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Minimization of Inspection Cost in a BLU Inspection System Using a Steady-State Flow Analysis*

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ABSTRACT

In this paper, we address a problem for minimizing the number of items inspected in a back-light-unit (BLU) inspection system, which includes a K-stage inspection system, a source inspection shop, and a re-inspection shop. In order to formulate our objective, we make a steady-state flow analysis between nodes (or shops), and derive the steady-state amount of flows between nodes and defective rates by solving a nonlinear balance equation. We provide an enumeration method for determining an optimal value of K which minimizes the number of items inspected. Our methodology could be applied and extended to similar situations with slight modification.

Keywords: Back-light Unit, Inspection Cost, K-stage Inspection System, Defective Rate

1. Introduction

A back-light unit (BLU) is attached at the back of a display unit as shown in Figure 1, and it illuminates a liquid crystal display (LCD) from the side or back. A BLU may include several cold cathode fluorescent lamps, a light guide panel (LGP), and several sheets. It can be made by assembling these parts sequentially into a mold frame. An LGP and several functional sheets guide light emitted from light sources to the whole surface of a BLU in order to attain an appropriate level of aimed uniform intensity of illumination.

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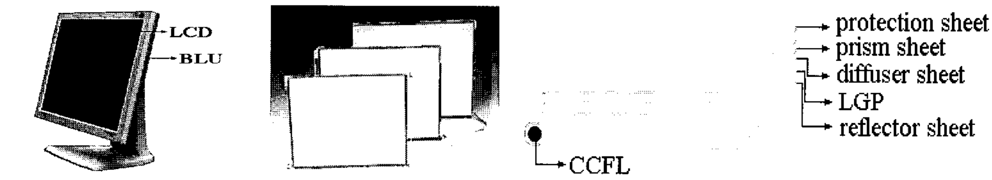

(a) an LCD and a BLU (b) various BLU's (c) a simplified structure of a BLU

Figure 1. An LCD, various BLU's, and a simplified structure of a BLU

The current issues focused by BLU suppliers include reducing both the outgoing quality rate of BLU's and the amount of work inspected and reworked throughout the BLU factory. In order to reduce the outgoing quality rate, BLU suppliers must reduce basically the defective rates of their assembly lines. However, they hardly do the activities because of insufficient investment cost and the absence of experts. The only way to survive has been just the strategy to select good BLU's and to rework bad items if they are detected. If we assign many workers at the end of assembly lines and let them select good items as many as possible and rework bad items when detected, then we may reduce more items inspected and reworked in the remaining processes after the assembly lines. If not, then we may inspect and/or rework much more items in the remaining processes. Thus, the BLU supplier would like to know how to design the selecting/reworking processes to reduce the outgoing quality rate as well as how to forecast in advance the amount of work inspected and reworked throughout the BLU factory.

Since most of papers related to a multiple inspection system assume different designs and operations in addition to limited constraints, and suggest their conclusions, it is not easy to search and utilize the published results from previous papers. However, some papers slightly related with multiple inspection system such as Raz and Thomas [1], Jaraiedi *et al.* [2], Avinadav and Raz [3] may be recommended. In order to select good items efficiently and to attain the specified outgoing quality rate, Yang [4] suggested a K-stage inspection system which was composed of K stages, each of which included an inspection process and a rework process. He determined the smallest integer value of K which could achieve a given target defective rate. However, he did not consider the number of items inspected and reworked throughout the BLU factory.

In this paper, partly utilizing the results of Yang's K-stage inspection system, we deal with an optimization problem and a case study of a BLU inspection system for minimizing the inspection cost. In Section 2, we describe briefly our problem for a BLU inspection system. In Section 3, we formulate our cost objective function by making some rational assumptions. First, we make a steady-state flow analysis between nodes, and we derive the amount of flows between nodes and steady-state defective rates by solving a nonlinear balance equation. Second, we show that the number of items reworked throughout the factory is invariant irrespective of the defective rate of items moved through the K-stage inspection system. Finally, we redefine the inspection cost as the number of items inspected, and we provide an enumeration method for determining an optimal value of K which minimizes the number of items inspected. In Section 4, a case study will be given and analyzed.

2. Problem Statement

In this paper, in order to represent processes such as operation or storage, we use the symbols suggested by the American Society of Mechanical Engineers as well as the if-else decision block. We regard the symbols and the blocks as nodes of a network. For example, a circle and a diamond which are nodes of network indicate an operation and an if-else decision block respectively.

Yang [4] suggested a K-stage inspection system consisting of K stages, each of which includes an inspection process and a rework process as shown in Figure 2. In the first stage, if an item coming off from an assembly line is classified as good by one of the inspectors allocated to the first stage, then it is sent to Node 2, a storage area represented by a reversed triangle. Otherwise, it is sent to the first rework process. After reworked by one of the reworkers allocated to the first stage, it is sent to one of the inspectors allocated to the second stage. If the reworked item is classified as good by one of the inspectors allocated to the second stage, then it is sent to Node 2. Otherwise, it is sent to the second rework process. At the last K-th stage, an item classified as good is sent to Node 2 and an item classified as bad is reworked and sent immediately to Node 2 without inspection. Assuming that inspectors are perfect in the sense that both type I error and type II error are zeros and using his results, we can express the average defective rate of items stored at Node 2 as

$$q_K = q_0 q_R^K \tag{1}$$

where q_0 = the average defective rate of items produced from BLU assembly lines, q_R = the average defective rate of items reworked. Throughout this paper, we assume that $0 < q_0 < 1$ and $0 < q_R < 1$. If follows that $0 < q_K < 1$.

As shown in Figure 3, our BLU inspection system consists of the BLU assembly lines, the K-stage inspection system, the source inspection shop, and finally the re-inspection shop, each of which includes several operations and/or storages. Items stored in Node 2 in the K-stage inspection system are packed into lots and transferred to Node 3 in the source inspection shop, where they are stored. If demands arrive, an inspector called as the source inspector, who is sent by a BLU buyer and works in the source inspection shop, starts to inspect samples drawn from each lot. If all the samples drawn from a lot are judged as good by the source inspector, the lot is accumulated and transported just in time to the consumer's assembly lines. Otherwise, lots are transferred to Node 5 in the re-inspection shop where all the items are re-inspected again. If those items in Node 5 are classified as good by several inspectors, they are transferred to Node 3, the storage area in the source inspection shop. If not, they are sent to Node 6 and reworked in Node 7, located in the re-inspection shop. The bad items returned from customers (or Node 9) are also reworked together with the items sent from Node 5. Note that Yang [4] estimated the defective rate of items stored in Node 2. However, considering the items returned from Node 5 and Node 7 to Node 3, we are interested partly in estimating the defective rate of items stored in Node 3.

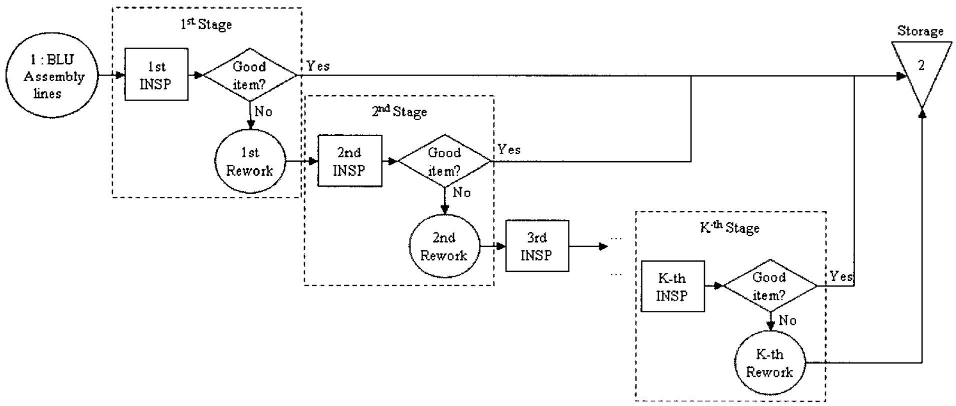

Figure 2. A conceptual process diagram of the K−stage inspection system

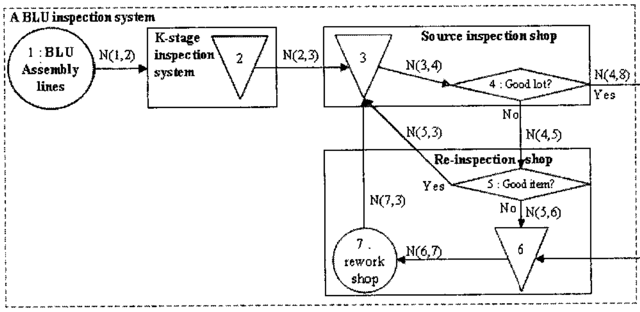

Figure 3. A conceptual process diagram of a BLU inspection system and a customer

Define $TC(K)$ to be the total inspection plus rework cost incurred at four nodes; the K-stage inspection system, Node 4 in the source inspection shop, and both Node 5 and Node 7 in the re-inspection shop. It may be conjectured that if K increases, the cost incurred at the K-stage inspection system increases while the cost incurred at Node 4, Node 5, and Node 7 decreases. Otherwise, reverse phenomenon will happen. Hence it can be expected that there exists an optimal value of K minimizing $TC(K)$, and our problem can be stated as follows; Find the optimal value of K, denoted by K^*, so that we minimize $TC(K)$.

3. Flow and Cost Analysis

In this section, we will make a steady-state flow analysis in order to estimate the number of items inspected or reworked in a node. We will make several assumptions so as to derive a set of flow balance equations, and to derive the steady-state amount of flows.

3.1 Flow Analysis

In order to facilitate our flow analysis, we define the "steady-state number or defective" as the number or defective which a system gives when its state has been reached in a steady state, and we define the following notations;

$NG(i, j) =$ the steady-state number of good items sent from Node i to Node j

$NB(i, j) =$ the steady-state number of bad items sent from Node i to Node j

$N(i, j) = NG(i, j) + NB(i, j)$

$NG(i) = NG(i, i) =$ the steady-state number of good items in Node i, produced or stored or inspected

$NB(i) = NB(i, i) =$ the steady-state number of bad items in Node i, produced or stored or inspected

$N(i) = N(i, i) = NG(i) + NB(i)$

$q(i) =$ the steady-state defective rate of items in Node i

Consider Node 8. We assume that as soon as the items, the amount of which is $N(4, 8)$, are moved to Node 8, they are temporarily stored in Node 8 and are inspected and loaded into an LCD assembly line. In other words, we assume that there are actually no stored BLU stocks in Node 8 due to a JIT production strategy, and we represent the series of these activities as $N(4, 8) = N(8) = N(8, 9)$. Assume that if BLU items are classified as good, the inspectors in Node 9 send them to Node 10. Otherwise the inspectors send them to Node 6. Suppose that $N(9, 10)$, the number of items required for the consumer's assembly lines, is Q items per day. Since the defective rate of items stored in Node 8 is $q(8)$, in the long run we have

$$N(4, 8) = N(8) = N(8, 9) = \frac{Q}{1 - q(8)} \tag{2}$$

$$N(9, 6) = \frac{q(8)Q}{1 - q(8)} \tag{3}$$

Consider Node 4. Assume that all the items available in Node 3 are packed into lots and are immediately sent to Node 4 for inspection, i.e., $N(3, 4) = N(3)$. Suppose that a lot is accepted and sent to Node 8 only if all the sampled n_S items are judged as good. Since the probability that an item can be judged as good is $q(3)$, the probability that a lot is rejected in Node 4 is

$$q_{LOT} = 1 - \{1 - q(3)\}^{n_S} \tag{4}$$

Define N_{LOT} to be the number of items in a lot and let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Assume that $N(3, 4)$ is very large enough. Since the average number of lots accepted by the source inspector given $N(3, 4)$ will be $\lfloor N(3, 4)/N_{LOT} \rfloor \{1-q(3)\}^{n_s}$, we have

$$N(4, 8) = \left\lfloor \frac{N(3, 4)}{N_{LOT}} \right\rfloor \{1-q(3)\}^{n_s} N_{LOT} \approx (1-q_{LOT})N(3, 4) = \frac{Q}{1-q(8)} \quad \text{(from Eq. (2))} \tag{5}$$

$$N(3, 4) = \frac{N(4, 8)}{(1-q_{LOT})} \approx \frac{Q}{(1-q_{LOT})(1-q(8))} \tag{6}$$

$$N(4, 5) = \frac{q_{LOT}N(4, 8)}{(1-q_{LOT})} \approx \frac{q_{LOT}Q}{(1-q_{LOT})(1-q(8))} \tag{7}$$

Since Q is big enough and the difference between two values computed by approximation and equal signs respectively is usually within an allowed tolerance, we will use an equal sign instead of an approximation sign from now on and we assume that the numbers used in this paper are real. In addition, note that we implicitly assume that the defective rates of items corresponding to $N(3, 4)$, $N(4, 5)$, and $N(4, 8)$ are the same as $q(8)$. That is, $q(3) = q(8)$.

Consider Node 5. Assume that all the good items coming into Node 5 are sent to Node 3 and all the bad items coming into Node 5 are sent to Node 6 by the inspectors in Node 5. Then since the defective rate of item coming into Node 5 is $q(3)$, we have,

$$N(5, 3) = (1-q(3))N(4, 5) = \frac{q_{LOT}Q}{(1-q_{LOT})} \tag{8}$$

$$N(5, 6) = q(3)N(4, 5) = \frac{q(3)q_{LOT}Q}{(1-q_{LOT})(1-q(3))} \tag{9}$$

Consider Node 6. Since all the items corresponding to $N(5, 6)$ and $N(9, 6)$ are bad, from Eq. (3) and Eq. (9), we have

$$N(6, 7) = N(5, 6) + N(9, 6) = \frac{q(3)Q}{(1-q_{LOT})(1-q(3))} \tag{10}$$

$$N(7, 3) = N(7) = N(6, 7) = \frac{q(3)Q}{(1-q_{LOT})(1-q(3))} \tag{11}$$

$$NG(7, 3) = (1-q_R)N(7, 3) = \frac{(1-q_R)q(3)Q}{(1-q_{LOT})(1-q(3))} \tag{12}$$

$$NB(7,3) = q_R N(7,3) = \frac{q_R q(3) Q}{(1-q_{LOT})(1-q(3))} \quad (13)$$

Consider Node 3. Since the steady-state amount of the flow into Node 3 must be equal to the steady-state amount of the flow out of Node 3, we have,

$$N(3) = N(3,4) = N(2,3) + N(5,3) + N(7,3) = \frac{Q}{(1-q(3))^{n_S+1}} \quad (14)$$

$$N(2,3) = N(3,4) - N(5,3) - N(7,3) = Q \quad \text{(from Eq. (6), Eq. (8) and Eq. (11))} \quad (15)$$

$$NG(2,3) = (1-q_K)Q \quad (16)$$

$$NB(2,3) = q_K Q \quad (17)$$

$$NB(3) = NB(2,3) + NB(7,3) = \left\{ q_K + \frac{q_R q(8)}{(1-q_{LOT})(1-q(8))} \right\} Q$$

$$\text{(from Eq. (13) and Eq. (17))} \quad (18)$$

Note that Eq. (15) can be obtained using a different approach. If we regard the source inspection shop and the re-inspection shop as a combined single block, we can construct a flow balance equation as $N(2,3) + N(9,6) = N(4,8)$. It follows that $N(2,3) = Q$. Now using Eq. (14) and Eq. (18), $q(3)$ can be expressed as

$$q(3) = \frac{NB(3)}{N(3)} = q_K(1-q_{LOT})(1-q(8)) + q_R q(8) \quad (19)$$

Using Eq. (4), Eq. (19) can be further reduced to

$$q_K(1-q(3))^{n_S+1} - (1-q_R)q(3) = 0 \quad (20)$$

Define $q_E(3)$ to be the steady-state defective rate of items available in Node 3, which satisfies Eq. (20). Then, we have the following property for $q_E(3)$, which says that Eq. (20) has only one root even though it is an (n_S+1) th-order polynomial equation, and that there exists only one value of $N(3)$ corresponding to $q_E(3)$.

Property 1: If $0<q_R,\ q_K<1$ and $0<q(3)<1$, then for positive integer n_S, there exists one and only one value $q_E(3)$ such that

(i) $q_E(3)$ satisfies the following equation; $f(q(3))=q_K(1-q(3))^{n_S+1}-(1-q_R)q(3)=0$, and

(ii) $N_E(3)=\dfrac{Q}{\{1-q_E(3)\}^{n_S+1}}=\dfrac{q_K Q}{(1-q_R)q_E(3)}$.

Proof: The first and second order derivatives of $f(q(3))$ can be derived respectively as

$$f'(q(3))=-\left\{ q_K(n_S+1)(1-q(3))^{n_S}+(1-q_R)\right\}$$

$$f''(q(3))=q_K n_S(n_S+1)(1-q(3))^{n_S-1}$$

Since $q_K>0,\ 1-q(3)>0,\ 1-q_R>0$, we have, $f'(q(3))<0$ and $f''(q(3))>0$. It follows that $f(q(3))$ is a strictly decreasing convex function of $q(3)$. Since $f(0)=q_K>0$ and $f(1)=-(1-q_R)<0$, there exists one and only one root, $q_E(3)$, in (0, 1) such that $f(q_E(3))=0$. Using Eq. (14) and Eq. (20), Property 1-(ii) holds. This completes the proof. □

From Property 1, we can compute the value of $N(i, j)$ for all i and j. Now in order to represent explicitly that both $N_E(3)$ and $q_E(3)$ depend upon K, we change them to $N_E(3:K)$ and $q_E(3:K)$ respectively for a nonnegative integer K. We have the following basic properties for them.

Property 2: If $0<q_R,\ q_K<1$, then

(i) both $q_E(3:K)$ and $N_E(3:K)$ are strictly decreasing functions of K respectively, and

(ii) $\lim_{K\to\infty} q_E(3:K)=0$,

(iii) $\lim_{K\to\infty} N_E(3:K)=Q$.

Proof: From Eq. (20), we have

$$\frac{\partial}{\partial K}q_E(3:K)=\frac{\left\{ 1-q_E(3:K)\right\}^{n_S+1}}{(n_S+1)q_K\left\{ 1-q_E(3:K)\right\}^{n_S}+(1-q_R)}\frac{\partial q_K}{\partial K}<0\ (\because \frac{\partial q_K}{\partial K}<0) \tag{21}$$

Thus, $q_E(3:K)$ is a strictly decreasing function of K. Using Property 1-(ii), we have,

$$\frac{\partial}{\partial K}N_E(3:K)=\frac{(n_S+1)Q}{\{1-q_E(3:K)\}^{n_S+2}}\frac{\partial q_E(3:K)}{\partial K}<0 \tag{22}$$

Hence, $N_E(3:K)$ is also a strictly decreasing function of K. Since q_K converges to zero as K goes to infinity, Property 2-(ii) holds from Eq. (20). It follows that Property 2-(iii) holds from Property 1-(ii). This completes the proof. □

From Property 2, the shapes of $q_E(3:K)$ and $N_E(3:K)$ may be drawn as in Figure 4. Note that the first left parts of the shapes can be slightly different from the figures since those values of $\frac{\partial^2}{\partial K^2}q_E(3:K)$ and $\frac{\partial^2}{\partial K^2}N_E(3:K)$ can be either positive or negative depending upon the input values.

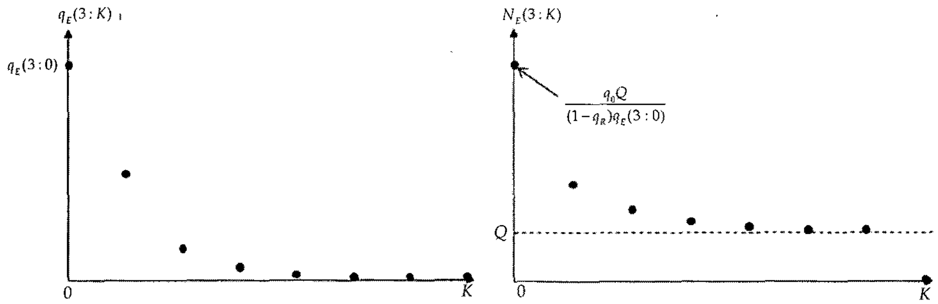

Figure 4. The shapes of $q_E(3:K)$ and $N_E(3:K)$

3.2 Cost Analysis

Define $NRW(1)$ and $NRW(2)$ to be the number of items reworked at the K-stage inspection system and the number of items reworked at the re-inspection shop respectively. Define $NRW(\cdot)$ to be the sum of $NRW(1)$ and $NRW(2)$. Then the following property indicates that $NRW(\cdot)$ is invariant irrespective of the value of K, and that we may exclude $NRW(\cdot)$ when making a cost analysis. In addition, it indicates that the only way to reduce $NRW(\cdot)$ is to reduce both q_0 and/or q_R.

Property 3: $NRW(\cdot)=\frac{q_0Q}{1-q_R}$.

Proof: We can express $NRW(1)$ and $NRW(2)$ respectively as,

$$NRW(1) = 0 \qquad \text{if K=0}$$

$$= \frac{(1-q_R{}^K)q_0Q}{1-q_R} \qquad \text{if } K \geq 1 \qquad \text{(from Yang [4])} \tag{23}$$

$$NRW(2) = N(6,7) = \frac{q(3)Q}{(1-q(3))^{n_S+1}} = \frac{q_K Q}{(1-q_R)}$$

$$\text{(from Eq. (4), Eq. (10) and Eq. (20))} \tag{24}$$

It can be easily proved that $NRW(\cdot) = \dfrac{q_0Q}{1-q_R}$ if we consider two cases of K=0 and $K \geq 1$. Note that $q_K = q_0 q_R^K$. This completes the proof. □

Since $NRW(\cdot)$ is constant irrespective of the value of K, we can exclude the rework cost and our total inspection plus rework cost, $TC(K)$, is now redefined as the only inspection costs incurred at three shops; the K-stage inspection system, the source inspection shop, and the re-inspection shop. Utilizing the results of Yang [4] again, the number of items inspected at the K-stage inspection system, denoted by $NINS(1)$, can be expressed as

$$NINS(1:K) = 0 \qquad \text{if K=0}$$

$$= \left\{1 + \frac{(1-q_R{}^{K-1})q_0}{1-q_R}\right\}Q \qquad \text{if } K \geq 1 \tag{25}$$

Assume that the source inspector must examine all of the n_S samples per lot even though he may happen to find a defective item and reject the lot without inspecting the remaining samples. Then, using Eq. (14), we can express $NINS(2)$, the number of items inspected in Node 4, as

$$NINS(2:K) = n_S \left\lfloor \frac{N(3,4)}{N_{LOT}} \right\rfloor = \frac{n_S Q}{N_{LOT}(1-q_E(3:K))^{n_S+1}} \tag{26}$$

From Eq. (4) and Eq. (7), the number of items inspected in Node 5, denoted by $NINS(3)$, can be expressed as,

$$NINS(3:K) = N(4,5) = \frac{\left\{1-(1-q_E(3:K))^{n_S}\right\}Q}{(1-q_E(3:K))^{n_S+1}} \tag{27}$$

Hence, we can express the total relevant inspection cost as $TC(K) = \sum_{i=1}^{3} NINS(i:K)$. It is not easy to make a graph of $TC(K)$. However, the following property might be useful to sketch and explain an approximated shape of $TC(K)$.

Property 4: If $0 < q_0, q_R < 1$, then we have,

(i) $NIN(1:K)$ is a strictly increasing concave function of K.

(ii) $\lim_{K\to\infty} NIN(1:K) = \left(1+\frac{q_0}{1-q_R}\right)Q$.

(iii) $NIN(2:K)$ is a strictly decreasing function of K.

(iv) $\lim_{K\to\infty} NIN(2:K) = \frac{n_S Q}{N_{LOT}}$.

(v) $NIN(3:K)$ is a strictly decreasing function of K.

(vi) $\lim_{K\to\infty} NIN(3:K) = 0$.

Proof: For K = 0 and 1, Property 4-(i) is clear since $NIN(1:0)=0$ and $NIN(1:1)=Q$ from Eq. (25). Since $\ln q_R < 0$, we have,

$$\frac{\partial}{\partial K} NIN(1:K) = -\frac{q_0 q_R^{K-1}(\ln q_R)Q}{1-q_R} > 0, \text{ and } \frac{\partial^2}{\partial K^2} NIN(1:K) = -\frac{q_0 q_R^{K-1}(\ln q_R)^2 Q}{1-q_R} < 0.$$

Hence, $NIN(1:K)$ is a strictly increasing concave function of K. Since $\lim_{K\to\infty} q_R^{K-1} = 0$, Property 4-(ii) holds. Taking the first derivative of $NIN(2:K)$, we have

$$\frac{\partial}{\partial K} NIN(2:K) = \frac{n_S(n_S+1)Q}{N_{LOT}\left\{1-q_E(3:K)\right\}^{n_S+2}} \frac{\partial q_E(3:K)}{\partial K} < 0 \quad \text{(from Eq. (21))}$$

Hence, $N_E(2:K)$ is a strictly decreasing function of K. From Eq. (26) and Property 2-(ii), Property 4-(iv) holds. Taking the first order derivative of $NIN(3:K)$, we have

$$\frac{\partial}{\partial K}NIN(3:K)=\frac{\left[\, n_S+\left[\, 1-\{1-q_E(3:K)\}^{n_S}\,\right]\,\right]Q}{\{\, 1-q_E(3:K)\}^{n_S+2}}\frac{\partial q_E(3:K)}{\partial K}<0 \quad \text{(from Eq. (21))}$$

It follows that $NIN(3:K)$ is a strictly decreasing function of K. From Property 2-(ii), Property 4-(vi) holds. This completes the proof. □

From Property 4, the shape of $NIN(\cdot:K)$ may be drawn as in Figure 5. Note that the first left parts of the shapes of $NIN(2:K)$ and $NIN(3:K)$ can be slightly different since those values of $\frac{\partial^2 NIN(2:K)}{\partial K^2}$ and $\frac{\partial^2 NIN(3:K)}{\partial K^2}$ can be either positive or negative depending upon the input values.

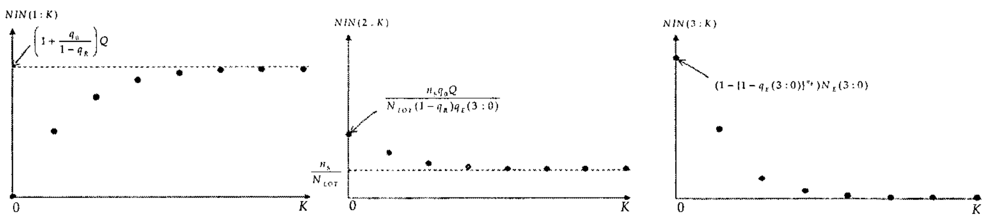

Figure 5. The shapes of $NIN(1:K)$, $NIN(2:K)$, and $NIN(3:K)$

3.3 A Procedure for Determining K^*

Since the candidate value of K is very limited, an enumeration method for determining K may work well. Hence we suggest the following procedure; For an appropriate value of K_{MAX},

Step 1. For K=1 to K_{MAX}

Begin

Find a solution of the equation in Eq. (20) and let $q_E(3:K)$ be the solution.

Compute $N_E(3:K)=\frac{Q}{\{\, 1-q_E(3:K)\}^{n_S+1}}$.

Compute $TC(K)$ using $q_E(3:K)$ and $N_E(3:K)$.

End

Step 2. Find K^* which minimizes $TC(K)$.

4. A Case Study of a BLU factory

After collecting the data accumulated for six months from a selected BLU supplier, we estimate $(Q, q_0, q_R, N_{LOT}, n_S)$ as (4,800 units/day, 16.1%, 5.0%, 240 units, 16 units). As shown in Table 1, as K increases, q_K decreases very rapidly up to zero. Similarly, as K increases, $q_E(3:K)$ and $N_E(3:K)$ also decrease and converge to zero and 4,800 units respectively as proved in Property 2. $NIN(1:K)$ increases up to 5,613 units. $NIN(2:K)$ and $NIN(3:K)$ decrease up to 320 units and zero respectively as proved in Property 4. The values of $TC(K)$ for $0 \le K \le 6$ are computed sequentially as 9,452, 5,779, 5,928, 5,933 units etc., and consequently $K^* = 1$ with $TC(K^*) = 5{,}779$ units and $q_E(3:K^*)$ = 7,461 PPM in this case study.

It can be observed that there is the greatest reduction of $TC(K)$ when K changes from zero to one as shown in Figure 6. In detail, the number of items inspected in the K-stage inspection system has increased from zero to only 4,800 units while the number of items inspected in the source inspection shop and the re-inspection shop has decreased from 9,452 units to even 979 units. Hence the number of inspected items reduced throughout the factory becomes as many as 3,673 units in total. After $K \ge 2$, $TC(K)$ increases a little and converges very rapidly to the value of 5,933 units. Note that $NRW(\cdot)$ is computed as 813 units and that it is invariant irrespective of K.

Table 1. Computational results of $TC(K)$

K	0	1	2	3	4	5	6
q_K (PPM)	161,000	8,050	403	20	1	0	0
$N_E(3:K)$	13,646	5,452	4,834	4,802	4,800	4,800	4,800
$q_E(3:K)$ (PPM)	59,611	7461	421	21	1	0	0
$NIN(1:K)$	0	4800	5,573	5611	5,613	5,613	5,613
$NIN(2:K)$	910	363	322	320	320	320	320
$NIN(3:K)$	8542	616	32	2	0	0	0
$TC(K)$	9,452	5,779	5,928	5,933	5,933	5,933	5,933

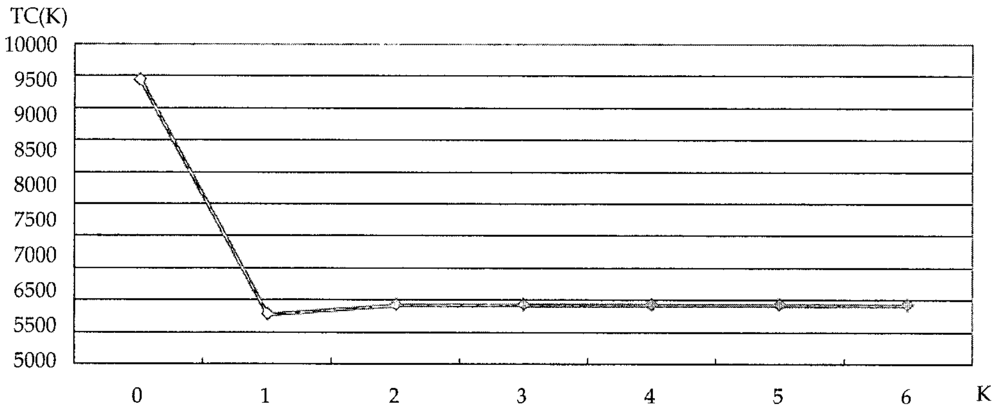

Figure 6. $TC(K)$ given $(Q, q_0, q_R, N_{LOT}, n_S)$ =(4,800, 16.1%, 5.0%, 240, 16)

5. Concluding Remarks

In this paper, we derived the steady-state amount of flows between nodes and the steady-state defective rate in Node 3 assuming that inspectors were perfect. Based on the amount of flows, we proved that the number of items which must be reworked throughout a BLU factory does not change irrespective of the value of K. Note that this result holds true only if our assumptions hold. Hence excluding the rework cost, we formulated and minimized our objective function in terms of the total number of items inspected throughout the factory.

However, when the defective rate coming off from the K-stage inspection system decreases in a real factory, we may measure other several benefits such as the decreased amounts of moving cost, storage cost, recall cost, and so on. Further research may be concentrated on the problems maximizing the combination of different benefits mentioned. Moreover, since one of our assumptions is that inspectors are perfect in the sense that both type I error and type II error are zeros, this assumption may be relaxed and very complicated results could be derived in the future. Our methodology used in this paper to derive the steady-state size of flows between nodes could be applied and extended to similar situations with slight modification.

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