

# An Estimation of Modeling Uncertainty for a Mechanical System in Actuators and Links in a Rigid Manipulator Using Control Theory

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## <Abstract>

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The goal of this work is to present an advanced method of an estimation of the Modeling Uncertainties coming up in industrial rigid robot's manipulator and actuators.

First, with the given physical robot model, the motion equation was derived. Considering a fictitious model, a new extended motion equation is developed. Based on this extended model, an observer and observer bank are designed for the estimation of modeling uncertainties which are involving the effects of gravity, friction, mass unbalance, and Coriolis which show the nonlinear characteristics in operation states

**Key words:** Actuator, rigid robot, Fault, estimation, nonlinearity effect, extended observer, observer bank.

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\* This paper was supported by the Daebul University Research Fund in 2009

## I . Introduction

Model uncertainties comprising the frictions, Coriolis forces, gravitational, centrifugal, backlash, parameter divergences, and the effects from the outside environment, are always troublesome task in system modeling. As the end effects are mixed, it is very hard to decouple the caused factors into the corresponding

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one. To understand the way of the method, the mode of the dynamics of a rigid robot is considered. As model uncertainties in terms of operation, the industrial robot comes to an issue and there are some ways to find them on the links, but no clues where coming from the effects. For example, some of them are analyzed as the vibration peaks and acoustics. Experts in these subjects often failed to articulate them on the link and their informations are misunderstood as signal effect from environment. A similar way to detect a split on the moving link is also Expert, for example, by [Link R. 1989, it the 1990, itremer, Preiffer, 1992, ] litut there has been not any method to get to know the characteristic feature of the uncertainties on the links. Therefore, ie movinstudy in envimethod b on the moe theory of disturbance rejection control [Mueller, 1993] is suggested for estimating uncertainties with respect to constant split depth on the crack. As an indicator for the existence of it, the nonlinear dynamic effects appeared by the change of the stiffness coefficients of link and the effects coming up in the actuators, are going to be investigated. These effects related to the measurement on the joints are one of the important clues to determine the existence of the uncertainties on the link. But setting up the relation between uncertainties and caused phenomena in the time domain operation is a tricky problem.

For this, first of all, the basic state observer is established in the way to modify the given system into the extended system with a linear fictitious model for the nonlinear link behavior and a harmonic behavior in the actuators. In this consideration, the effects of the extended system which may be nonlinear are interpreted as internal or external disturbance which is unknown at the initial stage. The unknown nonlinear effects are going to be approximated by the additional time signals yielded by elementary state observer.

When the observer guarantees the convergence, the estimate is successfully done. As an example of the physical model, the link is modeled into  $N(=6)$  finite sub-link and  $N-1$  actuators. Every one is called a subsystem. It is assumed that the material properties are homogeneous.

## II. Model and motion equation

There are various possibilities to derive the equation of motion of an industrial rigid robot in mechanics. Referring to the Fig. 1 [Spong, Vidyasagar, 1980, Spong,

1982], you can give the equation of motion of robot arm with  $N$  degrees-of-freedom as follows [Armstrong, Khatib, Burdick, 1986]:

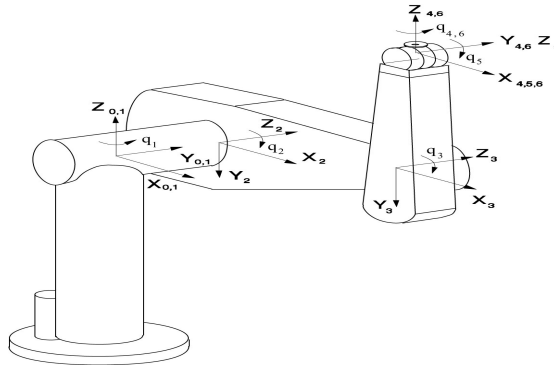


Fig. 1 Physical model of the Robot [Armstrong, Khatib, Burdick, 1986]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + f(\dot{q}) + g(q) = u \tag{1}$$

The elementary notations in the equation are denoted as follows:

- $q$  :  $N$  dimensional vector of the link coordinates of the concerned system
- $M(q)$ : the positive definite symmetric matrix of inertia moment of robot arm
- $C(q)$ : nonlinear coefficient matrix of Coriolis and centrifugal term
- $g(q)$ : vector of gravitation.
- $u$ : vector of actuator for every axis
- $f(g)$ : vector of friction

For the design of a controller, we can modify the equation (1) into the term

$$h(q, \dot{q}) = C(q, \dot{q})\dot{q} + f(\dot{q}) + g(q), \tag{2}$$

or into compact form as follows

$$M(q)\ddot{q} + h(q, \dot{q}) = u \tag{3}$$

The mass matrix  $M(q) = [M_{i,j}(q)]_{(6 \times 6)}$ ,  $i, j = 1, \dots, 6$  is symmetric and positive-definite with following elements: Assuming that there is only small deviation from motion and no redundant coordinate [Mueller, 1990], the system including unbalance in the 3rd, 4th, and 6th subsystem in the middle of the link, then the following equation can be accepted as linear system.

$$M_g \ddot{q}(t) + (D_{dg} + G_g) \dot{q}(t) + K_g q(t) = f_u(t) + f_g(t) Ls(j)(n(q(t), t)) + \hat{n}(q(t), t) \tag{4}$$

Here, the index  $g$  denotes the whole system. Equation (1) is able to be divided into  $N(=6)$  sub-finite system and its equation of motion with a subsystem  $j$  is described by

$$i_e = 1, \dots, N \tag{5}$$

$$j_k(i_e) = [(i_e-1)\frac{n}{2}+1] \quad (i_e = 1, \dots, N) \tag{6}$$

$$i = j_{k, \dots}, j_k + n - 1 \tag{7}$$

$$j = j_{k, \dots}, j_k + n - 1 \tag{8}$$

With  $i_e, j_k, i$  and  $j$  the vector in explicit form, the equation of motion can be given as follows:

$$q(i_e+1)(i)_{(i=1, \dots, \frac{n}{2}+1)} = q(i_e-1)(\frac{n}{2}+i) \tag{9}$$

$$\sum_{i_e=1}^N \sum_{j_k=j_{i_e}}^{j_k(i_e)+n-1} \left[ M \ddot{q}_{j_k(i_e)}(t) + (d_e + G_e) \dot{q}_{j_k(i_e)}(t) + K_e q_{j_k(i_e)}(t) \right] = [f_u(t)](i_e)_{i_e=3,4,5} + [f_g(t)](i_e)_{(i_e=1, \dots, N)} + Ls(n_f, i_e) [n(q(t), t)]_{(i_e=1, \dots, N)} \tag{10}$$

Where, the index  $e$  represents the elementary subsystem. The elementary notations in the equations are denoted as follows:

- $q(t), \dot{q}(t), \ddot{q}(t)$  : displacement vector, velocity vector, and acceleration vector of the system
- $M_g, K_g$ : mass matrix, stiffness matrix of undamaged section
- $d_{dq}, G_g = -G_g^T$  : matrix of the damping and gyroscopic matrix
- $q_e(t), \dot{q}_e(t), \ddot{q}_e(t)$  : displacement vector, velocity vector, and acceleration of the elementary subsystem.  $q_e(t) \in R^n$ ,  $n(=8)$  and  $nn(=32)$  are degrees of freedom of considered elementary subsystem and total system. The  $q_e(t)$  consists of ,  $q_e(t) = (x_l, y_l, \Theta_{yl}; x_r, y_r, \Theta_{yr})$  the indices  $l$  and  $r$  denote the left and the right node and  $x_r, y_r, \Theta_{xr}, \Theta_{yr}$  are the coordinates at the subsystem
- $f_u(t), f_g(t), n(q(t), t)$  : vector of unbalance, gravitation input vector and vector of the nonlinearities caused by unexpected influence(split)
- $M_e, K_e$ : mass matrix, stiffness matrix of undamaged section

•  $D_{de}, G_e = -G_e^T, Ls_{(n, i_e)}$  : matrix of damping, gyroscopic effects, and distribution vector with regard to the split at sub link number  $i_e$

All system matrices are constant in terms of time [Gasch, 1976] and the distribute matrix [Park, 2000] is given in the following way:

$$Ls(i_e) = \begin{bmatrix} 0000, \dots, 1000, \dots, 0000 \\ 0000, \dots, 0100, \dots, 0000 \end{bmatrix}^T \quad (11)$$

From now on, the index  $j$  will be left out with respect to the whole dynamic system. It is normally convenient for further operation to write the equation above via state space notation with  $x(t) = [q(t)^T, \dot{q}(t)^T]^T$  including the nonlinearities of the motion created by a split.

$$\dot{x}(t) = Ax(t) + Bu(t) + N_R(n_r(x(t))) + \hat{n} \quad (12)$$

The equation of the measurement is given by

$$y(t) = Cx(t) \quad (13)$$

where,  $A$  is  $(N_n \times N_n)$  dimensional system matrix which is responsible for the system dynamic with  $N_n = 2nn$ ,  $u(t)$  denotes dimensional vector of the excitation inputs to gravitation and unbalance and  $C$  presents  $m_e \times N_n$  - dimensional measurement matrix.  $W$  is the  $(N_n \times N_n)$  dimensional matrix and  $s(t)$  presents the plant vectors of noise that denotes the white measurement noise.  $x(t)$  is  $N_n$  - dimensional state vector and  $y(t)$  is  $M_e$  -dimensional vector of measurements, respectively. Here, the vector  $NR(x(t))$  characterizes the  $n_f$ - dimensional vector of nonlinear functions due to the split.  $NR$  is the input matrix of the nonlinearities the order of  $NR$  is of  $(N_n \times n_f)$ . It is presupposed that the matrices  $A, B, C, NR$  the vector  $u(t)$  and  $y(t)$  are already known. Where the weighting matrix  $Q$  corresponding to the plant and  $R_m$  regarding to the measurement should be suitably chosen by the trial and errors. Now it remains to reconstruct the unknown nonlinear vector. Now, it remains to reconstruct the unknown nonlinear vector  $NR(x(t), t)$  which mentions the disturbance force caused by a split. The basic idea is to get the signals from  $NR(x(t), t)$  approximated by the linear fictitious model [Mueller, 1990]

$$NR(x(t), t) \approx \hat{n}R(\hat{x}(t)) = Hv(t) \quad (14)$$

$$\dot{v}(t) = Vv(t) \quad (15)$$

$$\dim v(t) = s \quad (16)$$

that describes the time behavior of the nonlinearities due to the appearance of the split approximately as follows:

$$NR(x(t),t) \approx \hat{n}R(\hat{x}(t)) = H\hat{v}(t) \tag{17}$$

where  $\hat{v}(t)$  follows from [Mueller, 1993]. The matrices  $H$  and  $V$  have to be chosen according to the technical background considered in terms of oscillator model or integrator model [Park, Cho, 2000]. To make the signals  $\hat{n}(x(t))$  available, it needs the elementary observer (Beo) to be designed.

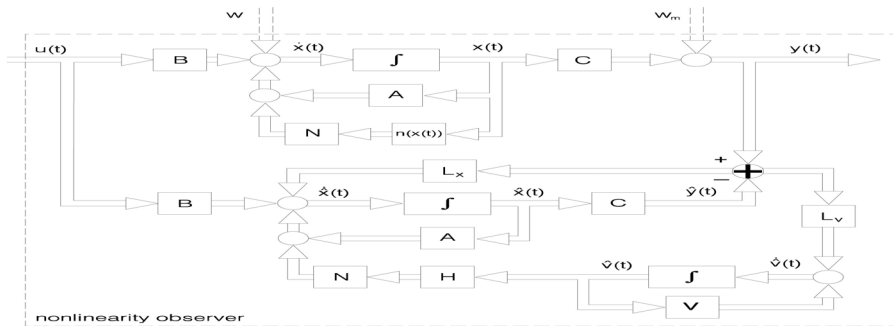


Fig. 2 Elementary observer (Beo)

At first, the given system (12-13) has to be extended with the fictitious model (4-17) into extended model

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix}}_{x_e(t)} = \underbrace{\begin{bmatrix} A & N_R H \\ 0 & V \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} x(t) \\ v(t) \end{bmatrix}}_{x_e(t)} + \begin{bmatrix} I \\ 0 \end{bmatrix} \dot{u}(t) \tag{18}$$

$$y(t) = \underbrace{\begin{bmatrix} C & 0 \\ \dots \\ \dots \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \tag{19}$$

here,  $N_R H$  couples the fictitious model(12, 13) to the whole system. To enable the successful estimate, it is obligatory to pay attention to the condition  $m_e \geq n_j$  i.e., the number of the measurements must be at least equal or greater then the modeled nonlinearities. In the case the above requirements are satisfied, the elementary observer in terms of an identity observer can be designed as follows:

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix}_{x_o(t)} = \underbrace{\begin{bmatrix} A-L_x C & N_R H \\ -L_v C & V \end{bmatrix}}_{A_0} \underbrace{\begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix}}_{x_o(t)} + \underbrace{\begin{bmatrix} I \\ 0 \end{bmatrix}}_{B_e} \hat{u}(t) + \underbrace{\begin{bmatrix} L_x \\ L_v \end{bmatrix}}_{L_o} y(t) \quad (20)$$

$$\hat{y}(t) = \underbrace{\begin{bmatrix} C \cdots 0 \end{bmatrix}}_{C_e} \begin{bmatrix} \hat{x}(t) \\ \vdots \\ \hat{v}(t) \end{bmatrix} \quad (21)$$

where matrices  $L_x$  and  $L_v$  are the gain matrix of the observer and white noise vector related to the state measurement respectively. The above equation (20,21) means that the observer consists of a simulated model with a correction feedback of the estimation error between real and simulated measurements. The matrix has  $A_o$  has  $((N_n + n_j) \times (N_n + n_j))$ -dimension and represents the dynamic behavior of the elementary observer. The asymptotic stability of the elementary observer can be guaranteed by a suitable design of the gain matrices  $L_x$  and  $L_v$  which are possible under the conditions of detect ability or observability of the extended system. To enable the successful estimation under the asymptotic, the eigenvalue of the considered observer  $A_0$  must be settled on the left side of the eigenvalue of the given system  $A_0$  to make the dynamic of the observer faster than the dynamic of the system. The fictitious model of the split behaviors is able to be designed using integrator model [Park, 2000] based on the chosen slit model [Gasch, 1976] as follows:

$$I H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & u \end{bmatrix}, \quad V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (22)$$

$$n_{(R1, x(t)_1)} + n(x(t)_1) \approx v_1(t) \quad (23)$$

$$n_{(R2, x(t)_2)} + n(x(t)_2) \approx v_2(t) \quad (24)$$

The observer gain matrices  $L_x$  and  $L_v$  can be calculated by pole assignment or by the Riccati equation [Park, Cho, 2000] as follows:

$$A + P + P A^T - P C^T R_m^{-1} C P + Q = 0 \quad (25)$$

$$\begin{bmatrix} L_x \\ \vdots \\ L_v \end{bmatrix} = P C^T R_m^{-1} \quad (26)$$

The weighting matrix  $Q$  and  $R_m$  have to be suitably chosen by the trial and error.

### III. Estimator Design for Uncertainty Mode

In the above section it has been studied at a given local position. It means that a certain place on the link is initially given as the position of the split, so the elementary observer has to survey not only the assigned local position but also any other place on the link and has to give the signals whether a split exists or not. As it has been known, it is possible to detect the split assigned certain place on the link. In the case a split appears at any subsystem in running time, it must be detected as well. But in many cases it has been shown that it is impossible or very difficult to estimate the position of the split at all subsystems on the link with one Beo. Generally, it depends on the number of the subsystem and the number of Beo. For the estimation of a split position a method based on Estimator is designed. The main idea is to feel the re-lated split forces from a certain local position to the arranged elementary observer. This is the main task in the section.

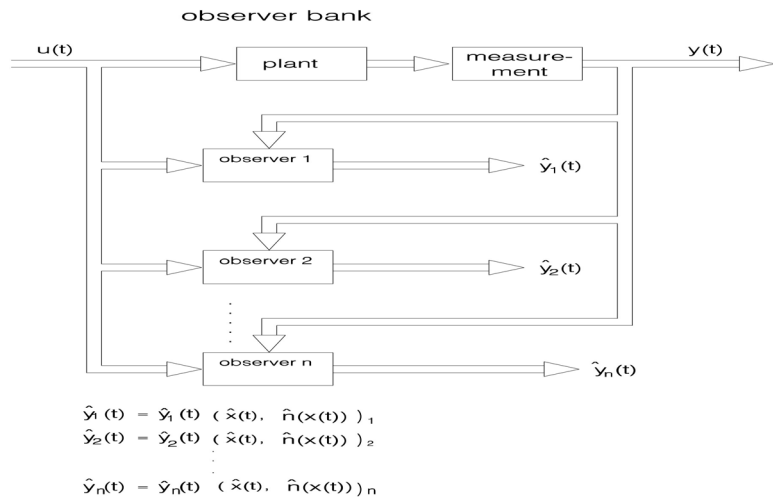


Fig. 3: Estimator (Observer Bank)

Fig. 3 shows the structure of the Estimator (Observer Bank) considered. It consists of a few elementary observer. The number of elementary observer depends on the number of the subsystems is modeled. Every elementary observer which is distinguished from the distribution matrix  $LS(i_e)$  gets the same input (excitation)  $u(t)$  and the feedback of the measurements, and is going to be set up at a suitable



place on the give system. For the appreciate arrangement of the Beo, the distribution matrix on the analogy of observer bank has been applied. In this way the Estimator(observer bank) is established with the Beo. To estimate the local place of the split, there are two steps. First of all, the Beo must be observer to certain local in the meaning of the asymptotical stability in the system. The requirement has been satisfied by the criteria from Hautus [Mueller, 1993, Park, Hu 2002].

$$\begin{aligned} & \text{Rank} \begin{bmatrix} \lambda I_{NA} - A & -N_R(Ls_{(i)}H) \\ 0 & \lambda I_{nf} - V \\ C_e & 0 \end{bmatrix} & (27) \\ & = \dim(x_e(t)) + \dim(v(t)) \\ & = N_n + n_f (= s) \end{aligned}$$

This means that the Beo has be capable of estimating the split at any location, where Beo is situated on the given system.

The unknown split position is to be found by the Beo arranged in a certain local place with the related split forces resulting from the split. To guarantee this the condition (27) is supposed to be fulfilled. In this work three Beos are arranged at the 2nd, 4th, subsystem and the 6th like this:

$$\begin{aligned} & Ls_{(2)}(i=2) = 1, \text{ otherwise } Ls_{(2)} = 0 & (28) \\ & Ls_{(4)}(i=15) = 1, \text{ otherwise } Ls_{(4)} = 0 \\ & Ls_{(6)}(i=30) = 1, \text{ otherwise } Ls_{(6)} = 0 \end{aligned}$$

The equation of the estimator with the 1st Beo A at the 2nd subsystem, the 2nd Beo C at the 6th subsystem and the 3rd Beo B 4th at subsystem are described by

$$\begin{aligned} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} &= \begin{bmatrix} A - L_{x2}C & N_v Ls(2) & H \\ -L_{v2}C & & V \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} b_r(t) \\ 0 \end{bmatrix} + \begin{bmatrix} L_{x2} \\ L_{v2} \end{bmatrix} y(t) \\ \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} &= \begin{bmatrix} A - L_{x4}C & N_v Ls(4) & H \\ -L_{v4}C & & V \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} b_r(t) \\ 0 \end{bmatrix} + \begin{bmatrix} L_{x4} \\ L_{v4} \end{bmatrix} y(t) & (29) \\ \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} &= \begin{bmatrix} A - L_{x6}C & N_v(Ls(6)) & H \\ -L_{v6}C & & V \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} b_r(t) \\ 0 \end{bmatrix} + \begin{bmatrix} L_{x6} \\ L_{v6} \end{bmatrix} y(t) \end{aligned}$$

## IV. Examples and Discussions

In this section, simulation results of the observers are given and discussed.

The Estimator consists of three Beo. The first Beo A is situated at the 2nd subsystem, the 2nd Beo B is at the 6th sub-system and the 3rd Beo C is placed at the 4th subsystem. The criteria to detect a split are the magnitude of the split

forces. In order to localize a split position, it is necessary to choose the maximal magnitude of the split force from all Beo by the comparison among the forces turn out. In the case, the estimator shows none of the split force, there is not any split in this system considered. If any one of the Beo gives the signal of a force the system has a split in a corresponding. As the 1st example, the give split is at the 1st of the link in the system considered.

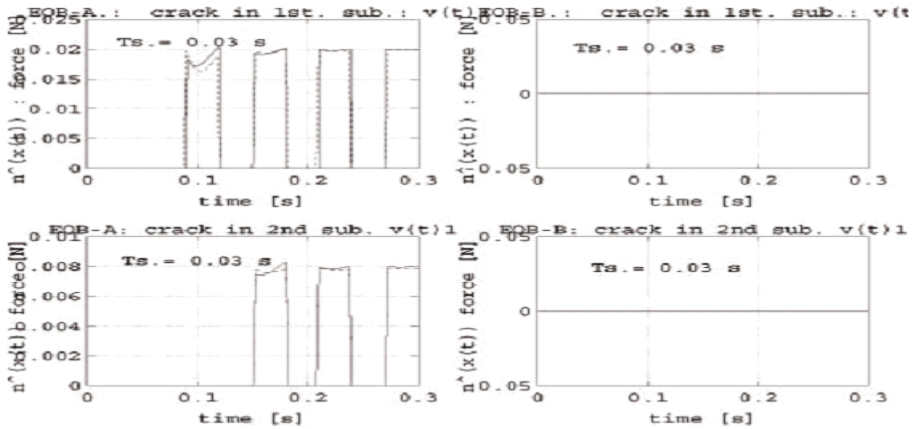


Fig. 4.1: Beo A. B. C: Split(crack) in the 1st Subsystem.

$$t_{(n;1)} = 0.135, t_{(e)} = 0.03[s].$$

*Y coordinate : force in N, X coordinate : time in sec*

Fig. 4.1 shows that the estimator recognizes the appearance of a defect at the time 0.135. Though the comparison of the forces, the elementary observer Beo A shows the largest split force. It means that split has been appeared closer or near to the 1st node(Beo A) than to the 6th node (Beo B) and the 4th node (Beo C). the detection and localization of the defect is successful. These forces related from certain position of a split to Beo A, Beo B and Beo C are supposed to be interpreted as mechanical forces due to the breathing and gapping from Gasch model[Gasch, 1976].

As the 2nd example, the given defect is situated at the 3rd node in the system and the 3rd actuator considered.

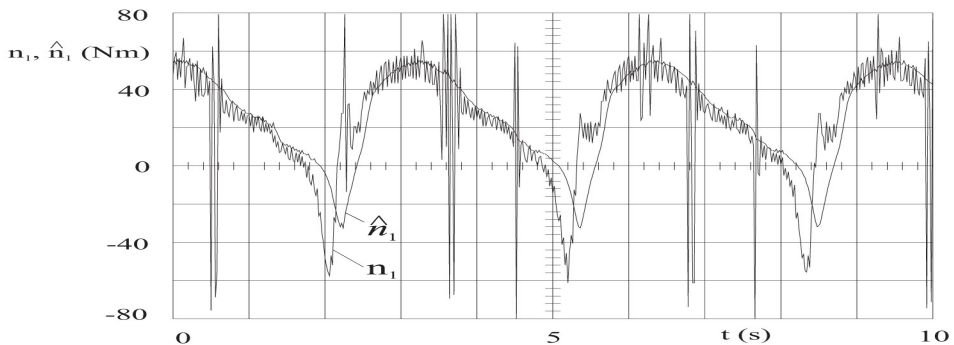


Fig. 4. 2 Beo, A, B, C: defect in the 3rd Subsystem  
 $t(ri) = 0.15$ ,  $t(s) = 0.03[s]$ , Y coordinate: force in N and  
 moment in NM, X coordinate : time in sec.

In the Fig. 4. 2, the elementary observer Beo A and Beo C show the same magnitude of the forces and moment. It denotes that the signals have been turned out between Beo A and Beo C. The estimator estimates the defect position between the 2nd node and the 4th node in the system. It is the 3rd node. In this way the Estimator estimates the existence of a defect by the split force and actuator moment and localize its position according to the magnitude of a forces and moment. In this case, the effects of the actuator plays domain role. These effects related from certain position of a defect to Beo A, Beo B and Beo C are supposed to be interpreted as mechanical forces. The numerical value of the  $\rho_o$  concerned with the weighting matrix Q is selected by tries and errors. The factor  $\rho_q$  of the weighting matrix  $R_m$  is of 0.975 and  $diag(i, j)$  is of 1. The matrices Q and  $R_m$  have been chosen by the trial and errors. It has been noticed that the observer estimates the signals very well. The external signal exists in case of the opened split. On analogy of the system model, the minimal and maximal values depend on the depth. If only the split is situated at the position where the Beo are located. Otherwise the position of the split plays a part in the values of the forces regarding to the excited inputs as well. However, the split forces are a clear indicator for the appearance of a split in operating time. The other figures which have been left out because of quantity of this paper, show that Beo B which is arranged at the right bearing, is not able to estimate the split in the 1st of the node in the system. In the simulation the given depth is of 2mm and the time of appearance of the split makes 0.2sec.

#### Comparison with another Methods:

There are some other methods in the area of the similar tasks. But the initial views of modeling are not the same as this topic. There are also not any consistence among the works. Therefore, they are not direct comparable each other. Anyway, there are some ways to treat these problems.

Soeffker et al.(1993) approached with the spectrum analysis of the vibration on turborotors. The robot's link and turborotors have different kinematic and kinetic. So, it gave some other aspects of phenomena.

Wang(1987) investigated the vibration on the moving robot without the actuator dynamic acting on the link. The author in this work illustrated vibration peak during the operation and didn't care about the modeling uncertainty. Due to the limitation of paper's magnitude, it's not suitable to cite more works.

## V. Summary and Conclusion

This paper has dealt with modeling and finding out the uncertainties of modeling.

Using the FEM, the mathematical model of the link and actuator with their effects has been presented. Based on the mathematical model, the elementary observer and an estimator have been developed. With this estimator, the task estimating the mode of uncertainties has been done with the effects of actuator. The above methods without actuator domain effect give a clear relation between the damaged link by a split and the caused phenomena in vibration by means of the measurement at the joints. Theoretical results have been given. The forces in the result are the internal forces which have been reconstructed as disturbance forces created by the split. From the given examples, it has been theoretically shown that a split on the link can be detected. The Estimator is able to estimate the location of a split. But with the harmonic effects by the actuator domain the simulation gave a kind of the twisted figure. This can lead to a false diagnosis. The method considered can be applied to the similar area with the nonlinear dynamic effects from a split problem by the suitable of an Estimator.

Anyway, the suggested methods are very significant not only for the further theoretical research and development but also for the transfer in the experiments.

For the exact estimation of defect and localization it must be considered how to model the link including the actuators precisely. This is one of the remained tasks to be researched furthermore.

### V. I Contributions

Some of the main contributions of this paper are :

- A method for describing and representing the modeling in motion equations that is, among others, capable of handling the frictions, coriolis, gravity, loss in general form has been developed.
- modeling considers actuator dynamics and joint elasticity.
- An observer and observer bank for the estimating the states, and nonlinearity of uncertainties has been developed
- Concepts and tools for synthesizing a system for compensation of uncertainty

### V. II Extensions and Future Directions

- Experimental implementation and analysis of the proposed concepts
- Developing a programming language and interface tools to bridge the gap between task planning and gains.

- Investigating the use of descriptor models and methods for this problem
- Involving the joint elasticity in modeling directly

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### Acknowledgment

This paper is dedicated to honor the retirement of Dr. Sanghyuk Lee. Dr. Sanghyuk Lee, professor of Korea National University of Education, has been devoting himself to the improvement of The KIIE(Korean Institute of Industrial Educators) and his following scholars.

## <국문초록>

### 시스템 모델링의 불확실성 추정과 보상

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이 논문은 산업용 로봇의 모델링을 할 때 일어나는 불확실성을 측정하여 불확실성이 야기하는 비선형 문제를 해결하는 데 필요한 정보를 얻는 데 목적이 있다. 우선 주어진 로봇모델에서 수학적 운동방정식을 유도하고 불확실성의 물리적 현실에 가능한 가상모델을 접목하여 수학적 확장 모델을 세우고 이를 바탕으로 불확실성을 측정 할 수 있는 관측자를 설계한다. 이 불확실성에는 모델링을 하기 어려운 모델링 오차, 중력, 마찰, 질량의 불균일 분포, 코리올리스 힘이 포함된다. 관측자와 관련된 조건들을 관측가능성 및 수렴 관계를 분석한다.

**주요어:** 시스템 모델링, 구동기, 불확실성, 보상, 제어이론, 관측자

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