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SOME COMMON FIXED POINT THEOREMS USING COMPATIBLE MAPS OF TYPE(α) ON INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT. In this paper, we prove some common fixed point theorems for six maps satisfying compatible maps of type(α) on intuitionistic fuzzy metric spaces in sense of Park et al.[7]. Our research are generalization and extension for the results of [1], [2], [3] and [13].

1. INTRODUCTION

Kaleva and Seikkala [5], Kramosil and Michalek[6] etc Several authors have introduced the concept of fuzzy metric space in different ways. Grabiec [2] obtained the Banach contraction principle in setting of fuzzy metric spaces introduced by Kramosil and Michalek [6]. Park and Kim [8] proved a fixed point theorem in a fuzzy metric space. Jungck et al. [4] introduced the concept of compatible maps of type(α) in metric space and proved common fixed point theorems in metric space. Also, Cho[1] introduced the concept of compatible maps of type(α) in fuzzy metric spaces. Furthermore, Park et al. [7] defined the intuitionistic fuzzy metric space, Park et al.[10] introduced the some properties in intuitionistic fuzzy metric space and obtain the common fixed point theorems in intuitionistic fuzzy metric space.

Recently, Sharma [13] proved common fixed point theorems for six maps satisfying some conditions in fuzzy metric spaces.

In this paper, we prove common fixed point theorems for six maps satisfying some conditions on intuitionistic fuzzy metric space in sense of Park et al. [7]. Our research are generalization and extension for the results of [1], [2], [3] and [13].

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2. Preliminaries

Now, we give some definitions, properties on intuitionistic fuzzy metric space as following:

Definition 2.1 ([14]). A operation $*: [0,1] \times [0,1] \to [0,1]$ is *continuous t-norm* if $*$ is satisfying the following conditions:

(a) $*$ is commutative and associative,

(b) $*$ is continuous,

(c) $a * 1 = a$ for all $a \in [0,1]$,

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.2 ([14]). A operation $\diamond: [0,1] \times [0,1] \to [0,1]$ is *continuous t-conorm* if $\diamond$ is satisfying the following conditions:

(a) $\diamond$ is commutative and associative,

(b) $\diamond$ is continuous,

(c) $a \diamond 0 = a$ for all $a \in [0,1]$,

(d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.3 ([7]). The 5-tuple $(X, M, N, *, \diamond)$ is said to be an *intuitionistic fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a) $M(x, y, t) > 0$,

(b) $M(x, y, t) = 1 \Longleftrightarrow x = y$,

(c) $M(x, y, t) = M(y, x, t)$,

(d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(e) $M(x, y, \cdot): (0, \infty) \to (0, 1]$ is continuous,

(f) $N(x, y, t) > 0$,

(g) $N(x, y, t) = 0 \Longleftrightarrow x = y$,

(h) $N(x, y, t) = N(y, x, t)$,

(i) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,

(j) $N(x, y, \cdot): (0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Lemma 2.4 ([8]). *In an intuitionistic fuzzy metric space X, $M(x,y,\cdot)$ is nondecreasing and $N(x,y,\cdot)$ is nonincreasing for all $x,y \in X$.*

Definition 2.5 ([11]). Let X be an intuitionistic fuzzy metric space.

(a) $\{x_n\}$ is said to be *convergent to a point* $x \in X$ by $\lim_{n\to\infty} x_n = x$ if $\lim_{n\to\infty} M(x_n,x,t) = 1$, $\lim_{n\to\infty} N(x_n,x,t) = 0$ for all $t > 0$.

(b) $\{x_n\}$ is called a *Cauchy sequence* if

$$\lim_{n\to\infty} M(x_{n+p},x_n,t) = 1, \quad \lim_{n\to\infty} N(x_{n+p},x_n,t) = 0$$

for all $t > 0$ and $p > 0$.

(c) X is *complete* if every Cauchy sequence converges in X.

In this paper, X is considered to be the intuitionistic fuzzy metric space with the following condition:

$$\lim_{t\to\infty} M(x,y,t) = 1, \quad \lim_{t\to\infty} N(x,y,t) = 0 \tag{2.1}$$

for all $x,y \in X$ and $t > 0$.

Remark 2.6. Since $*, \diamond$ are continuous, it follows from (d), (i) that the limit of the sequence in intuitionistic fuzzy metric space is uniquely determined.

Lemma 2.7 ([9]). *Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space X with the condition (2.1). If there exist a number $k \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,*

(2.2) $M(x_{n+2},x_{n+1},kt) \geq M(x_{n+1},x_n,t), \quad N(x_{n+2},x_{n+1},kt) \leq N(x_{n+1},x_n,t)$

for all $t > 0$ and $n = 1,2,\cdots$, then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.8 ([8]). *Let X be an intuitionistic fuzzy metric space. If there exists a number $k \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,*

$$M(x,y,kt) \geq M(x,y,t), \ N(x,y,kt) \leq N(x,y,t),$$

then $x = y$.

3. Compatible Maps of Type(α)

In this part, we give the concepts of compatible maps of type(α) on intuitionistic fuzzy metric space and some properties of these maps.

Definition 3.1 ([10]). Let A, B be mappings from intuitionistic fuzzy metric space X into itself. The mappings are said to be *compatible* if

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \quad \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 3.2 ([10]). Let A, B be mappings from intuitionistic fuzzy metric space X into itself. The mappings are said to be compatible of type(α) if

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1,$$
$$\lim_{n\to\infty} N(ABx_n, BBx_n, t) = 0 \text{ and } \lim_{n\to\infty} N(BAx_n, AAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Proposition 3.3 ([10]). *Let X be an intuitionistic fuzzy metric space and A, B be continuous mappings from X into itself. Then A and B are compatible iff they are compatible of type(α).*

Proposition 3.4 ([10]). *Let X be an intuitionistic fuzzy metric space and A, B be mappings from X into itself. If A, B are compatible of type(α) and $Az = Bz$ for some $z \in X$, then $ABz = BBz = BAz = AAz$.*

Proposition 3.5 ([10]). *Let X be an intuitionistic fuzzy metric space and A, B be mappings from X into itself. If A, B are compatible of type(α) and $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$, then*

(a) *$\lim_{n\to\infty} BAx_n = Ax$ if A is continuous at $x \in X$,*

(b) *$ABx = BAx$ and $Ax = Bx$ if A and B are continuous at $x \in X$.*

4. Some Common Fixed Point Using Compatible Maps of Type(α)

In this part, we prove some common fixed point theorems for six maps satisfying some conditions.

Theorem 4.1. *Let X be a complete intuitionistic fuzzy metric space with $t * t \geq t$, $t \diamond t \leq t$ for all $t \in [0, 1]$ and satisfy the condition (2.1). Let A, B, S, T, P and Q be maps from X into itself such that*

(a) *$P(X) \subset AB(X)$, $Q(X) \subset ST(X)$,*

(b) *$AB = BA$, $ST = TS$, $PB = BP$, $QS = SQ$ and $QT = TQ$,*

(c) *A, B, S and T are continuous,*

(d) *(P, AB) and (Q, ST) are compatible of type(α),*

(e) *There exist* $k \in (0,1)$ *such that for all* $x, y \in X$, $\beta \in (0,2)$ *and* $t > 0$,

$$\begin{aligned}
&M(Px, Qy, kt)\\
&\geq M(ABx, Px, t) * M(STy, Qy, t) * M(STy, Px, \beta t)\\
&\quad * M(ABx, Qy, (2-\beta)t) * M(ABx, STy, t),\\
&N(Px, Qy, kt)\\
&\leq N(ABx, Px, t) \diamond N(STy, Qy, t) \diamond N(STy, Px, \beta t)\\
&\quad \diamond N(ABx, Qy, (2-\beta)t) \diamond N(ABx, STy, t).
\end{aligned}$$

Then A, B, S, T, P *and* Q *have a unique common fixed point in* X.

Proof. By (a), since $P(X) \subset AB(X)$, we can choose a point $x_1 \in X$ such that $Px_0 = ABx_1$. Also, since $Q(X) \subset ST(X)$, we can choose $x_2 \in X$ for this point x_1 such that $Qx_1 = STx_2$. Inductively construct sequence $\{y_n\} \subset X$ such that $y_{2n} = Px_{2n} = ABx_{2n+1}$, $y_{2n+1} = Qx_{2n+1} = STx_{2n+2}$ for $n = 1, 2, \cdots$. By (b), we have for all $t > 0$ and $\beta = 1 - q$ with $q \in (0,1)$,

$$\begin{aligned}
&M(y_{2n+1}, y_{2n+2}, kt)\\
&= M(Px_{2n+1}, Qx_{2n+2}, kt)\\
&\geq M(ABx_{2n+1}, Px_{2n+1}, t) * M(STx_{2n+2}, Qx_{2n+2}, t) * M(STx_{2n+2}, Px_{2n+1}, \beta t)\\
&\quad * M(ABx_{2n+1}, Qx_{2n+2}, (2-\beta)t) * M(ABx_{2n+1}, STx_{2n+2}, t)\\
&= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+1}, y_{2n+1}, (1-q)t)\\
&\quad * M(y_{2n}, y_{2n+2}, (1+q)t) * M(y_{2n}, y_{2n+1}, t)\\
&\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n}, y_{2n+1}, qt),\\
&N(y_{2n+1}, y_{2n+2}, kt)\\
&= N(Px_{2n+1}, Qx_{2n+2}, kt)\\
&\leq N(ABx_{2n+1}, Px_{2n+1}, t) \diamond N(STx_{2n+2}, Qx_{2n+2}, t) \diamond N(STx_{2n+2}, Px_{2n+1}, \beta t)\\
&\quad \diamond N(ABx_{2n+1}, Qx_{2n+2}, (2-\beta)t) \diamond N(ABx_{2n+1}, STx_{2n+2}, t)\\
&= N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t) \diamond N(y_{2n+1}, y_{2n+1}, (1-q)t)\\
&\quad \diamond N(y_{2n}, y_{2n+2}, (1+q)t) \diamond N(y_{2n}, y_{2n+1}, t)\\
&\leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t) \diamond N(y_{2n}, y_{2n+1}, qt).
\end{aligned}$$

Letting $q \to 1$,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t),$$

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t).$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+2}, y_{2n+3}, t)$$
$$N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t) \diamond N(y_{2n+2}, y_{2n+3}, t).$$

Thus,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_n, y_{n+1}, t) * M(y_{n+1}, y_{n+2}, t),$$
$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_n, y_{n+1}, t) \diamond N(y_{n+1}, y_{n+2}, t)$$

for $n = 1, 2, \cdots$. And for positive integers n, p,

$$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t) * M\Big(y_{n+1}, y_{n+2}, \frac{t}{k^p}\Big)$$
$$N(y_{n+1}, y_{n+2}, kt) \leq N(y_n, y_{n+1}, t) \diamond N\Big(y_{n+1}, y_{n+2}, \frac{t}{k^p}\Big).$$

Since $\lim_{n\to\infty} M(y_{n+1}, y_{n+2}, \frac{t}{k^p}) = 1$ and $\lim_{n\to\infty} N(y_{n+1}, y_{n+2}, \frac{t}{k^p}) = 0$, we have

$$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t), \ \ N(y_{n+1}, y_{n+2}, kt) \leq N(y_n, y_{n+1}, t).$$

By Lemma 2.7, $\{y_n\}$ is a Cauchy sequence in X and since X is complete, $\{y_n\}$ converges to $z \in X$. Also, since $\{Px_{2n}\}$, $\{Qx_{2n+1}$, $\{ABx_{2n+1}\}$ and $\{STx_{2n+2}\}$ are subsequences of $\{y_n\}$, hence

$$\lim_{n\to\infty} Px_{2n} = z = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} ABx_{2n+1} = \lim_{n\to\infty} STx_{2n+2}.$$

Since A, B are continuous and (P, AB) are compatible maps of type(α), by Proposition 3.5(a), we have

$$\lim_{n\to\infty} P(AB)x_{2n+1} = ABz \text{ and } \lim_{n\to\infty} (AB)^2 x_{2n+1} = ABz.$$

Also, since S, T are continuous and (Q, ST) is compatible maps of type(α), by Proposition 3.5(2), we have $\lim_{n\to\infty} Q(ST)x_{2n+2} = STz$ and $\lim_{n\to\infty} (ST)^2 x_{2n+2} = STz$.

First, let $x = (AB)x_{2n+1}$ and $y = x_{2n+2}$ with $\beta = 1$ in (e), we obtain

$$\begin{aligned}
&M(P(AB)x_{2n+1}, Qx_{2n+2}, kt) \\
&\geq M((AB)^2 x_{2n+1}, P(AB)x_{2n+1}, t) * M(STx_{2n+2}, Qx_{2n+2}, t) \\
&\quad * M(STx_{2n+2}, P(AB)x_{2n+1}, t) * M((AB)^2 x_{2n+1}, Qx_{2n+2}, t) \\
&\quad * M((AB)^2 x_{2n+1}, STx_{2n+2}, t),
\end{aligned}$$

$$
\begin{aligned}
&N(P(AB)x_{2n+1}, Qx_{2n+2}, kt) \\
&\leq N((AB)^2 x_{2n+1}, P(AB)x_{2n+1}, t) \diamond N(STx_{2n+2}, Qx_{2n+2}, t) \\
&\diamond N(STx_{2n+2}, P(AB)x_{2n+1}, t) \diamond N((AB)^2 x_{2n+1}, Qx_{2n+2}, t) \\
&\diamond N((AB)^2 x_{2n+1}, STx_{2n+2}, t).
\end{aligned}
$$

Letting $n \to \infty$,

$$
\begin{aligned}
&M(ABz, z, kt) \\
&\geq 1 * 1 * M(z, ABz, t) * M(ABz, z, t) * M(ABz, z, t) \\
&\geq M(ABz, z, t), \\
&N(ABz, z, kt) \\
&\leq 0 \diamond 0 \diamond N(z, ABz, t) \diamond N(ABz, z, t) \diamond N(ABz, z, t) \\
&\leq N(ABz, z, t).
\end{aligned}
$$

Hence, by Lemma 2.8, $ABz = z$.

Second, let $x = Px_{2n}$ and $y = x_{2n+1}$ with $\beta = 1$ in (e), we have

$$
\begin{aligned}
&M(P(Px_{2n}), Qx_{2n+1}, kt) \\
&\geq M(AB(Px_{2n}), P(Px_{2n}), t) * M(STx_{2n+1}, Qx_{2n+1}, t) \\
&*M(STx_{2n+1}, P(Px_{2n}), t) * M(AB(Px_{2n}), Qx_{2n+1}, t) \\
&*M(AB(Px_{2n}), STx_{2n+1}, t), \\
&N(P(Px_{2n}), Qx_{2n+1}, kt) \\
&\leq N(AB(Px_{2n}), P(Px_{2n}), t) \diamond N(STx_{2n+1}, Qx_{2n+1}, t) \\
&\diamond N(STx_{2n+1}, P(Px_{2n}), t) \diamond N(AB(Px_{2n}), Qx_{2n+1}, t) \\
&\diamond N(AB(Px_{2n}), STx_{2n+1}, t).
\end{aligned}
$$

Taking the limit $n \to \infty$, we have

$$
\begin{aligned}
&M(Pz, z, kt) \\
&\geq M(Pz, z, t) * M(z, z, t) * M(z, Pz, t) * M(z, z, t) * M(z, z, t) \\
&\geq M(Pz, z, t), \\
&N(Pz, z, kt) \\
&\leq N(Pz, z, t) \diamond N(z, z, t) \diamond N(z, Pz, t) \diamond N(z, z, t) \diamond N(z, z, t) \\
&\leq N(Pz, z, t).
\end{aligned}
$$

Therefore by Lemma 2.8, $Pz = z$. Hence $Pz = z = ABz$.

Third, we show that $Bz = z$. Let $x = Bz$ and $y = x_{2n+1}$ with $\beta = 1$ in (e), and using (b), we obtain

$$\begin{aligned} & M(P(Bz), Qx_{2n+1}, kt) \\ & \geq M(AB(Bz), P(Bz), t) * M(STx_{2n+1}, Qx_{2n+1}, t) * M(STx_{2n+1}, P(Bz), t) \\ & \quad * M(AB(Bz), Qx_{2n+1}, t) * M(AB(Bz), STx_{2n+1}, t), \\ & N(P(Bz), Qx_{2n+1}, kt) \\ & \leq N(AB(Bz), P(Bz), t) \diamond N(STx_{2n+1}, Qx_{2n+1}, t) \diamond N(STx_{2n+1}, P(Bz), t) \\ & \quad \diamond N(AB(Bz), Qx_{2n+1}, t) \diamond N(AB(Bz), STx_{2n+1}, t). \end{aligned}$$

Letting $n \to \infty$,

$$\begin{aligned} & M(Bz, z, kt) \\ & \geq M(Bz, Bz, t) * M(z, z, t) * M(z, Bz, t) * M(Bz, z, t) * M(Bz, z, t), \\ & N(Bz, z, kt) \\ & \leq N(Bz, Bz, t) \diamond N(z, z, t) \diamond N(z, Bz, t) \diamond N(Bz, z, t) \diamond N(Bz, z, t). \end{aligned}$$

Therefore, by Lemma 2.8, $Bz = z$. Also, since $ABz = z$, hence $Az = z$.

Forth, let $x = z$ and $y = STx_{2n+2}$ with $\beta = 1$ in (e), we have

$$\begin{aligned} & M(Pz, Q(ST)x_{2n+2}, kt) \\ & \geq M(ABz, Pz, t) * M((ST)^2 x_{2n+2}, Q(ST)x_{2n+2}, t) * M((ST)^2 x_{2n+2}, Pz, t) \\ & \quad * M(ABz, Q(ST)x_{2n+2}, t) * M(ABz, (ST)^2 x_{2n+2}, t), \\ & N(Pz, Q(ST)x_{2n+2}, kt) \\ & \leq N(ABz, Pz, t) \diamond N((ST)^2 x_{2n+2}, Q(ST)x_{2n+2}, t) \diamond N((ST)^2 x_{2n+2}, Pz, t) \\ & \quad \diamond N(ABz, Q(ST)x_{2n+2}, t) \diamond N(ABz, (ST)^2 x_{2n+2}, t) \end{aligned}$$

As $n \to \infty$, we have

$$\begin{aligned} & M(z, STz, kt) \\ & \geq M(z, z, t) * M(STz, STz, t) * M(STz, z, t) * M(z, STz, t) * M(z, STz, t) \\ & \geq M(z, STz, t), \\ & N(z, STz, kt) \\ & \leq N(z, z, t) \diamond N(STz, STz, t) \diamond N(STz, z, t) \diamond N(z, STz, t) \diamond N(z, STz, t) \\ & \leq M(z, STz, t). \end{aligned}$$

By Lemma 2.8, $STz = z$.

Fifth, by putting $x = z$ and $y = Qx_{2n+1}$ with $\beta = 1$ in (e) and using (b), we have

$$\begin{aligned} & M(z, Qz, kt) \\ \geq\ & M(z, z, t) * M(z, Qz, t) * M(z, z, t) * M(z, Qz, t) * M(z, z, t) \\ \geq\ & M(z, Qz, t), \\ & N(z, Qz, kt) \\ \leq\ & N(z, z, t) \diamond N(z, Qz, t) \diamond N(z, z, t) \diamond N(z, Qz, t) \diamond N(z, z, t) \\ \leq\ & N(z, Qz, t). \end{aligned}$$

Therefore $Qz = z$ and hence $STz = z = Qz$.

Sixth, we prove that $Tz = z$. Let $x = z$ and $y = Tz$ with $\beta = 1$ in (e) and using (b), we have

$$\begin{aligned} & M(Pz, Q(Tz), kt) \\ \geq\ & M(ABz, Pz, t) * M(ST(Tz), Q(Tz), t) * M(ST(Tz), Pz, t) \\ & * M(ABz, Q(Tz), t) * M(ABz, ST(Tz), t) \\ \geq\ & M(Tz, z, t), \\ & N(Pz, Q(Tz), kt) \\ \leq\ & N(ABz, Pz, t) \diamond N(ST(Tz), Q(Tz), t) \diamond N(ST(Tz), Pz, t) \\ & \diamond N(ABz, Q(Tz), t) \diamond N(ABz, ST(Tz), t) \\ \leq\ & N(Tz, z, t). \end{aligned}$$

Therefore, $Tz = z$. Since $STz = z$, hence $STz = z = Sz$. Thus, $Az = Bz = Sz = Tz = Pz = Qz = z$, that is, z is a common fixed point of A, B, S, T, P and Q.

Finally, we prove the uniqueness of common fixed point. Let $u (u \neq z)$ be another common fixed point of A, B, S, T, P and Q, and $\beta = 1$, then by (e),

$$\begin{aligned} & M(Pz, Qu, kt) \\ \geq\ & M(ABz, Pz, t) * M(STu, Qu, t) * M(STu, Pz, t) \\ & * M(ABz, Qu, t) * M(ABz, STu, t), \\ & N(Pz, Qu, kt) \\ \leq\ & N(ABz, Pz, t) \diamond N(STu, Qu, t) \diamond N(STu, Pz, t) \\ & \diamond N(ABz, Qu, t) \diamond N(ABz, STu, t). \end{aligned}$$

It follows that

$$\begin{aligned}
&M(z,u,kt)\\
&\geq M(z,z,t)*M(u,u,t)*M(u,z,t)*M(z,u,t)*M(z,u,t)\\
&\geq M(z,u,t),\\
&N(z,u,kt)\\
&\leq N(z,z,t)\diamond N(u,u,t)\diamond N(u,z,t)\diamond N(z,u,t)\diamond N(z,u,t)\\
&\leq N(z,u,t).
\end{aligned}$$

Hence $z=u$. That is, z is unique common fixed point of A,B,S,T,P and Q. □

Corollary 4.2. *Let X be a complete intuitionistic fuzzy metric space with $t*t\geq t$, $t\diamond t\leq t$ for all $t\in[0,1]$ and satisfy the condition (2.1). Let A,S,P and Q be maps from X into itself such that*

(a) $P(X)\subset A(X)$, $Q(X)\subset S(X)$,

(b) (P,A) *and* (Q,S) *are compatible of type(α),*

(c) A *and* B *are continuous,*

(d) *There exist $k\in(0,1)$ such that for all $x,y\in X$, $\alpha\in(0,2)$ and $t>0$,*

$$\begin{aligned}
&M(Px,Qy,kt)\\
&\geq M(Ax,Px,t)*M(Sy,Qy,t)*M(Sx,Px,\beta t)\\
&\quad *M(Ax,Qy,(2-\beta)t)*M(Ax,Sy,t),\\
&N(Px,Qy,kt)\\
&\leq N(Ax,Px,t)\diamond N(Sy,Qy,t)\diamond N(Sx,Px,\beta t)\\
&\quad \diamond N(Ax,Qy,(2-\beta)t)\diamond N(Ax,Sy,t).
\end{aligned}$$

Then A,S,P and Q have a unique common fixed point in X.

Proof. Let I_X be the identity map on X. Then the proof follows from Theorem 4.1 with $B=T=I_X$. □

Corollary 4.3. *Let X be a complete intuitionistic fuzzy metric space with $t*t\geq t$, $t\diamond t\leq t$ for all $t\in[0,1]$ and satisfy the condition (2.1). Let $P$and Q be a maps from X into itself. If there exists a constant $k\in(0,1)$ such that*

$$\begin{aligned}
&M(Px,Qy,kt)\\
&\geq M(x,Px,t)*M(y,Qy,t)*M(y,Px,\beta t)*M(x,Qy,(2-\beta)t)*M(x,y,t),\\
&N(Px,Qy,kt)\\
&\leq N(x,Px,t)\diamond N(y,Qy,t)\diamond N(y,Px,\beta t)\diamond N(x,Qy,(2-\beta)t)\diamond N(x,y,t)
\end{aligned}$$

for all $x, y \in X$, $\beta \in (0,2)$ and $t > 0$. Then P and Q have a common fixed point in X.

Proof. The proof follows from Theorem 4.1 with $A = B = S = T = I_X$. □

Corollary 4.4. *Let X be a complete intuitionistic fuzzy metric space with $t * t \geq t$, $t \diamond t \leq t$ for all $t \in [0,1]$ and satisfy the condition (2.1). Let P, S be compatible maps of type(α) on X such that $P(X) \subset S(X)$. If S is continuous and there exists a constant $k \in (0,1)$ such that*

$$\begin{aligned}
&M(Px, Py, kt) \\
&\geq M(Sx, Px, t) * M(Sy, Py, t) * M(Sy, Px, \beta t) \\
&\quad * M(Sx, Py, (2-\beta)t) * M(Sx, Sy, t), \\
&N(Px, Py, kt) \\
&\leq N(Sx, Px, t) \diamond N(Sy, Py, t) \diamond N(Sy, Px, \beta t) \\
&\quad \diamond N(Sx, Py, (2-\beta)t) \diamond N(Sx, Sy, t)
\end{aligned}$$

for all $x, y \in X$, $\beta \in (0,2)$ and $t > 0$. Then P and S have a unique common fixed point in X.

Proof. The proof follows from Theorem 4.1 with $P = Q$, $S = A$ and $B = T = I_X$. □

Example 4.5. Let $X = [0,1]$ with the metric d defined by $d(x,y) = |x - y|$ and for each $t \in [0,1]$, define

$$M(x,y,t) = \frac{t}{t + d(x,y)}, \quad N(x,y,t) = \frac{d(x,y)}{t + d(x,y)}$$

for all $x, y \in X$ and $t > 0$.

Clearly, X is a complete intuitionistic fuzzy metric space where $*$ and $\diamond$ is defined by $a * b = ab$ and $a \diamond b = \min\{1, a + b\}$.

Let A, B, S, T, P and Q be defined as $Ax = x$, $Bx = \frac{x}{2}$, $Sx = \frac{x}{5}$, $Tx = \frac{x}{3}$, $Px = \frac{x}{6}$ and $Qx = 0$ for all $x \in X$. Then $P(X) = [0, \frac{1}{6}] \subset [0, \frac{1}{2}] = AB(X)$. Also, since $Q(X) = \{0\}$ and $ST(X) = [0, \frac{1}{15}]$, hence

$$Q(X) \subset ST(X).$$

If we take $k = \frac{1}{2}$, $t = 1$ and $\beta = 1$, we see that (e) of Theorem 4.1 is satisfied. Furthermore, (b) and (c) od Theorem 4.1 are satisfied, and (P, AB) is compatible maps of type(α) if $\lim_{n \to \infty} x_n = 0$ where $\{x_n\} \subset X$ such that $\lim_{n \to \infty} Px_n =$

$\lim_{n\to\infty} ABx_n = 0$ for some $0 \in X$. Similarly, (Q, ST) is also compatible maps of type(α). Thus all conditions of Theorem 4.1 are satisfied and 0 is the unique common fixed point of A, B, S, T, P and Q.

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