T}$, i.e., $\mathbf{Q}^{\mathrm{H}}\mathbf{Q}=\mathbf{I}$, and an upper triangular matrix $\mathbf{R}\in\mathbb{C}^{n_T\times n_T}$ such that $\mathbf{H} = \mathbf{QRU}$, where $\mathbf{U}$ is the column permutation matrix. Then, the received vector is multiplied by the Hermitian transpose of $\mathbf{Q}$

$$\begin{aligned}\mathbf{y}&=\mathbf{Rx}+\mathbf{v}\\&=(\mathbf{D}+\mathbf{B})\mathbf{x}+\mathbf{v},\end{aligned}\tag{12}$$

where $\mathbf{y} = \mathbf{Q}^{\mathrm{H}}\mathbf{r}$ and $\mathbf{v} = \mathbf{Q}^{\mathrm{H}}\mathbf{n}$. Note that the noise statistics are unchanged due to the orthogonality of matrix $\mathbf{Q}$. The matrix $\mathbf{D}$ is the diagonal matrix whose elements are the diagonal elements of $\mathbf{R}$, and $\mathbf{B}$ is a strictly upper triangular matrix such that $(\mathbf{D} + \mathbf{B}) = \mathbf{R}$. As a consequence, the MIMO system becomes spatially causal, which implies that:

$$y_k = R_{k,k}\tilde{x}_k + \sum_{i=k+1}^{n_T} R_{k,i}\hat{x}_i \tag{13}$$

and

$$\hat{x}_k = Q\left(\frac{y_k - \sum_{i=k+1}^{n_T} R_{k,i}\hat{x}_i}{R_{k,k}}\right). \tag{14}$$

Note that due to the structure of matrix $\mathbf{R}$, the last component of $\mathbf{x}$, i.e., $x_{n\mathrm{T}}$, is interference-free; hence, it is detected first. The already-detected component of $\mathbf{x}$ is canceled out from the received vector. This technique is repeated up to the first component of $\mathbf{x}$, i.e., x_1.

In the case of MMSE-SQRD detection, the extended channel matrix

$$\tilde{\mathbf{H}}=\begin{bmatrix}\mathbf{H}^T & \sigma_n\mathbf{I}\end{bmatrix}^T \tag{15}$$

is decomposed into $\mathbf{Q}$ and $\mathbf{R}$ matrices such that $\tilde{\mathbf{H}} = \mathbf{QRU}$, with $\mathbf{U}$ as the column permutation matrix. The assorted ZF-QRD and MMSE-QRD detection schemes are obtained by skipping the sorting stage in the QRD.

Table 2 gives the pseudocode of the ZF-SQRD detection algorithm. The MMSE-SQRD scheme is obtained by simply replacing $\mathbf{H}$ by $\tilde{\mathbf{H}}$.

Fig. 6 shows the BER performance of the QRD-based detection schemes as well as that of the optimum detector. The best performance is achieved by the MMSE-SQRD detection scheme, where it lags the optimum performance by about 9dB at target BER of 10^{-4}.

In general, DFD detection algorithms without sorting have a diversity order of $(n_{\mathrm{R}} - n_{\mathrm{T}} + 1)$ [27]. That is, the diversity order of DFD without sorting equals one for an equal number of transmit and receive antennas regardless of the number of receive antennas. This is because signals are detected independently, where the ZF solution or MMSE solution of each component of **x** is demodulated and considered as error-free in the following detection levels.

Since one of the main reasons for the inaccuracy of both the linear detection and DFD is the ill-conditionality of the channel matrix, we introduce in the following section the method of lattice bais reduction-aided detection.

Table 2. Pseudocode for the ZF-SQRD detection algorithm.

Input : $\mathbf{H}$, n_T, $\mathbf{p} = 1, 2, \cdots, n_T$
$\mathbf{R} = \mathbf{0}$, $\mathbf{Q} = \mathbf{H}$
for i = 1 to n_T **do**
$\mathbf{norms}_i = \|\mathbf{q}_i\|^2$
end
for i = 1 to n_T **do**
$k_i = \arg\min_{j=i,\cdots,n_T} (\mathbf{norms}_j)$
Exchange i-th and k_i-th columns in $\mathbf{R}$, $\mathbf{Q}$, $\mathbf{p}$, and $\mathbf{norms}$
$R_{i,i} = \sqrt{\mathbf{norms}_i}$
$\mathbf{q}_i = \mathbf{q}_i \, / \, R_{i,i}$
for $k = i + 1$ to n_T **do**
$R_{i,k} = \mathbf{q}_i^H \cdot \mathbf{q}_k$
$\mathbf{q}_k = \mathbf{q}_k - R_{i,k} \cdot \mathbf{q}_i$
$\mathbf{norms}_k = \mathbf{norms}_k - \|R_{i,k}\|^2$
end
end
$\mathbf{y} = \mathbf{Q}^H \mathbf{r}$
for $k = n_T, \cdots, 1$ **do**
$\hat{d}_k = \sum_{i=k+1}^{n_T} R_{k,i} \hat{x}_i$
$\hat{x}_k = \mathcal{Q}\left(\frac{y_k - \hat{d}_k}{R_{k,k}}\right)$
end
Permutate $\hat{\mathbf{x}}$ according to $\mathbf{p}$

5. Lattice Reduction-Aided Detection (LRAD)

When the basis vectors of the lattice defined in (2) are orthogonal or semi-orthogonal, the channel matrix overall power is fairly distributed among its singular values, i.e., the singular values of the channel matrix have almost equal powers. In this case, the noise enhancemenet is overcome and the detection regions have square or rectangular shapes, depending on the lengths of the basis vectors. On the other hand, when the basis vectors are correlated, the power of the channel matrix is concentrated in a few singular values. This leaves other singular values with small powers. Therefore, the filtering matrix will have high power, leading to noise enhancement.

Since the lattice can be described by different generating matrices, i.e., **H**, lattice reduction techniques aim to find a "nicer" generating matrix that leads to better detection performance. The idea behind lattice reduction techniques is to otain

$$\mathbf{H}_{\text{red}} = \mathbf{HP}, \tag{16}$$

where **P** is a unimodular matrix with integer entries that has determinant 1. The reduced matrix $\mathbf{H}_{\text{red}}$ is much better conditioned than the channel matrix **H**.

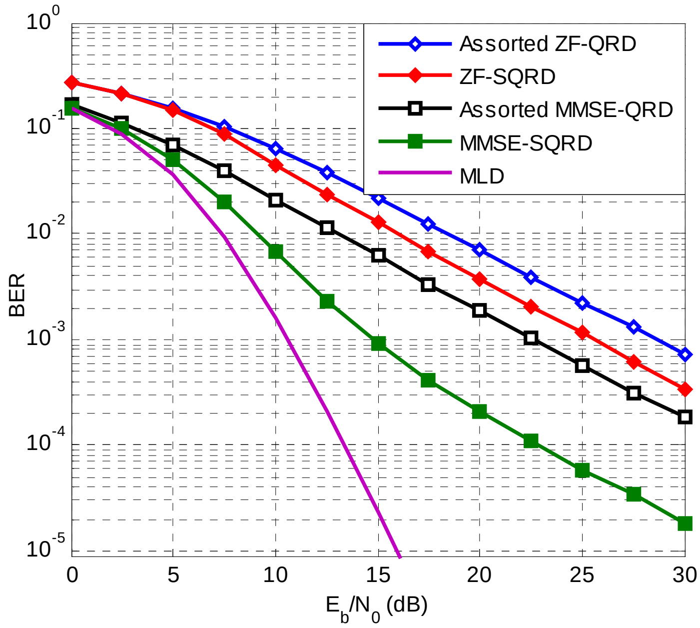

Fig. 6. BER performance of the DFD QRD-based detection schemes in 4×4 system using 4-QAM.

The Lenstra–Lenstra–Lovász (LLL) lattice basis reduction algorithm [10] and Seysen's algorithm (SA) [11] were extensively used in communication systems to improve the detection performance. It is shown that the optimum diversity can be obtained when the LR technique is followed by a sub-optimum low-complexity detection scheme such as LZF or LMMSE ([28][29] and references therein). In [30], better performance was achieved by combining the LR technique with successive interference cancelattion detection schemes.

Given the lattice $\Delta(\mathbf{H})$ with basis vectors $\left(\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{n_T}\right)$, the LLL algorithm finds a set of vectors $\left(\mathbf{h}_1^*, \mathbf{h}_2^*, \cdots, \mathbf{h}_{n_T}^*\right)$, with shorter lengths and better mutual orthogonality properties, using the following approach:

$$\mathbf{h}_i^* = \mathbf{h}_i - \sum_{j=1}^{i-1} \mu_{i,j}\, \mathbf{h}_j^*, \quad (17)$$

$$\mu_{i,j} = \frac{\left\langle \mathbf{h}_i, \mathbf{h}_j^* \right\rangle}{\left\langle \mathbf{h}_j^*, \mathbf{h}_j^* \right\rangle}, \quad (18)$$

where $\langle \, , \rangle$ is the inner product. A basis $\mathbf{H}$ is said to be LLL reduced if

1. $\left|\mu_{i,j}\right| \le \frac{1}{2}$ for $1 \le i, j \le n_T$, and
2. $p\left\|\mathbf{h}_i^*\right\|^2 \le \left\|\mathbf{h}_{i+1}^* + \mu_{i+1,i}\, \mathbf{h}_i^*\right\|^2$,

where $0.25 < p < 1$. Large values of p results in better basis reduction but also higher complexity. A tradeoff between reduction accuracy and complexity can be achieved by setting a suitable value for p. The orthogonality defect is then defined as:

$$\delta(\mathbf{H}^*) = \frac{\prod_{i=1}^{n_T} \left\| \mathbf{h}_i^* \right\|^2}{\det(\mathbf{H}^H \mathbf{H})}, \tag{19}$$

where $\delta(\mathbf{H}^*) \geq 1$, with equality if $\mathbf{H}^*$ is orthogonal.

The complexity of the LLL algorithm was investigated in several studies (see [12], [13] and references therein). The number of iterations of the LLL algorithm is upper-bounded by $n^2 \log(\kappa(\mathbf{H})) + n$ where $n = 2 \times n_T$ with log base $1/p$ [13]. This implies that for an ill-conditioned channel matrix, the computational complexity of the LLL algorithm becomes high. Therefore, in the worst-case, the number of iterations of the LLL algorithm is large, and it is affected by the conditionality of the channel matrix in addition to the size of the lattice.

SA is introduced to simultaneously reduce the lattice $\Delta(\mathbf{H})$ and its dual lattice $\Delta(\mathbf{H}^\#)$, where $\mathbf{H}^{*\#} = \mathbf{G}^H$, and $\mathbf{G} = \left(\mathbf{H}^{*H}\mathbf{H}^*\right)^{-1} \mathbf{H}^{*H}$. *Seysen's orthogonality measure* is defined as follows [11]:

$$S(\mathbf{H}^*) = \sum_{i=1}^{nT} \left\| \mathbf{h}_i^* \right\|^2 \left\| \mathbf{h}_i^{*\#} \right\|^2, \tag{20}$$

where $\mathbf{h}_i^{*\#}$ is the *i*-th basis vector of the dual lattice $\Delta(\mathbf{H}^\#)$. The minimum value of $S(\mathbf{H}^*)$ is n_T which is achieved if the basis $\mathbf{H}^*$ is orthogonal. In this case, $\mathbf{H}^{*\#}$ is also orthogonal, which is the main goal of SA. Thus, the SA algorithm finds a local minumum of $S(\mathbf{H}^*)$ in an iterative manner. At each iteration, a pair of basis vectors are selected and one of them is updated so that the *Seysen's orthogonality measure* is reduced. The SA algorithm terminates when no further reduction in $S(\mathbf{H}^*)$ can be achieved. It is important to mention herein that the SA algorithm obtains a more orthogonal basis while requiring fewer iterations to reduce the lattice basis.

In [31], a list of candidates (LoC) lattice reduction-aided detection scheme using the LLL algorithm was introduced, where a quasi-maximum-likelihood (quasi-ML) performance is achieved. This is done by obtaining a reduced basis for each symbol hypothesis. In [32], this technique was extended to SA to achieve the Quasi-ML performance. Although the list of candidates scheme improves the BER performance, its complexity is roughly ($|\Omega| \times$nT) times that of the plain lattice reduction techniques. For a high number of transmit antennas and high order modulation schemes, this complexity becomes infeasable in latency and computational complexity limited communication systems.

After obtaining the reduced basis of the lattice, the detection is carried over the obtained generating matrix and then we compensate for unimodular matrix $\mathbf{P}$. If we apply LZF equalization of $\mathbf{H}_{\text{red}}$, we obtain

$$\begin{aligned} \tilde{\mathbf{x}} &= \mathbf{H}_{\text{red}}^{-1} \mathbf{r}, \\ &= \mathbf{P}^{-1}\mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}_{\text{red}}^{-1} \mathbf{n}, \\ &= \mathbf{P}^{-1}\mathbf{x} + \mathbf{v}, \end{aligned} \tag{21}$$

where $\mathbf{v} = \mathbf{H}_{\text{red}}^{-1} \mathbf{n}$ and $\mathbf{P}^{-1}$ is also a unimodular matrix. Since $\mathbf{H}_{\text{red}}$ is much more orthogonal than $\mathbf{H}$, only a small noise amplification is present as compared to the case of the conventional LZF. Since matrix $\mathbf{P}$ only includes integer elements, the vector $\tilde{\mathbf{x}}$ is quantized and then the effect of $\mathbf{P}^{-1}$ is compensated for.

Fig. 7 shows the cumulative distribution function of the natural lograithm of the condition number of the lattice basis before and after lattice reduction in 4×4 and 8×8 MIMO-SM systems. We notice that the lattice basis obtained by the SA is more orthogonal than that obtained by the LLL algoritthm. Moreover, we remark that as n_T increases, the condition number of the unreduced lattice basis increases. This is because the smallest singular value of matrix **H** vanishes as the matrix dimension increases [21][22]. Also, the supeiority of the SA over the LLL one becomes more evident as the lattice dimension increases.

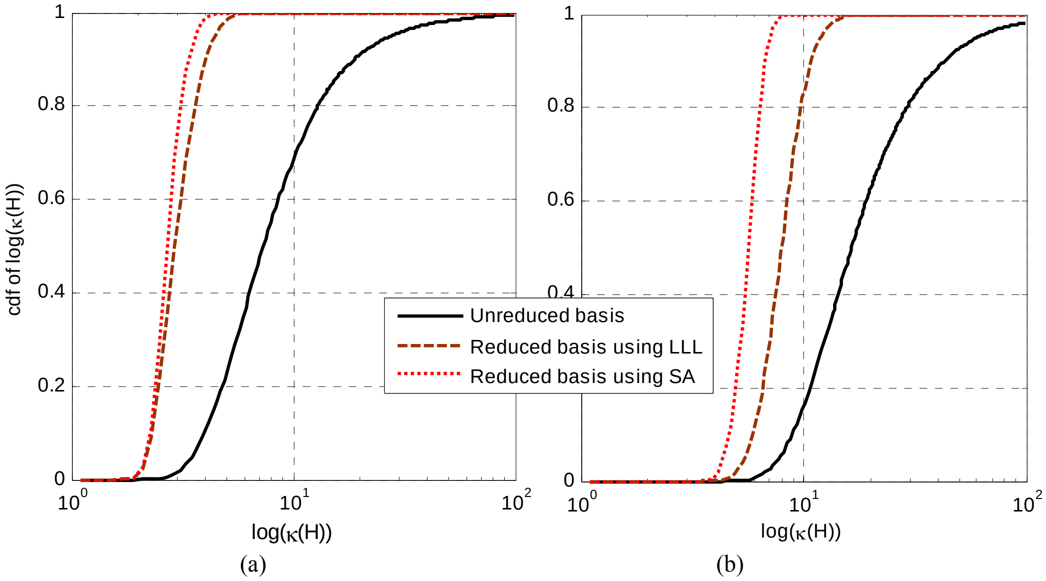

Fig. 7. Cumulative distribution function of the natural logarithm of the channel matrix for LLL and Seysen algorithms in (a) 4×4 and (b) 8×8 MIMO-SM system.

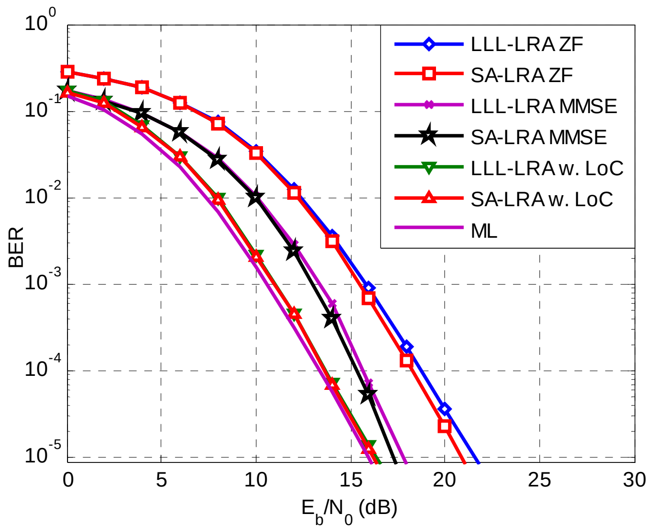

Fig. 8. BER performance of the lattice reduction aided (LRA) linear detection in 4×4 system using 4-QAM. In LLL algorithm, p is set to 0.75.

Fig. 8 shows the BER performance of the LRA linear detection schemes. Because SA obtaines a more orthogonal lattice basis, it leads to better performance compared to the LLL algorithm. Using the LMMSE detector leads to better performance for both lattice reduction techniques. Also, the diversity order of the optimum detector is achieved by the LRAD schemes.

The BER performance of the LRAD with list of candidates (LoC) algorithm [31] achieves a quasi-ML performance at the expense of additional computational complexity. The computational complexity of the lattice basis reduction is random and can be high when the channel matrix is ill-conditionned. In such a case, a large number of iterations are required to orthogonalize the lattice basis. In the following, we introduce several detection schemes that achieve quasi-ML performance with low respective average complexity.

6. Tree-search Detection

Several tree-search detection algorithms have been proposed in the literature that achieve quasi-ML performance while requiring lower computational complexity. In these techniques, the lattice search problem of (3) is presented as a tree where nodes represent the symbols' candidates. In the following, we introduce three tree-search algorithms and discuss their advantages and drawbacks.

6.1 Sphere Decoder (SD)

SD was proposed in the literature to solve several lattice search problems [14]. Based on Hassibi's and Vikalo analysis, SD achieves quasi-ML performance with polynomial average computational complexity for a large range of signal-to-noise ratios [33]. Hence, instead of testing all the hypotheses of the transmitted vector, SD restricts the search in (3) to the lattice points that reside in the hypersphere of radius d and are centered at the received vector $\mathbf{r}$. Therefore,

$$\hat{\mathbf{x}}_{SD} = \arg\min_{\mathbf{x}\in\Omega^{n_T}} \left(\|\mathbf{y} - \mathbf{R}\mathbf{x}\|^2 \le d^2 \right). \tag{22}$$

The accumulative metric in (22) is then calculated successively, where the metric at the n_T-th detection level is given by:

$$E_{n_T} = \left(y_{n_T} - R_{n_T, n_T} \hat{x}_{n_T} \right)^2, \tag{23}$$

and the accumulative metric at the (n_T - 1)-th detection level is given as follows:

$$E_{n_T - 1} = E_{n_T} + \left(y_{n_T - 1} - R_{n_T - 1, n_T - 1} x_{n_T - 1} - R_{n_T - 1, n_T} x_{n_T} \right)^2, \tag{24}$$

and so forth.

The order in which hypotheses are tested at each detection level is defined by the employed search strategy. For details about a comparison between these strategies, refer to [14].

It is shown; however, that SD has variable complexity which depends on the channel condition and the instantaneous noise power, where the worst-case complexity of SD is consequently comparable with that of MLD. That is, the worst-case complexity of SD is exponential. In fact, Jalden and Otterson have shown in [34] that even the average complexity of SD is exponential for a fixed SNR value. Also, in terms of implementation complexity, SD is inefficient due to its sequential nature in the tree search stage that limits the possibility of pipelining, where consequently the detection latency is increased [35].

Fig. 9(a) shows an example of SD for $n_T = n_R = 3$. Note that we use the SE enumeration method in the search phase, which leads to reduced complexity [14]. At first d^2 is initiated to infinity. Then, it is updated to the accumulative metric of the first-found lattice point. In this manner, after each iteration of the SD, the search square radius can be tightened to exclude more unnecessary nodes. The SD is terminated when no more lattice points can be obtained with an accumulative metric smaller than that of the already-found one. The thick line in **Fig. 9(a)** represents the solution obtained by the SD.

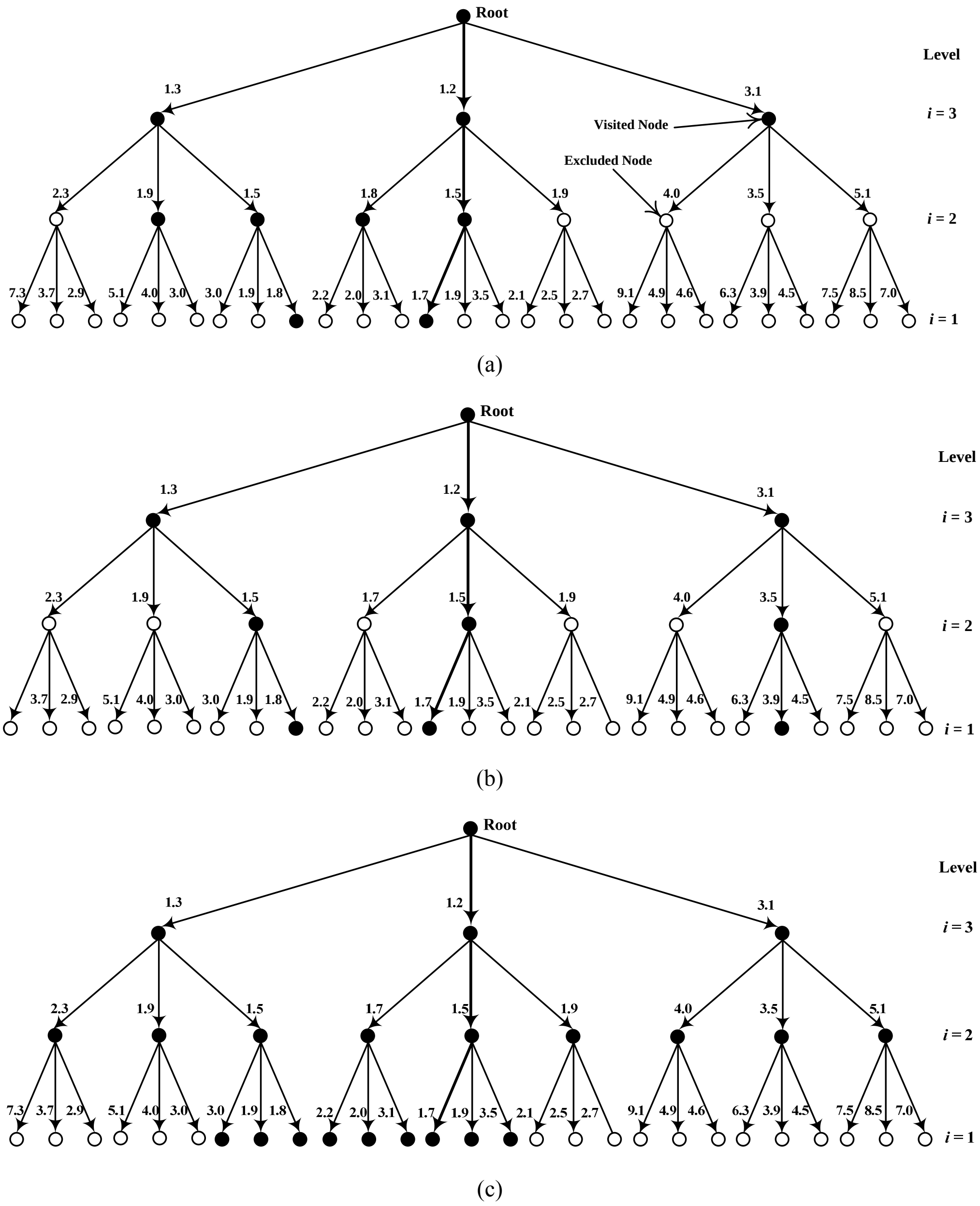

Fig. 9. Tree-search detection algorithms. (a) Sphere decoder, (b) Fixed-complexity sphere decoder (FSD) for $p = 1$, and (c) QRD-M algorithm, for $n_T = n_R = M = 3$.

Figs. 10, **11** Show the BER performance of the SD which coincides with the optimum performance.

Although SD achieves a quasi-ML performance, it has the following drawbacks:

1. The complexity of SD is random and depends on the conditionality of the channel matrix and the noise variance. The worst-case complexity of SD is therefore exponential, which is infeasible in computational power limited communciation systems [36].
2. The SD has a sequential nature because it requires the update of the search radius every time a new lattice point with a smaller accumulative metric is found. This limits the possibility of parallel processing and hence reduces the detection throughput, i.e., increases the detection latency.

Barbero *et al.* have proposed a fixed complexity sphere decoder to overcome the aforementioned drawbacks of the SD.

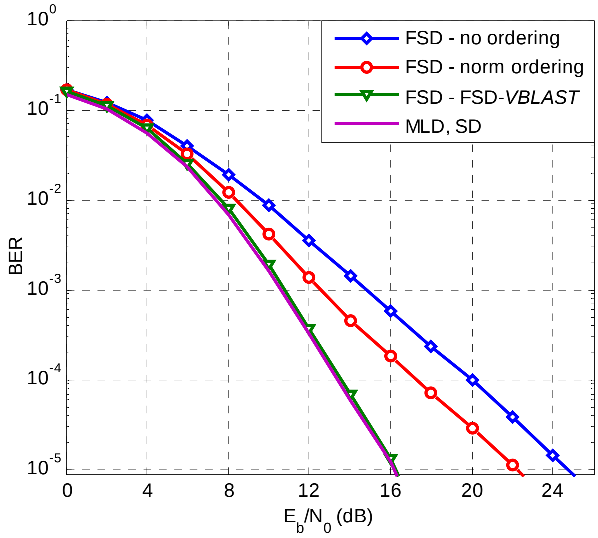

Fig. 10. BER performance of the SD and FSD in 4 × 4 system with $p = 1$ for several ordering schemes.

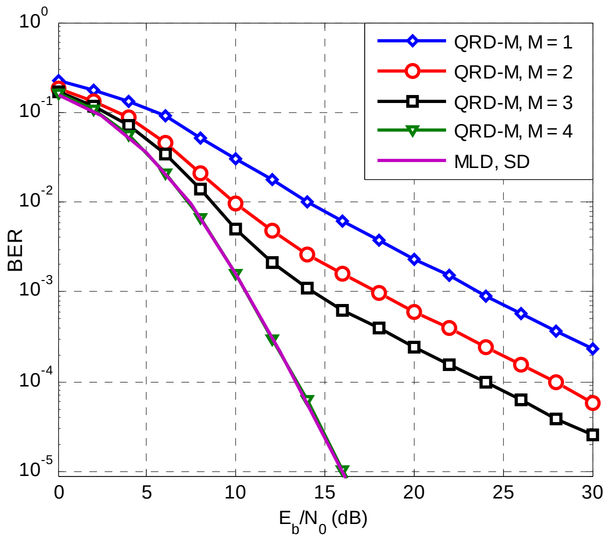

Fig. 11. BER performance of SD and QRD-M algorithms in 4 × 4 MIMO-SM system with $M = 4$.

At the second detection level, the retained M candidates at the previous level are extended to all possible candidates. The resulting ($M\Omega$) branches are sorted based on their accumulative metrics, where the M branches with the smallest accumulative metrics are retained for the next detection level. This strategy is repeated down to the last detection level, i.e., $i = 1$. **Fig. 9(c)** depicts an example of the QRD-M algorithm for $n_T = n_R = M = 3$. The thick line represents the obtained estimate by the QRD-M algorithm.

6.2 Fixed-complexity Sphere Decoder (FSD)

The Fixed-complexity sphere decoder (FSD) was proposed by Barbero *et al.* to overcome the aforementioned drawbacks of SD. FSD achieves a quasi-ML performance by performing the following two-stage tree search [15][37]:

- **Full expansion:** In the first p levels, a full expansion is performed, where all symbols replicas candidates are retained for the following levels.
- **Single expansion:** A single expansion of each retained branch is done in the remaining ($n_T - p$) levels, where only the symbol replica candidate with the lowest accumulative metric is considered for the next levels.

Because all possible symbols candidates are retained in the first p levels, the reliability of signals detected in these levels does not affect the final detection performance. Therefore, signals with the least robustness are detected in the full expansion stage. On the other hand, in the remaining ($n_T - p$) levels, signals are sorted based on their reliability, where signals with the least noise amplification are detected first.

In the conventional FSD, the V-BLAST algorithm is employed to obtain the required signal ordering by the FSD.

Fig. 9(b) depicts an example of the FSD for $n_T = n_R = 3$ and $p = 1$. This means the full set of candidates are retained at the first detection level in the so called the *full expansion stage*. In the two remaining detection levels, each retained branch is expanded independently, where the resulting branch with the smallest accumulative metric is the only retained one. The obtained solution is indicated by the thick line. **Fig. 10** depicts the BER performance of the FSD for $p = 1$ in the 4×4 MIMO-SM system using 4-QAM. Results show that the ordering has a crucial effect on the the performance of the FSD. For instance, both the performance and the attained diversity order are degraded when the ordering stage is skipped or when a non-optimal signal ordering is used. A low complexity FSD ordering scheme that requires a fraction of the computations of the V-BLAST scheme was proposed in [38]. A close to optimum performance was achieved in [38] by embedding the signal sorting stage in the QR factorization of the channel matrix.

6.3 QRD-M Detection

In the QRD-M detection algorithm, only a fixed number of symbol candidates, M, is retained at each detection level [16]. At the first detection level, the root node is extended to all the possible Ω candidates of x_{nT}, the accumulative metrics of the resulting branches are calculated and the best M candidates with the smallest metrics are retained for the next detection level.

Fig. 11 shows the BER of the QRD-M algorithm in 4 x 4 MIMO-SM system for several values of M. The QRD-M algorithm achieves the ML performance for $M = |\Omega|$ which equals 4 in the case of 4-QAM.

There are two drawbacks of the conventional QRD-M algorithm: (i) it employs a systematic tree-search without considering the noise power or the channel conditionality and (*ii*) for high n_T and high order modulations, detection throughput is reduced due to the increase in the size of the search tree. Several algorithms were proposed in the literature to overcome these drawbacks by introducing a method that adaptively selects the number of retained candidates per detection level (see [17] and references therein).

In [18], two independent detection algorithms were proposed to overcome the aforementioned drawbacks of the conventional QRD-M algorithms. These detection algorithms are based on set grouping and are called adaptive parallel QRD-M (APQRDM) and adapative iterative QRD-M (AIQRDM). As a result of employing set grouping of the modulation set, the tree-search is divided into smaller independent search problems that are refered to as *partial detection phases* (PDPs). The APQRDM algorithm increases the detection throughput by employing a parallel tree-search for the PDPs, whereas the AIQRDM algorithm reduces the hardware requirements of the conventional QRD-M by employing an iterative tree-search of the PDPs.

7. Summary

Table 3 shows a comparison between the detection schemes introduced throughout this technical survey. Note that the the diversity order achieved by the DFD algorithms without sorting is equivalent to that of the linear detection schemes. Employing ordering leads to an improvement in the diversity order [8]. Therefore, the attained diversity order is dependent on the accuracy of the employed signal ordering scheme.

Table 3. Comparison between several detection schemes.

Category	Scheme	Diversity Order	Remarks
LD	ZF	$(n_R - n_T + 1)$	Low complexity, Degraded diversity order, Independent detection
	MMSE	$(n_R - n_T + 1)$	
DFD	Assorted V-BLAST	$(n_R - n_T + 1)$	Improved performance and diversity order, High complexity
	V-BLAST	$(n_R - n_T + 1) \sim n_R$	
	Assorted QRD	$(n_R - n_T + 1)$	Improved performance and diversity order, Moderate complexity
	SQRD	$(n_R - n_T + 1) \sim n_R$	
LRAD	LLL and SA (with LD or DFD techniques)	n_R	Quasi-ML performance and diversity order, High complexity (worst-case)
TSD	SD	n_R	Quasi-ML performance and diversity order, Low average complexity, High complexity(worst-case), sequential tree-search
	FSD		Quasi-ML performance and diversity order, Parallel tree-search, Sensitive to signal ordering
	QRD-M		Quasi-ML performance and diversity order, High complexity and latency for high n_T and Ω
	Improved QRD-M (AQRDM [17], AIQRDM, APQRDM [18] etc.)		Quasi-ML performance and diversity order, Reduced complexity and latency

8. Conclusions

In this paper, we presented the state-of-the-art detection techniques for MIMO multiplexing systems. Detection techniques were categorized into LD, DFD, and TSD. Also, we introduced LRAD and its improved LoC techniques. LD algorithms are favorable in terms of computational complexity, but they lead to degradation in both the performance and diversity order. DFD techniques detect signals in an iterative manner such that already-detected signals are subtracted out. DFD techniques outperform the LD ones, but they are still far from achieving the optimal performance. The performance of both the LD and DFD schemes can be improved by obtaining a more orthogonal channel matrix via LR techniques. Nevertheless, LR techniques have random complexity that is inapplicable in the worst-case. For computational complexity and latency limited communication systems, we conclude that the TSD techniques are the most favorable. All TSD techniques achieve the quasi-ML performance with different search strategies and therefore they have different computational complexities. Among these techniques, the QRD-M algorithm was extensively studied in the literature and introduced as a potential candidate for signal detection in future communication systems. Also, several works have been conducted to improve the efficiency of the QRD-M algorithm by reducing its complexity, either processing it in an iterative manner to reduce the hardware requirements, or in parallel to reduce the detection latency.

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