# 직교 기저행렬을 이용하는 직교 주파수분할다중화의 수학적 구현

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# A Mathematical Implementation of OFDM System with Orthogonal Basis Matrix

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## 요 약

본 논문에서는 직교 기저행렬을 이용한 직교 주파수분할다중화 시스템의 새로운 구현방안이 수학적으로 개발된다. 직교 기저행렬은 Haar 기저행렬을 기본으로 하고 있으나 직교 주파수분할다중화의 다중 부채널 신호를 변조하기에 적당한 형태를 갖추고 있다. 여기서는 새로운 기저행렬이 간단한 재귀알고리즘에 의하여 확장될 수 있음이 증명된다. 그리고 송신기 조합행렬의 차수는 확장에 의하여 두 배로 증가된다. 수신기에서 복조는 직교 기저행렬의 재귀에 의하여 생성되는 조합행렬의 역행렬에 의하여 수행된다. 따라서 제안된 직교 주파수분할다중화 시스템에서는 원 신호의 완벽한 재생이 가능함을 알 수 있다.

## ABSTRACT

In this paper, a new implementation method of OFDM (orthogonal frequency division multiplexing) system with an orthogonal basis matrix is developed mathematically. The basis matrix is based on the Haar basis but has an appropriate form for modulation of multiple subchannel signals of OFDM. It is verified that the new basis matrix can be expanded with a simple recursive algorithm. The order of synthesis matrix in the transmitter is increased by the factor of two with every expansion. Demodulation in the receiver is carried out by its inverse matrix, which can be generated recursively with the orthogonal basis matrix. It is shown that perfect reconstruction of original signals is possibly achieved in the proposed OFDM system.

## 키워드

직교 주파수분할다중화, 직교 기저행렬, Haar 기저, 무선통신

## Key word

Orthogonal frequency division multiplexing (OFDM), orthogonal basis matrix, Haar basis, wireless communications

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## Ⅰ. Introduction

Orthogonal frequency division multiplexing (OFDM) has been considered as one of the most promising transmission scheme in wireless communications. It has been adopted for various wireless communication standards such as IEEE 802.11g [1] of high-speed wireless local area networks (WLANs), European digital audio/video broadcasting (DAB/DVB) [2],[3]. ECMA- 368 [4] defines in detail the structure of multi-band OFDM (MB-OFDM) for WiMedia ultra-wideband (UWB) communication systems.

Due to orthogonality between the transform pair and feasibility of a fast algorithm, discrete Fourier transform (DFT) has usually been employed to modulate a block of parallelized signals in the transmitter and separate each subchannel signal out of the received sampled waveforms [5].

The DFT is only a special case of orthogonal basis matrix. In this paper, a new implementation method of OFDM system with orthogonal basis matrix is studied in mathematical manner. The new basis matrix is similar to the Haar matrix but has an appropriate form to modulate multiple subchannel signals. And a generalized algorithm to find inverse matrix of the basis has been presented. The orders of synthesis matrix in the transmitter and analysis matrix in the receiver can be increased with a simple recursive algorithm. It is proved that perfect reconstruction of original data can be achieved in the proposed OFDM system.

This paper is organized as follows. In Section II, a mathematical structure of the conventional OFDM is introduced briefly. An orthogonal basis matrix and its recursive expansion are studied in Section III. In Section IV, a new method of implementing an OFDM system with the orthogonal basis matrix is provided and analyzed. Finally, some concluding remarks and further studies about the proposed system are stated in Section V.

## Ⅱ. The Conventional OFDM System

An orthogonal basis matrix of great interest in digital signal processing (DSP) is DFT matrix [6]. It is an N×N square matrix defined as

$$W=\begin{pmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 \\ 1 & W_N & \cdots & W_N^k & \cdots & W_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & W_N^n & \cdots & W_N^{nk} & \cdots & W_N^{n(N-1)} \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & \cdots & W_N^{k(N-1)} & \cdots & W_N^{(N-1)^2} \end{pmatrix} \tag{1}$$

for $0 \le k,\ n \le N\text{-}1$, where $W_N^{nk}=e^{j2\pi nk/N}$ is the element of the $k$th row and the $n$th column, respectively.

In contemporary implementation of an OFDM system, IDFT is exploited to modulate a set of complex signals in the transmitter. Thus, the transmitted signals of an OFDM symbol, $\boldsymbol{x}=[x_0, x_1, \cdots, x_{N\text{-}1}]^T$ is

$$x=\frac{1}{N}W^{*}X, \tag{2}$$

where $N$ is the number of subchannels and $W^{*}$ denotes the conjugate of $W$. And $\boldsymbol{X}=[X_0, X_1, \cdots, X_{N\text{-}1}]^T$ is parallelized input data. The superscript, $T$, denotes transpose of a matrix.

Demodulation of received OFDM symbols is carried out by the DFT. Hence, the recovered signals are given as

$$\hat{X}=W\cdot y^T, \tag{3}$$

where $\boldsymbol{y}=[y_0, y_1, \cdots, y_{N\text{-}1}]$ represents sampled waveform of the received signals. Thus, the overall transfer function of the conventional OFDM system is

$$T_{\mathrm{DFT}}=\frac{1}{N}W\cdot W^{*}. \tag{4}$$

Since the orthogonality principle is satisfied with DFT pair, that is, $WW^{*} = NI$, where $I$ is the $N\times N$ identity matrix, perfect reconstruction of the original signals can be achieved.

## Ⅲ. A New Basis Matrix

Another well known orthogonal basis in signal processing is Haar matrix. It has been known that the matrix of order $n$ has $2^n$ elements in each row and column. And order of the matrix can be increased recursively [7] as

$$H_n = H_{n-1} \otimes H_1, \tag{5}$$

where

$$H_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{6}$$

The new orthogonal basis matrix based on the Haar basis should have an appropriate form of frequency allocation for multiple subchannels of an OFDM symbol in the transmitter. Let us define a new set of basis matrices as follow;

$$P_1(z) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & z^{-1} \\ 1 & -z^{-1} \end{pmatrix}, \tag{7}$$

$$P_2(z) = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1(z) & z^{-2}P_1(z) \\ P_1(z) & -z^{-2}P_1(z) \end{pmatrix}, \tag{8}$$

$$\vdots$$

$$P_N(z) = \frac{1}{\sqrt{2}} \begin{pmatrix} P_{N-1}(z) & z^{-2(N-1)}P_{N-1}(z) \\ P_{N-1}(z) & -z^{-2(N-1)}P_{N-1}(z) \end{pmatrix}. \tag{9}$$

It is verified that the matrices are expanded with a simple recursive algorithm. And the order of the basis matrices is increased by a factor of two with every expansion. Since $P_n(z)$ consists of $P_{n-1}(z)$ and $z^{-2^{(n-1)}}$, each $P_n(z)$ can be found by $z^{-2^{(N-1)}}P_{n-1}(z)$ and sign inversion. Hence, the number of complex computations can be reduced significantly.

Let $P_{(N,k)}(z)$ for $0 \le k \le K-1$, where $K = 2^N$, be the $k$th row vector of $P_N(z)$. Then, the inner product of $P_{(N,i)}(z)$ and $P_{(N,j)}(z)$ is computed as

$$\langle P_{(N,i)}, P_{(N,j)} \rangle = \sum_{r=0}^{K-1} P_{(N,i,r)} P^{*}_{(N,j,r)} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}, \tag{10}$$

where $P_{(N,j,r)}$ is the $r$th element of $P_{(N,j)}(z)$. Thus,

$$P_{(N,0)}, P_{(N,1)}, \cdots, P_{(N,K-1)} \in C^{(K)} \tag{11}$$

are linearly independent over $C$, where $C^{(K)}$ is the set of all $K$-tuple complex numbers [8]. In addition, the new basis is an orthogonal matrix in $M^N(C)$ which is the set of all $K\times K$ matrices over the set of all complex numbers.

## Ⅳ. OFDM System with the Orthogonal Basis Matrix

As it has been analyzed, the presented basis matrices have an appropriate structure for parallel transmission system. An OFDM system designed with the new basis matrices is illustrated in Fig. 1, where S/P and P/S represent serial-to-parallel and parallel-to-serial converter, respectively. The signal mapper converts each parallelized $L(=\log_2 M)$-tuple bit stream to a complex signal on a constellation of $M$-ary modulation such as quadrature amplitude modulation (QAM).

Transmitted sequence of the OFDM system, $S'(z) = \left[S_0'(z), S_1'(z), \cdots, S_{K-1}'(z)\right]$, is composed of $S_k'(z) = z^{-k}S_k(z)$, where $S_k(z)$ is the $k$th element of

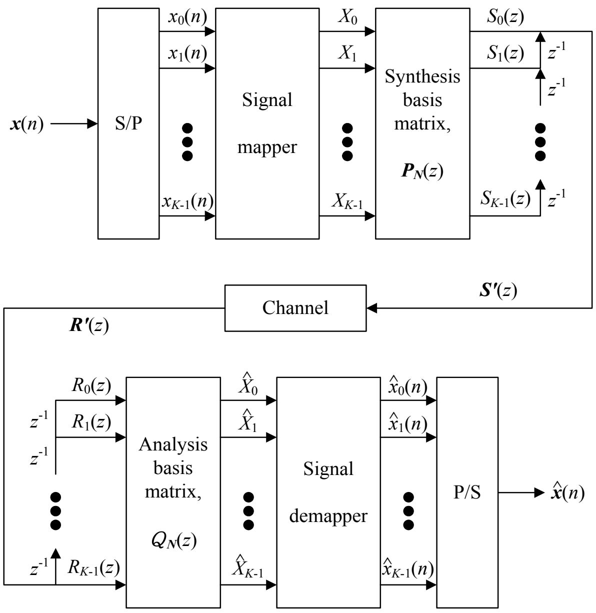


그림 1. 새로운 기저행렬을 이용하는 직교 주파수분할다중화의 블록도.
Fig. 1. A block diagram of OFDM system with the new basis matrix.

$P_N(z)X(z)$ and can be represented as

$$S_k(z) = \frac{1}{\sqrt{2^N}} \sum_{i=0}^{K-1} P_{(N,k,i)} X_i. \tag{12}$$

$X(z) = [X_0(z), X_1(z), \cdots, X_{K-1}(z)]^T$ is parallelized $K$-tuple complex data. The sampled waveform of received signals, $R(z) = [R_0(z), R_1(z), \cdots, R_{K-1}(z)]$, where

$$R_k(z) = z^{-(K-1)+k} R_k^{'},\ 0 \le k \le K\text{-}1, \tag{13}$$

can be demodulated by analysis basis matrix $Q_N(z)$ as follow;

$$T_{\mathrm{OFDM}}(z) = \frac{\hat{X}(z)}{X(z)} = z^{-(K-1)} Q_N(z)\, P_N(z). \tag{14}$$

Let $T_N(z) = Q_N(z)P_N(z)$ in (14), then $T_N(z)$ can be divided into two parts as

$$T_N(z) = \sum_{i=0}^{K-1} \sum_{k=0}^{K-1} Q_{(N,i,k)}(z) P_{(N,i,k)}(z) + \sum_{i=0}^{K-1} \sum_{\substack{j=0,\\ j\neq i}}^{K-1} \sum_{k=0}^{K-1} Q_{(N,i,k)}(z) P_{(N,i,k)}(z). \tag{15}$$

The second term of right-hand side of (15) is interchannel interference, which is effect of the $j$th subchannel signal to the $i$th subchannel.

To achieve perfect reconstruction of the original signals, the transfer function $T_{\mathrm{OFDM}}(z)$ should be represented as

$$T_{OFDM}(z) = T(z) \cdot I, \tag{16}$$

where $I$ is the $K \times K$ identity matrix and $T(z) = [T_0(z), T_1(z), \cdots, T_{K-1}(z)]^T$ for

$$T_i(z) = \sum_{k=0}^{K-1} Q_{(N,i,k)}(z)\, P_{(N,i,k)}(z) \tag{17}$$

$$Q_N(z) = \frac{1}{\sqrt{2}} \left( \begin{array}{c|c} \left( \begin{array}{c|c} \ddots & \left\{ z^{-2^{(N-2)}} P_{N-2}(z) \right\}^{-1} \\ \hline P_{N-2}^{-1}(z) & \left\{ -z^{-2^{(N-2)}} P_{N-2}(z) \right\}^{-1} \end{array} \right) & \left\{ z^{-2^{(N-1)}} P_{N-1}(z) \right\}^{-1} \\ \hline P_{N-1}^{-1}(z) & \left\{ -z^{-2^{(N-1)}} P_{N-1}(z) \right\}^{-1} \end{array} \right). \tag{18}$$

represents coefficients of the transfer function. Hence, the analysis matrix at the receiver may be inverse of the synthesis matrix given as (18).

Let $z = e^{jw}$, then

$$Q_{N-k}(z) = P_{N-k}^{\dagger}(z), \tag{19}$$

where $P_{N-k}^{\dagger}(z) = P_{N-k}^{T}(z^{-1}) = \left\{ z^{2(N-k)} P_{N-k}^{-1}(z) \right\}^{T}$. And $\dagger$ denotes transpose of complex conjugate of a matrix. It is noted that the recursive algorithm can be applied to build the inverse matrix, too. The analysis matrix can be found from the corresponding synthesis matrix with little computational complexity. It is, therefore, considered that the proposed implementation method does not require serious increase in the computational complexity.

## Ⅴ. Conclusions and Further Studies

In this paper, a new implementation method of OFDM system with an orthogonal basis matrix has been studied mathematically. The matrix has a similar form to the Haar basis but can be well adapted for modulation of multiple subchannel signals. With a simple recursive algorithm, the order of synthesis matrix is increased by the factor of two. The analysis matrix of order n can be found by the inverse of the corresponding synthesis matrix. This implies that perfect reconstruction of the original data is possibly achieved.

For practical applications, error performance of the proposed OFDM system in additive white Gaussian noise and multipath fading channels has to be computed. In addition, computational complexity and spectral characteristics of the proposed system should be analyzed in future.

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