BRGC *M*-PSK 신호의 비트 정보 발생

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Bit Metric Generation of BRGC *M*-PSK Signals

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요 약

본 논문에서는 좌표 회전 방법을 이용하여 binary reflected Gray coded (BRGC) M-PSK 신호의 비트별 매트릭 계산 함수를 제안하였다. 이 함수는 BRGC 매핑 함수의 특징을 이용하여 좌표축 회전을 최소화 하는 방법을 찾아 적용하였다. 제안된 비트별 매트릭 발생 함수는 채널 상태 정보를 필요로 하지 않으면 구현을 할 때 낮은 복잡도를 갖는 장점이 있다.

Key Words : BRGC, High Order Modulation, PSK, Coordinate Rotation, Bitwise Metric

ABSTRACT

In this paper, we present a bitwise metric generating function for a binary reflected Gray coded (BRGC) M-PSK signal by means of coordinate rotation. Using the properties of the BRGC mapping, we minimize the number of coordinate rotations. The proposed function does not rely on channel state information (CSI), and provides low implementing complexity.

Ⅰ. Introduction

High-order modulated signals should combat with high signal to noise ratio (SNR) requirement, signal-to-noise fluctuations over dispersive fading channels, and other physical environmental limitations for more wireless channel capacity. Iterative decoding is actively considered to overcome these limitations. Even high order modulation with iterative decoding simultaneously provides both large coding gain and high bandwidth efficiency, it is essential to calculate the symbol-to-bit metric information of the high order modulated signals for the utilization of well designed binary iterative decoders. In order to reduce the complexity of the soft demapper/ demodulator for bit metric generation, several methods were proposed for BRGC PSK signals [1]-[2] such as a pragmatic approach with signal space concepts, a log likelihood ratio (LLR) approach, and other approaches [3]-[7]. In [7], using demodulated signal a new LLR expression was presented with the virtual constellation (VC). Wang of [3] presented the iterative decoding performances for *M*-PSK signals between several types of soft metric functions including Log-MAP, Max-Log-Map, and coordinate rotation (CR), and showed that the CR has better performance than other functions and the CR does not require CSI with less implementing complexity. This work provided how to rotate the signal space and how to get the soft metric value for 8- and 16-PSK

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signals with coordinate rotation algorithm which rotates signal space depending on the different received symbol position.

In this paper, we present a general bitwise metric generating functions for BRGC *M*-PSK signals with coordinate rotation of Wang's algorithm in [3]. In order to obtain the final mathematical function, we use the BRGC properties for bit value placement on a BRGC *M*-PSK constellation.

Ⅱ. System Model

In this discussion, we assume that the modulated BRGC *M*-PSK signal is to be transmitted over an AWGN channel. In an *M*-PSK, $m(=\log_2 M)$ bits of serial stream are mapped at a certain symbol position on a 2-dimensional signal space with in-phase (*I*) and quadrature (*Q*) axes using BRGC, where *M* is the number of symbols in the PSK constellation.

The transmitted *M*-PSK symbol $s = s_I + js_Q$ belongs to a set of the *M*-ary alphabet $\{S_{-M/2}, \ldots, S_{-1}, S_1, \ldots, S_{M/2}\}$, $s = f(b_0, b_1, \cdots, b_k)$, where $k \in \{0, \ldots, m-1\}$, and $f(\cdot)$ is a BRGC mapping function with *m*-tuples. Figure 1 shows the examples of the perfectly BRGC mapped signal constellation for 8- and 16-PSK signals with their reference angles θ_8 and θ_{16} for symbol region partitioning, respectively, where the shaded areas represent the symbol regions for the corresponding symbol.

With the BRGC mapping function with bit set

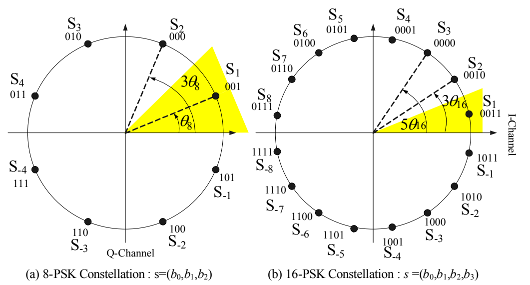

Fig. 1. Signal constellation for BRGC 8- and 16-PSK signals

$(b_0, b_1, \cdots, b_{m-1})$ for a symbol, the constellation has some peculiar properties of symmetric for bit placement except b_0 and b_1 as follows.

a) *k*-th bit ($k>1$) for a symbol has $M/2^k$ consecutive symbols with the same bit value 1/0, and the signal space is partitions into 2^k regions by the $2n\pi/2^k$, where n is an integer.

b) A rotational property: When we rotate the signal space of each bit by amount of specific angle, its result shows exactly the same with the original signal space.

c) A symmetric property: All quadrants in a PSK signal space has line-symmetric relationship between each other. For examples, the first quadrant is the quadrature axis line-symmetric with the second quadrant; the second is the in-phase axis line-symmetric with the third, and so on.

Figure 2 shows bit decision regions of each bit for bit set of a 16-PSK symbol. With the Fig. 2, we can confirm the above described three properties.

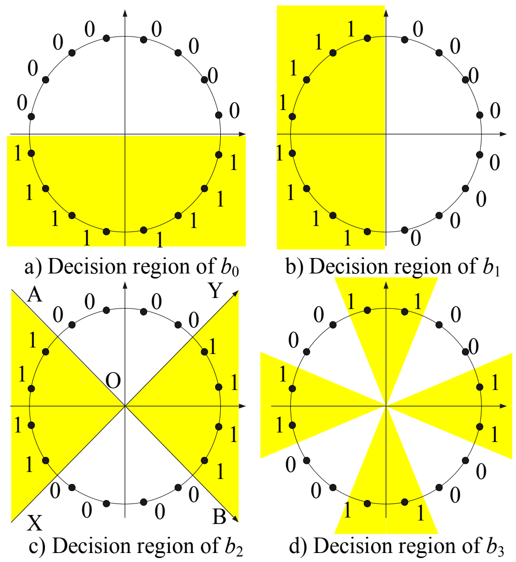

Fig. 2. Bit decision regions for each bit of 16-PSK symbols with bit set $(b_0, b_1, \cdots, b_3)$.

Ⅲ. Bitwise metric function

In order to use a well-designed binary iterative decoder for Turbo codes or LDPCs, we need to decompose the received symbol to bitwise metrics obtained soft bit decision values through a demapper for the input of the iterative decoder.

The received complex signal $r = r_I + jr_Q$ is assumed that the frequency and the phase are perfectly synchronized at the receiver.

For BPSK signal, because the bit decision region is the same as Fig. 2 (a) or (b), the received symbol value can be the bitwise metric as it is. For QPSK signal, as the bit decision regions are Fig. 2 (a) and (b), the metric $\Lambda(b_0)$ for b_0 is the value of the quadrature component of the received signal and the metric $\Lambda(b_1)$ for b_1 is the value of the in-phase component, or vice versa. Note that, for b_1, the metric can be also obtained with taking the new quadrature component after coordinate rotation by $\pi/2$ which value is the same with the original in-phase component.

Higher modulated signals have the same bit decision regions for b_0 and b_1 as Fig.2 (a) and (b) resulting in the same bit metrics as BPSK and QPSK.

$$\begin{aligned} \Lambda(b_0) &= -r_Q \\ \Lambda(b_1) &= -r_I \end{aligned}. \tag{1}$$

However the remained bit metrics $\Lambda(b_k)$ for $k>1$ should be found out with considering more complicated bit decision regions as given example in Fig. 2(c) and (d). For example, for b_2 in Fig. 2(c), depending on the received symbol position, we can obtain the bitwise metrics with taking new in-phase or quadrature component after rotating I or Q axis to $\overline{XY}$ or $\overline{AB}$. In the same way, the bit metric for b_3 in Fig.2(d) can be calculated.

The coordinate rotation is performed with

$$\begin{bmatrix} r_I' \\ r_Q' \end{bmatrix} = \begin{bmatrix} \cos(\xi) & \sin(\xi) \\ -\sin(\xi) & \cos(\xi) \end{bmatrix} \begin{bmatrix} r_I \\ r_Q \end{bmatrix}, \tag{2}$$

where ξ is the angle of rotation.

When we calculate the soft metrics for BRGC M-PSK signals using Wang's algorithm in [3], first of all, we have to find the amount of the angle and decide the reference axis for coordinate rotation for all symbols. And then we rotate the signal space and obtain the metrics.

However, when we consider the characteristics of BRGC, we don't need to consider all signal space to obtain the soft bitwise metrics. For example, in the case of 16-PSK, we can find that the soft metric of b_2 over the first quadrant is the same with that of the third quadrant when we rotate the third quadrant through π and the second quadrant is also with the forth quadrant. Thus, taking absolute value of input in-phase and quadrature component, we can calculate the all bitwise metrics in the first quadrant space. As a result, we can constitute a new function for the k-th bit ($k>1$) soft metric generation with taking the new quadrature component r_Q' as

$$\Lambda(b_k) = (-1)^{(D_k+1)}\{-|r_I|\sin(\xi_k) + |r_Q|\cos(\xi_k)\} \tag{3}$$

where $k>1$, the rotation angle $\xi_k = \pi(1+2D_k)/2^k$, $\psi = \tan^{-1}(|s_Q|/|s_I|)$ in radian, the symbol location index $D_k = \left\lfloor \frac{\lfloor \psi \times 2^k/\pi \rfloor}{2} \right\rfloor$, and $\lfloor \bullet \rfloor$ is the mathematical floor function. The result (3) is a general bitwise metric generation function for BRGC M-PSK signals. If the value of the elements of bit set for a symbol is all inverted, the result (3) is changed to

$$\Lambda(b_k) = (-1)^{(D_k)}\{-|r_I|\sin(\xi_k) + |r_Q|\cos(\xi_k)\} \tag{4}$$

where $k>1$.

Note that we can obtain the modified version of (3) and (4) using the equation for the in-phase component r_I' instead of using the equation for the quadrature component r_Q' from (2).

For 8-PSK, $D_2 = 0$, and $\xi_2 = \pi/4$. And, from Fig.3 of a 16-PSK partial signal space, a) for b_2, the parameters of (3) are $D_2 = 0$ and $\xi_2 = \pi/4$,

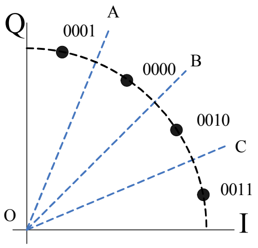

Fig. 3. The first quadrant of BRGC 16-PSK

the decision line of $\overline{OB}$; b) for b_3, the parameters with the region $0 \le \psi < \pi/4$ are $D_3 = 0$ and $\xi_3 = \pi/8$, the decision line of $\overline{OC}$, and the parameters with the region $\pi/4 \le \psi < \pi/2$ are $D_3 = 1$ and $\xi_3 = 3\pi/8$, the decision line of $\overline{OA}$.

Using (3) for the constellation in Fig.1, we can obtain the bitwise metrics for $k > 1$ as

1) Bitwise metric of b_2 for BRGC 8- and 16-PSK is

$$\Lambda(b_2) = -\left[-|r_I|\sin\left(\frac{\pi}{4}\right) + |r_Q|\cos\left(\frac{\pi}{4}\right)\right], \quad (5)$$

2) Bitwise metrics of b_3 for BRGC 16-PSK are

$$\Lambda(b_3) = -\left[-|r_I|\sin\left(\frac{\pi}{8}\right) + |r_Q|\cos\left(\frac{\pi}{8}\right)\right], \quad (6)$$

where $0 \le \psi < \pi/4$, and

$$\Lambda(b_3) = -|r_I|\sin\left(\frac{3\pi}{8}\right) + |r_Q|\cos\left(\frac{3\pi}{8}\right), \quad (7)$$

where $\pi/4 \le \psi < \pi/2$.

Note that with the equivalents of $\sin(3\pi/8) = \cos(\pi/8)$ and $\cos(3\pi/8) = \sin(\pi/8)$, we can change (7) into

$$\Lambda(b_3) = -|r_I|\cos\left(\frac{\pi}{8}\right) + |r_Q|\sin\left(\frac{\pi}{8}\right), \quad (8)$$

which enables (7) to share the sine and cosine table memory of (6).

Figure 4 shows the Turbo-coded performance plots for a BRGC 16-PSK signal over AWGN with virtual constellation (VC) in [7] and coordinate rotation (CR) in this paper. In this example, a BICM (Bit-Interleaved Coded Modulation) Turbo code was used with the generating polynomail [1 1 1; 1 0 1] and a random interleaver for the encoder.

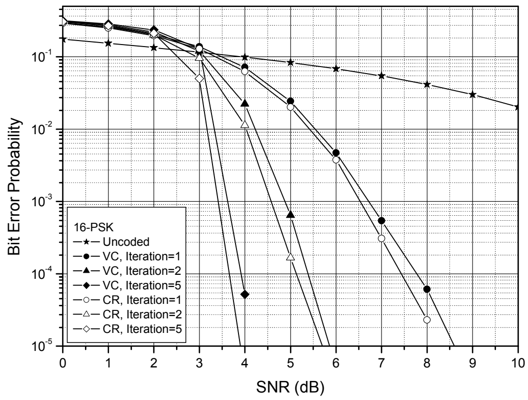

Fig. 4. 16-PSK Turbo decoding performances comparison between VC and CR

From this figure, we can find that CR shows better performance than VC because soft demodulating process to acquire the phase angle pollutes Euclid distance impacting on iterative decoding process negatively.

Ⅳ. Conclusions

We presented a general function for the soft bitwise metric calculation with coordinate rotation. The proposed function is general for BRGC M-PSK signals and minimizes the size of sine or cosine look-up table memory for coordinate rotation comparing to the Wang's algorithm because we consider only the first quadrant of the signal space. Also, since the Euclidean distances of the received signal perpendicular to each reference axis for rotation can represent the reliability information for the corresponding bit, this function does not require the CSI especially in the high SNR case. We also proposed how to easily obtain the parameters in the proposed function with the received symbol value and the shared information with the receiver and the transmitter, such as the modulation order and the mapping information.

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