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Evaluation of Failure Theories to Determine the Wood Strength Variation with Grain Slope*1

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ABSTRACT

Three failure theories were studied to evaluate the wood strength variation with grain slope. Maximum stress theory, Tsai-Hill theory and Hankinson formula were presented to hypothesize the failure of wood according to grain slope to loading direction. Red pine and Japanese larch were used as materials to simulate failure strength prediction with grain slope. Calculation of strength results was that the strength of wood drops rapidly between parallel to grain orientation (0 degree) and 15 degree grain orientation. The strength of wood with grain orientation were somewhat different at small grain angles among failure theories, and this tendency was due to tension and compression distinction, and shear accounting in each theories. For the above 45 degree grain orientation, the predicted failure strength of wood with grain variation were very close in each failure theories and were useful in assessing failure strength of wood. The applicable these theories should be considered that the wood has different behavior in tension and compression, and this lead to different strength at small grain angles in each theories. Furthermore, reconsideration is needed to assess the failure strength of wood at small grain angles in Hankinson formula and further studies are necessary to accounting for shear behavior at small grain angles.

Keywords : Failure theories, maximum stress theory, Tsai-Hill theory, Hankinson formula, grain angles

1. INTRODUCTION

Wood is an unique material and may be considered as an orthotropic material due to it's natural configuration. Wood has three principal directions and they are called longitudinal, radial and tangential direction. The mechanical properties of wood are quite different with directions and also have different engineering design values according to it's mutual directions.

The strength of wood are highly dependant on the directions, and in some wood product applications, the directions of important stresses may not coincide with the natural axes of fiber orientation (Wood Handbook, 1999). In design methodology, the strength prediction is essential and several theories for predicting the strength of orthotropic materials were presented (Tsai and Wu, 1971; Narayanswami and Adelman, 1977; Liu, 1984., etc) However, many of these

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theories are complicated and difficult to apply.

Prediction of wood strength by uniaxial tensile stress along the material symmetry axes is elementary, as these strength are determined through simple uniaxial tensile stress tests (Clouston, etc., 1998). The failure associated with tension is often occurred in wood joints and compression is occurred in post and columns, bearing part of wood joint. The strength drops severely as the angle between the direction of applied load and grain orientation increases. The rapid grain orientation of wood is considered as a irregularities that occurred while the tree was growing and sawing patterns were not good. Also, the lumber is graded by several defects, and grain orientation is one of the most important factors in lumber grading system. Therefore, the failure theory employed should accurately predict strength of wood with grain deviation and this lead to determine the allowable stress determination safely and predictably. The failure theory is also needed to design structural component and calculate margin of safety and helps to guide in materials development.

Many theories have been proposed in recent years and of all theories available, the following theories are more widely used. Especially, the hankinson formula is now in general use for the determination of wood strength with grain deviation. But this theory does not contain the shear strength component of the wood but merely the compressive strength parallel and perpendicular to the grain of wood. The compressive strength perpendicular to the grain is difficult to determine because normally the crushing strength of wood perpendicular to the grain does not exist. The wood only will be densified under the influence of compressive strength perpendicular to the grain (Kollmann and Côte, 1968). Also other failure theories have different physical basis and required some convenience to predict failure of wood and wood composite simply.

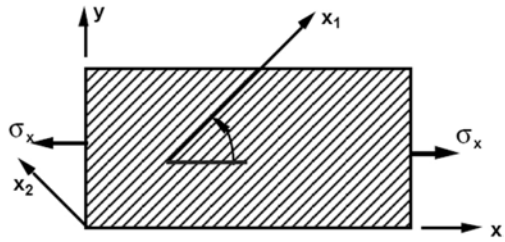


Fig. 1. Schematic of principal axes of wood.

The purpose of this paper was to present and evaluate the usefulness of 3-failure theories, Maximum failure theory, Tsai-Hill failure theory which are based on macromechanical failure theories, and hankinson formula for considering the macroscopic behavior of the failure of wood with grain deviation.

2. FAILURE FORMULAS

2.1. Maximum Stress Theory

Jenkins (1920) extended the concept of maximum stress theory to predict the strength of planar orthotropic materials such as wood. With this theory, it is postulated that failure in materials will occur when any one of the stresses, axial tension, axial compression and shear attains a respective value of maximum tension, compression and shear. The maximum stress criterion is the simplest criterion. Because of its simplicity, it is the most widely used in design and analysis of composite material structure.

Wood is subjected to uniaxial tension or compression at an angle shown in Fig. 1, the failure occurs when at least one stress component along the principal axes exceeds the corresponding strength in that direction;

$$\sigma_x = \frac{F_1}{\cos^2\theta} \quad (1)$$
$$\sigma_x = \frac{F_2}{\sin^2\theta}$$
$$\sigma_x = \frac{F_{12}}{\cos\theta\sin\theta}$$

where F_1 is the strength of longitudinal direction of wood
F_2 is the strength of transverse direction of wood
F_{12} is the shear strength of wood in 1-2plane

If the applied axial load is tension or compression, the following failure criteria obtained: when $\sigma_x > 0$

$$\sigma_1 = F_{1t} \text{ or } \sigma_x = \frac{F_{1t}}{\cos^2\theta} \quad (2)$$
$$\sigma_2 = F_{2t} \text{ or } \sigma_x = \frac{F_{2t}}{\sin^2\theta}$$
$$\sigma_{12} = F_{12} \text{ or } \sigma_x = \frac{F_{12}}{\cos\theta\sin\theta}$$

and when $\sigma_x < 0$

$$\sigma_1 = F_{1c} \text{ or } \sigma_x = -\frac{F_{1c}}{\cos^2\theta}$$
$$\sigma_2 = F_{2c} \text{ or } \sigma_x = -\frac{F_{2c}}{\sin^2\theta}$$
$$\sigma_{12} = F_{12} \text{ or } \sigma_x = \frac{F_{12}}{\cos\theta\sin\theta}$$

where σ_1 is longitudinal tension strength (=F_{1t}) or compression strength (=F_{1c})
σ_2 is transverse tension strength (=F_{2t}) or compression strength (=F_{2c})
σ_{12} is shear strength (=F_{12})

For this criterion, the failure surface becomes a rectangular box of height 2F($_{c\ or\ t}$) and whose flat sides are parallel to the coordinate axes of materials symmetry σ_1 and σ_2 shown in Fig. 2.

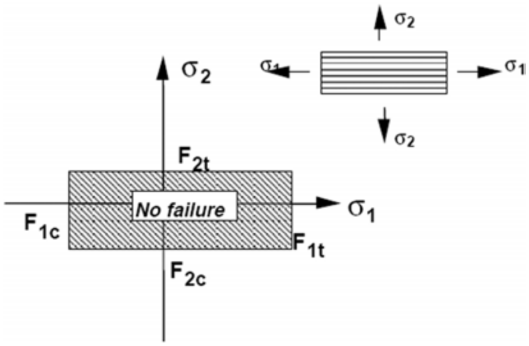


Fig. 2. Strength envelope for wood in Maximum stress theory.

Three regions can be identified, corresponding to three different modes of failure, tensile, compressive and in-plane shear failure.

2.2. Tsai-Hill Theory

The Tsai-Wu failure criterion is a phenomenological failure theory which is widely used for anisotropic composite materials which have different strengths in tension and compression (Tsai and Wu, 1971). When only one is available, the Tsai-Hill theory is used (Liu, 2001). Hill extended the von Mises criterion for ductile anisotropic material, and proposed following equation:

$$1 = \frac{\sigma_1^2}{F_1^2} - \frac{\sigma_1\sigma_2}{F_1^2} + \frac{\sigma_2^2}{F_2^2} + \frac{\sigma_{12}^2}{F_{12}^2} \quad (3)$$

No distinction is made between tensile and compressive strengths. However, the appropriate strength values can be used equation (3) according to the signs of normal stress σ_1 and σ_2. Thus

$F_1 = F_{1t}$ when $\sigma_1 > 0$, or F_{1c} when $\sigma_1 < 0$

Table 1. Functions relating mechanical properties to specific gravity of clear straight grained wood (Wood Handbook, 1999)

Species	Specific gravity	Property	Equation**	Strength (kPa)
Red pine	0.47*	Tension (//)	$166{,}000G^{0.85}$	87,375
		Tension (⊥)	$6{,}000G^{1.11}$	2,595
		Compression (//)	$93{,}700G^{0.97}$	45,047
		Compression (⊥)	$16{,}500G^{1.57}$	5,042
		Shear	$16{,}600G^{0.85}$	8,737
Japanese larch	0.61*	Tension (//)	$166{,}000G^{0.85}$	109,053
		Tension (⊥)	$6{,}000G^{1.11}$	3,466
		Compression (//)	$93{,}700G^{0.97}$	58,010
		Compression (⊥)	$16{,}500G^{1.57}$	7,593
		Shear	$16{,}600G^{0.85}$	10,905

*adopted from Wood properties of the useful tree species grown in Korea (KFRI, 2008)
**specific gravity (G) based on 12% moisture content

$F_2 = F_{2t}$ when $\sigma_2 > 0$, or F_{2c} when $\sigma_2 < 0$

In the case of uniaxial off-axis loading condition, $\sigma_1 = \sigma_x \cos^2\theta$, $\sigma_2 = \sigma_x \sin^2\theta$ and $\sigma_{12} = -\sigma_x \cos\theta \sin\theta$ are substituting equation (3), we obtain the following equation for axial strength.

$$\frac{1}{\sigma_x^2} = \frac{\cos^4\theta}{F_1^2} + \left(\frac{1}{F_{12}^2} - \frac{1}{F_1^2}\right)\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{F_2^2} \quad (4)$$

From the equation (4), the failure stress equation for the tensile and compressive stress are obtained as follows;

$$\frac{1}{\sigma_x^2} = \frac{\cos^4\theta}{F_{1t}^2} + \left(\frac{1}{F_{12}^2} - \frac{1}{F_{1t}^2}\right)\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{F_{2t}^2}$$

(tensile stress) (5)

$$\frac{1}{\sigma_x^2} = \frac{\cos^4\theta}{F_{1c}^2} + \left(\frac{1}{F_{12}^2} - \frac{1}{F_{1c}^2}\right)\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{F_{2c}^2}$$

(compressive stress)

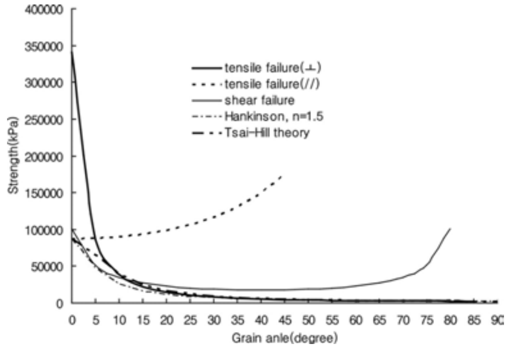
2.3. Hankinson Formula

Strength properties in directions ranging from parallel to perpendicular to the fibers can be approximated using Hankinson formula (Wood handbook, 1999). He proposed the compressive strength of wood in principal materials plane:

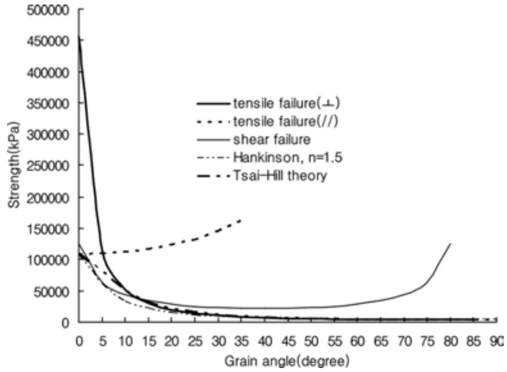
$$F_x = \frac{F_1 F_2}{F_1 \sin^n\theta + F_2 \cos^n\theta} \quad (6)$$

where F_1 = compressive strength parallel to grain
F_2 = compressive strength perpendicular to grain
$n = 2$
θ = angle from fiber direction

This formula is an empirically derived equation and has been used almost exclusively for the predicting the ultimate strength of wood under the compression loading. Hankinson used $n = 2$ for predicting the compressive strength of wood with grain variation. Later, values of n may vary between 2 and 2.5 from available literature. If the F_1 and F_2 are tensile stresses, the n = 1.5 was proposed (Kollmann and Côte, 1968). It also varies between 1.5 and 2 from the literature (Wood Handbook, 1999).



(a)



(b)

Fig. 3. Failure curves of uniaxial tensile strength in each failure criterion for Red pine (a) and Japanese larch (b).

3. FAILURE ANALYSIS and NUMERICAL RESULTS

3.1. Selected Wood Species and Strength Properties

Red pine (*Pinus densiflora* S.Z.) and Japanese larch (*Larix kaempferi* C.) were selected in this study. These species were predominant domestic wood species and generally used as wood construction materials. The strength properties of selected woods were derived by specific gravity based on 12% moisture content because the mechanical properties tend to be linearly related to specific gravity. Functions relating mechanical properties to specific gravity of clear straight grained wood was adopted from Wood Handbook (1999). The derived strength data are presented in Table 1. The tension strength parallel to grain was estimated 10 times shear strength of wood (Kollmann and Côte, 1968).

3.2. Numerical Results

The failure curves of uniaxial tensile strength in each failure theories are shown respectively in Fig. 3.

The strength drops rapidly between parallel to grain orientation (0 degree) and 15 degree grain orientation. Also, the predicted failure strengths of wood in three failure theories were very close at the grain orientation above 30 degree. In maximum stress theory, the failure of wood embodies 3 failure surfaces, tensile failure parallel or perpendicular to the grain and shear failure of which the lowest value controls the predicted failure. Also it will show 3 different mode failure corresponding to its failure envelope and it is important to note that the tensile failure was predominant around 0 degree, whereas the shear strength is a critical failure criteria around 15 degree (Table 2). Laufenberg (1984) presented that the tensile strength along the fibers controls the mode of failure from a fiber orientation of 0 degree to 2.6 degree and the shear strength controls the failure from a fiber orientation of 2.6 degree to 17 degree. This is the similar results in presented failure theory. The hankinson formula data showed the smallest strength values comparing to other failure theories. From the numerical results, and previous results (Woodward and Minor, 1988), it was possible to misstate the strength at small grain angles and it needs to account for shear strength component in small grain angles be-

Table 2. Predicted ultimate strength derived by failure theories relating to tension (unit;kPa)

Species	Failure theories	Grain angle (degrees)					
		0	15	30	45	60	90
Red pine	Maximum stress*	87,375	34,978	10,380	5,190	3,460	2,595
	Tsai-Hill	87,375	25,070	9,210	4,975	3,410	2,595
	Hankinson, n=1.5	87,375	16,232	6,874	4,238	3,178	2,595
Japanese larch	Maximum stress	109,053	46,320	13,864	6,932	4,624	3,466
	Tsai-Hill	109,053	32,157	12,117	6,606	4,545	3,466
	Hankinson, n=1.5	109,053	21,415	9,141	5,649	4,241	3,466

*Each strength values in Maximum stress theory is the lowest value among the tensile failure (//), tensile failure (⊥), and shear failure stress

cause the strength of wood is dramatically changed even though small grain angle deviations.

Hankinson formula requires only tensile/compressive strength parallel or perpendicular to the grain whereas maximum stress theory and Tsai-Hill theory require shear stength in addition to axial strength. It is to note that all 3-theories is more applicable except for small angles, but hankinson formula are needed to be modified to account for shear strength at small angles. Hyperbolic formula derived by Wood ward and Minor (1988) showed considerable results comparing to hankinson formula.

The failure curves of uniaxial compressive strength in each failure theories are shown in Fig. 4.

The compressive strength also drops rapidly between parallel to grain orientation (0 degree) and 15 degree grain orientation. Also, the predicted failure strength of wood in three failure theories were very close at the grain orientation above 30 degree and showed the same tendency to tensile strength. In maximum stress theory, the shear strength of wood is a critical failure criteria at small angles between 0 and 15 degree (Table 3). From the numerical results, it was different in strength properties among 3-failure theories at small grain angles. The strength prediction value was greater in Maximun stress

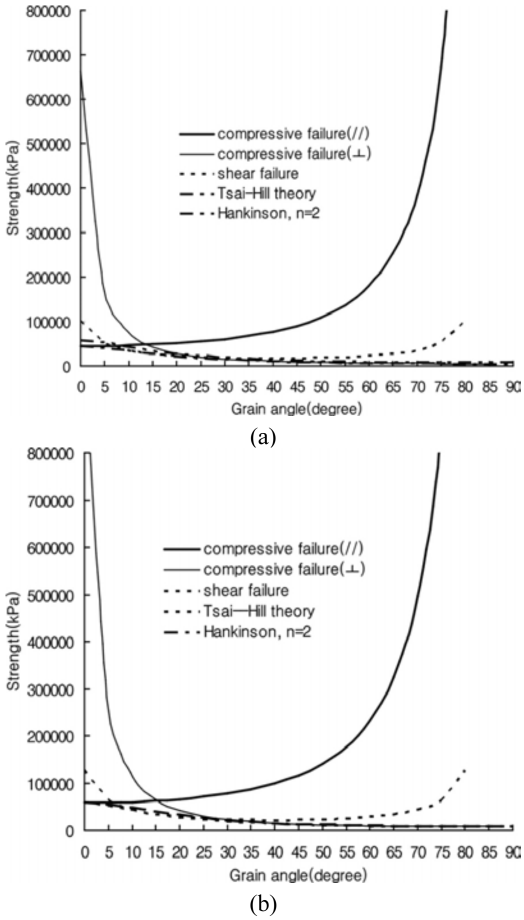


Fig. 4. Failure curves of uniaxial compressive strength in each failure criterion for Red pine (a) and Japanese larch (b).

Table 3. Predicted ultimate strength derived by failure theories relating to compression

(unit ; kPa)

Species	Failure theories	Grain angle (degrees)					
		0	15	30	45	60	90
Red pine	Maximum stress*	45,047	34948	20,168	10,084	6,722	5,042
	Tsai-Hill	45,047	26,788	14,003	8,733	6,385	5,042
	Hankinson, n = 2	45,047	29413	15,098	9,068	6,480	5,042
Japanese Larch	Maximum stress	58,010	43,620	30,372	15,186	10,124	7,593
	Tsai-Hill	58,010	34,431	18,992	12,462	9,408	7,593
	Hankinson, n = 2	58,010	40,128	21,783	13,409	9,686	7,593

*Each strength values in Maximum stress theory is the lowest value among the tensile failure (//), tensile failure (⊥), and shear failure stress

theory than Tsai-Hill theory and hankinson formula and Tsai-Hill theory showed the lowest value. But in the case of tensile stress, the strength prediction value was greater in Maximun stress theory, whereas Hankinson formula showed the lowest value. This different tendency is resulted from wood behavior of different strength values between tension and compression, so applicable these theories is considered to be accounted for shear behavior at small grain angles.

3.3. Evaluation of Failure Theories and Stress Component in Uniaxial Stress

The stress components of parallel to grain (σ_1), perpendiculat to grain (σ_2) and shear (σ_{12}) are;

$$\sigma_1 = \sigma_x \cos^2\theta \quad (7)$$
$$\sigma_2 = \sigma_x \sin^2\theta$$
$$\sigma_{12} = \sigma_x \cos\theta \sin\theta$$

By equatiuon (7), the ratio of stress was calculated to study the dominating failure at a given grain angles and the generated calculation are shown in Fig. 5. The longitudinal stress is a crit-

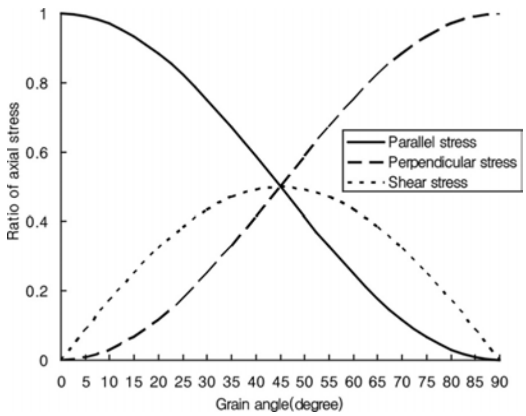


Fig. 5. Stress component and ratio of uniaxial stress.

ical factor at around 0 degree whereas transverse stress and shear stress is not the dominating factor. So the tensile strength parallel to grain controls the mode of failure in Maximum stress theory. It is important to note that the shear stress component controls the failure at small grain angles and tensile strength perpendicular to grain controls the failure above 45 degree (Fig. 5). However, hankinson formula does not contain shear stress component, it is likely to be different to other theories at small grain angles. The Tsai-Hill theory does not distinguish between the compressive strength and tensile strength in its equation. But the wood has the much different values and this can results in underestimation of the compressive stress comparing to other failure theories (Table 3).

Maximum stress theory and Tsai-Hill theory are the stress based theories and generally based on the normal and shear strength of uniaxial direction, but hankinson formula is emprically derived. As these failure theories are phenomenological and based on macromechanical analysis, the failure can be characterized by these theories except for hankinson formula. The hankinson formula merely predicts the strength of wood with grain variation, but do not present the mode of failure. This equation is not applicable to predict of failure mode of wood composite like engineered wood products that each element is layered and aligned to major axis. When the failure theories are employed into an analysis of laminated wood products like glulam, LVL and PSL, the failure mode of the laminate can be predicted conveniently by simple testing.

4. CONCLUSION

3 failure theories, Maximum stress theory, Tsai-Hill theory and hankinson formula were evaluated the wood strength variation with grain slope. The strength drops rapidly between parallel to grain orientation (0 degree) and 15 degree grain orientation. The tensile strength of wood with grain orientation were very close in each failure theories above 45 degree orientation, but the difference is somewhat greater at small angles below 15 degree. In axial tension condition, it was possible to overestimate the strength at small grain angles and it needs to account for shear strength in small grain angles because the strength of wood is dramatically changed even though small grain angle deviations.

In Maximum stress theory, the shear strength of wood is a critical failure criteria at small angles under 15 degree for compressive loading condition. And this is not the same tendency in the tensile strength. Further studies are needed to estimate the materials behavior and account for shear behavior at small grain angle deviations. This will be needed to verify the strength of engineered wood products composed of veneer or strand aligned to axial direction because each component will be composed with some grain deviation to major axis of composite.

The strength prediction values were greater in Maximun stress theory than Tsai-Hill theory and hankinson formula, and Tsai-Hill theory showed the lowest value.

The applicable these theories should be considered that the wood has different behavior in tension and compression, so this lead to different tendency in each theories. Also Further study is needed to accounting for shear behavior at small grain angles.

Maximum stress theory and Tsai-Hill theory are the stress based theories and Hankinson formula is emprically derived equation. When the failure theories are employed into an analysis of laminated wood products like glulam, LVL and PSL, the failure of the laminate can be predicted conveniently by simple testing.

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