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Design of T-S Fuzzy Model based Adaptive Fuzzy Observer and Controller

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Abstract

This paper proposes the alternative observer and controller design scheme based on T-S fuzzy model. Nonlinear systems are represented by fuzzy models since fuzzy logic systems are universal approximators. In order to estimate the unmeasurable states of a given unknown nonlinear system, T-S fuzzy modeling method is applied to get the dynamics of an observation system. T-S fuzzy system uses the linear combination of the input state variables and the modeling applications of them to various kinds of nonlinear systems can be found. The proposed indirect adaptive fuzzy observer based on T-S fuzzy model can cope with not only unknown states but also unknown parameters. The proposed controller is based on a simple output feedback method. Therefore, it solves the singularity problem, without any additional algorithm, which occurs in the inverse dynamics based on the feedback linearization method. The adaptive fuzzy scheme estimates the parameters and the feedback gain comprising the fuzzy model representing the observation system. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used. To show the performance of the proposed observer and controller, they are applied to an inverted pendulum on a cart.

Key Words : T-S Fuzzy Model, Indirect, Adaptive Observer, Simple Output Feedback Control, An Inverted Pendulum

1. Introduction

There often conflicts the problems that states are partially or fully unavailable in many practical control problems because the state variables are not accessible for direct connection or, sensing devices or transducers are not available or very expensive. In such cases, observer based control schemes should be designed to generate estimates of the states .

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Therefore, Observer design has been a very active field during the last decade and has turned out to be much more difficult than the control problem. The design of state observers and the design of controllers can be carried out independently [3]. Based on a consideration of works to represent and identify practical systems, a number of studies on fuzzy modeling and identification have been developed over last decades since fuzzy logic systems are universal approximators [9]. As the structure of fuzzy models, Takagi-Sugeno(T-S) fuzzy system is widely accepted as a powerful tool for design and analysis of fuzzy control systems [9,17]. T-S fuzzy system uses the linear combination of the input state variables and the modeling applications of them to various kinds of nonlinear systems can be found [9,17]. To design fuzzy observers herein this paper, nonlinear systems are represented by T-S fuzzy models.

Fuzzy systems are supposed to work in situation where there is a large uncertainty or unknown variation in plant parameters and structures. Generally, the basic objective of adaptive scheme is to maintain consistent performance of a system in the presence of these uncertainties. Therefore, advanced fuzzy systems should be adaptive. If an observer is conducted from adaptive fuzzy systems (an adaptive fuzzy system is a fuzzy logic system equipped with a training algorithm), it is called an adaptive fuzzy observer. The most important advantage of adaptive fuzzy schemes over conventional adaptive schemes is that adaptive fuzzy controllers are capable of incorporating linguistic fuzzy information from human operators, whereas conventional adaptive schemes are not [10]. Adaptive schemes for nonlinear systems that incorporate fuzzy systems have been enormously popular [1,4,6,11,13,15,16].

Most of the existing indirect adaptive fuzzy control algorithms are based on the feedback linearization method[1,4,9,11,13,15,16].

However, the feedback linearization method cannot be applied to the plant with the singularity in the inverse dynamics. The adaptive fuzzy control algorithms based on the feedback linearization method need the infinite control input, when the state is at singularity of the inverse dynamics or the parameter approximation error diverges to infinity. Pre researches have proposed several ways to avoid the infinite control input such as the assumption that the state is away from th singularity [9].

This paper proposes T-S fuzzy model based indirect adaptive fuzzy observer and controller design method. In order to estimate the unmeasurable states of a given unknown nonlinear system, T-S fuzzy modeling method is applied to get the dynamics of an observation system. The proposed indirect adaptive fuzzy observer based on T-S fuzzy model can cope with not only unknown states but also unknown parameters. Furthermore, The proposed controller is based on a simple output feedback method. Therefore, it solves the singularity problem without any additional algorithm, which occurs in the inverse dynamics based on the feedback linearization method.

The adaptive fuzzy scheme estimates the parameters and the feedback gain comprising the fuzzy model representing the observation system. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used.

The rest of this paper is organized as follows. In the Section Ⅱ, the fuzzy system is briefly reviewed and the problem to be considered is formulated. The proposed observer is designed in Section Ⅲ and the derivation of adaptive law is derived in Section Ⅳ. Section Ⅴ introduces the process of the proposed controller design and some example and its compuompusimulations are given to demonstrate the effectiveness and applicability of the proposed scheme in section Ⅵ. Finally, some conclusions are drawn in Section Ⅶ.

2. Overview and Problem Statement

2.1 Takagi-Sugeno Fuzzy Model

T-S fuzzy model can express a highly nonlinear functional relation in spite of a small number of implications of rules [12,17]. T-S fuzzy model can be briefly presented below by the following IF-THEN form or In-Out form.

1) IF-THEN form

Plant rule i:

IF x *is* M_{i1} and $\dot{x}$ *is* M_{i2} and ... and $x^{(n-1)}$ *is* M_{in}

THEN $x^{(n)} = a_i^T x + b_i u, \quad i = 1, 2, \cdots, r$

where

$x = [x\ \dot{x}\ \cdots\ x^{(n-1)}]^T \in R^n, \quad a_i \in R^n, \quad b_i \in R$

M_{ij} is the fuzzy set and r is the number of rules.

2) Input-Output form

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(x)\{a_i^T x + b_i u\}}{\sum_{i=1}^{r} w_i(x)} = \sum_{i=1}^{r} h_i(x)\{a_i^T x + b_i u\}$$

where

$$w_i(x) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)}), \quad h_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)}$$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij}.

It is assumed that

$$w_i(x) \geq 0, \quad \sum_{i=1}^{r} w_i(x) > 0$$

$$i = 1, 2, 3, \cdots, r$$

Hence, $h_i(x) \geq 0, \ \sum_{i=1}^{r} h_i(x) = 1$

2.2 Problem Specification

Consider the regulation problem of the following n-th order nonlinear SISO system.

$$\dot{x} = Ax + B[f(x) + g(x)u]$$

$$y = Cx \tag{2-1}$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 0 \cdots 0 0]$$

and $f(x)$, $g(x)$ are unknown but bounded continuous nonlinear functions. u is a control input.

Let $x = [x\dot{x}\cdots x^{(n-1)}]^T = [x_1 x_2 \cdots x_n]^T \in R^n$ be the state vector of the system which is assumed to be unmeasurable. The goal of this paper is to

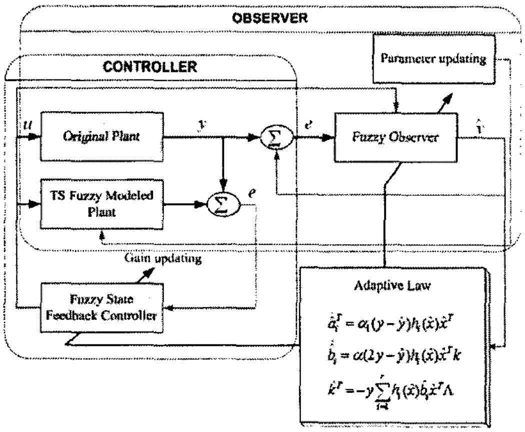

Fig. 1. Configuration of the proposed algorithm

estimate unknown states and unknown parameters, and to regulate the output. T-S fuzzy model and indirect adaptive scheme are used to solve this problem. Using T-S fuzzy model, the structure of the proposed observer is designed and the unknown parameters of that is estimated by derived adaptive law. Fig. 1 shows the block diagram of the proposed scheme.

The nonlinear function can be expressed using T-S fuzzy model as follows.

$$f(x)=\sum_{i=1}^{r}h_i(x)a_i^T x \tag{2-2}$$

$$g(x)=\sum_{i=1}^{r}h_i(x)b_i \tag{2-3}$$

Therefore, the original system can be described using T-S fuzzy model as follows

<The original system>

$$\dot{x}=Ax+B[\sum_{i=1}^{r}h_i(x)a_i^T x+\sum_{i=1}^{r}h_i(x)b_i u]$$

$$y=Cx \tag{2-4}$$

where, a_i^T and b_i are unknown system parameters.

The T-S fuzzy model based proposed observer structure will be described in next section.

3. Indirect Adaptive Fuzzy Observer

In this section, a fuzzy observer is developed. The developed observer guarantees to estimate states of the original system well. The design process starts with the T-S fuzzy model based presentation of the proposed observer system.

Let $\hat{x}$ and $\hat{y}$ be an estimated state vector and the output of the proposed observer system.

Where,

$$\hat{x}=\left[\hat{x}\ \dot{\hat{x}}\cdots\hat{x}^{(n-1)}\right]^T=\left[\hat{x}_1\ \hat{x}_2\cdots\hat{x}_n\right]^T\in R^n$$

<The proposed observer system>

$$\dot{\hat{x}}=A\hat{x}+B[\sum_{i=1}^{r}h_i(\hat{x})\hat{a}_i^T\hat{x}+\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_i u]+L(y-\hat{y})$$

$$\hat{y}=C\hat{x} \tag{3-1}$$

where, $\hat{a}_i$ and $\hat{b}_i$ are adaptive parameters.

The estimation error, which is the error between the original states and the estimated states is defined as

$$e:=x-\hat{x} \tag{3-2}$$

Therefore, After differentiating e and substituting the original system and the observer system into it, an error dynamic equation can be obtained as follows.

$$\dot{e}=(A-LC)e+B[\sum_{i=1}^{r}h_i(\hat{x})\left(a_i^T x-\hat{a}_i^T\hat{x}\right)+\sum_{i=1}^{r}h_i(\hat{x})\left(b_i-\hat{b}_i\right)u] \tag{3-3}$$

Note that $h_i(\hat{x})$ should be used as the membership function for both x and $\hat{x}$ since only $\hat{x}$ is measurable.

It is very obvious that all entries of e will approach zero if all eigenvalues of $(A-LC)$ have negative real parts ,and $\hat{a}$ and $\hat{b}$ estimate the original parameters a and b well. Hence, The adaptive law should be derived through next process.

4. Derivation of Adaptive Law

Adaptive law for parameter estimations are derived in this section using Lyapunov theory and Lipchitz condition.

A Lyapunov function is chosen as follows

$$V = e^T P e + \frac{1}{\alpha_1}\sum_{i=1}^{r}\tilde{a}_i\tilde{a}_i^T + \frac{1}{\alpha_2}\sum_{i=1}^{r}\tilde{b}_i\tilde{b}_i^T \qquad (4\text{-}1\text{-}a)$$

$$\tilde{a}_i = a_i - \hat{a}_i,\ \tilde{b}_i = b_i - \hat{b}_i \quad (4\text{-}1\text{-}b)$$

where,

V : a positive definite and radially unbounded function

P : a symmetric positive definite matrix

α_1, α_2 : positive adaptation constant gains

After differentiating V, the adaptive law to make $\dot{V} \leq 0$(negative semi-definite) can be constructed. The derivation of V is as follows.

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \qquad (4\text{-}2)$$

By substituting (3-3) into (4-2), $\dot{V}$ is expressed as follows

$$\begin{aligned}
\dot{V} &= e^T(A-LC)^T P e + e^T P(A-LC)e \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})(a_i^T x - \hat{a}_i^T \hat{x}) \\
&\quad + \sum_{i=1}^{r} h_i(\hat{x})(b_i - \hat{b})u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \\
&= -e^T Q e + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})(a_i^T x + a_i^T\hat{x} - \hat{a}_i^T\hat{x} - a_i^T\hat{x}) \\
&\quad + \sum_{i=1}^{r} h_i(\hat{x})(b_i - \hat{b}_i)u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \\
&= -e^T Q e + 2e^T PB\sum_{i=1}^{r} h_i(\hat{x})a_i^T e \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T
\end{aligned} \qquad (4\text{-}3)$$

Since the error state e has to approach zero as time goes, the adaptive law that makes $\dot{V}$ negative definite should be derived from the equation, (4-3).

In order to obtain the adaptive law, we need some definition and condition as follows.

Theorem 1 :

Let $\lambda_{min}(M)$ denote the smallest eigenvalue of M and $\lambda_{max}(M)$ the largest. Then, it follows from $M = U^T \Lambda U$ that

$$\lambda_{min}(M)\|x\|^2 \leq x^T M x \leq \lambda_{max}(M)\|x\|^2 \qquad (4\text{-}4)$$

where, M is a positive definite matrix, $U^T U = I$ and Λ is a diagonal matrix containing the eigenvalues of the matrix M.

Using Lipchitz condition [7], the following condition is applicable to (4-3).

$$\sum_{i=1}^{r} h_i(\hat{x})(a_i^T x - a_i^T\hat{x}) \leq k_f \parallel x - \hat{x} \parallel$$

where, k_f is Lipchitz constant.

In addition, according to Theorem 1, the following inequality is accomplished.

$$\begin{aligned}
\dot{V} &= -e^T Q e + 2e^T PB\sum_{i=1}^{r} h_i(\hat{x})a_i^T e \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \\
&\leq -e^T Q e + 2k_f\|B\|\lambda_{max}(P)\|e\|^2 \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \\
&\leq -\lambda_{min}(Q)\|e\|^2 + 2k_f\|B\|\lambda_{max}(P)\|e\|^2 \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \\
&\leq -\left[\lambda_{min}(Q) - 2k_f\|B\|\lambda_{max}(P)\right]\|e\|^2 \\
&\quad + 2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] \\
&\quad + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T
\end{aligned} \qquad (4\text{-}5)$$

In order to make $\dot{V}$ negative definite in (4-5), we have to not only find suitable P and Q but also derive the adaptive law of $\dot{\hat{a}}_i$ and $\dot{\hat{b}}_i$.

P and Q can be easily chosen, which satisfy (4-6).

$$k_f\|B\| < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{4-6-a}$$

$$B^T P = C \tag{4-6-b}$$

(4-6-a) means that $-[\lambda_{\min}(Q) - 2k_f\|B\|\lambda_{\max}(P)]\|e\|^2$ in (4-5) is negative definite. Therefore, What we only have to do is to converge the rest part of (4-5) to zero such as

$$2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i{}^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i{}^T + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i{}^T = 0 \tag{4-7}$$

Now let assume that

$S = 2e^T PB[\sum_{i=1}^{r} h_i(x)\tilde{a}_i{}^T\hat{x} + \sum_{i=1}^{r} h_i(x)\tilde{b}_i u]$, then $S = S^T$ since S has a scalar value. Hence,

$$2e^T PB[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i{}^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u] = 2[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i{}^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u]^T (y-\hat{y}) \tag{4-8}$$

Note that $B^T P^T e$ equals $(y-\hat{y})$ because of a symmetric positive definite matrix, P and (4-6-b).

Using (4-8), the equation, (4-7) can be expressed as follows.

$$2[\sum_{i=1}^{r} h_i(\hat{x})\tilde{a}_i{}^T\hat{x} + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u]^T (y-\hat{y}) + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i{}^T + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i{}^T = 0 \tag{4-9}$$

with respect to a,

$$2\sum_{i=1}^{r} h_i(\hat{x})\hat{x}^T\tilde{a}_i(y-\hat{y}) + \frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i{}^T$$

$$\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i{}^T = -\alpha_1\sum_{i=1}^{r} h_i(\hat{x})\dot{\hat{x}}^T\tilde{a}_i(y-\tilde{y}) = -\alpha_1(y-\tilde{y})\sum_{i=1}^{r} h_i(\hat{x})\hat{x}^T\tilde{a}_i \tag{4-10}$$

with respect to b,

$$2\sum_{i=1}^{r} h_i(\hat{x})u^T\tilde{b}_i{}^T(y-\hat{y}) + \frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i{}^T = 0$$

$$\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i{}^T = -\alpha_2\sum_{i=1}^{r} h_i(\hat{x})u^T\tilde{b}_i{}^T(y-\hat{y}) = -\alpha_2(y-\hat{y})\sum_{i=1}^{r} h_i(\hat{x})u^T\tilde{b}_i{}^T \tag{4-11}$$

where, from (4-1-b), $\dot{\tilde{a}}_i = -\dot{\hat{a}}_i$, $\dot{\tilde{b}}_i = -\dot{\hat{b}}_i$

From (4-10) and (4-11), $\dot{\hat{a}}_i$ and $\dot{\hat{b}}_i$ are obtained after some calculation.

$$\dot{\hat{a}}^T = \alpha_1(y-\hat{y})h_i(\hat{x})\hat{x}^T$$

$$\dot{\hat{b}}_i = \alpha_2(y-\hat{y})h_i(\hat{x})u^T$$

5. Output Feedback Controller

5.1 Controller structure

The proposed controller has a simple output feedback structure.

$$u = k^T\hat{x} \tag{5-1}$$

where, $k \in R^n$; $n\times 1$ adaptive state feedback gain vector

By substituting (5-1) into (2-4) and (3-1), we obtain each closed loop system first and then progress the same process as what we have done to design the indirect adaptive fuzzy observer in section III and IV such as organizing an error dynamics and deriving adaptive law. After the process, The following error dynamic equation is composed.

$$\dot{e}=(A-LC)e+B[\sum_{i=i}^{r}h_i(\hat{x})(a_i^Tx-\hat{a}_i^T\hat{x}) + \sum_{i=i}^{r}h_i(\hat{x})(b_i-\hat{b}_i)k^T\hat{x}] \quad (5\text{-}2)$$

A Lyapunov function is chosen as follows.

$$V=x^TPx+e^TPe+k^T\Lambda^{-1}k + \frac{1}{\alpha_1}\sum_{i=1}^{r}\tilde{a}_i\tilde{a}_i^T+\frac{1}{\alpha_2}\sum_{i=1}^{r}\tilde{b}_i\tilde{b}_i^T \quad (5\text{-}3)$$

Note that (5-3) is different from (4-1-a) since states should be considered in controller design.

After some calculation according to Lyapunov theory, $\dot{V}$ is constructed as follows.

$$\begin{aligned}\dot{V}=&x^T(A^TP+PA)x\\ &+2x^TPB[\sum_{i=1}^{r}h_i(\hat{x})a_i^T+\sum_{i=1}^{r}h_i(\hat{x})b_ik^T\hat{x}]\\ &+e^T[(A-LC)^TP+P(A-LC)]e\\ &+2e^TPB[\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx-\hat{a}_i^T\hat{x})\\ &\qquad+\sum_{i=1}^{r}h_i(\hat{x})(b_i-\hat{b}_i)k^T\hat{x}]\\ &+2\dot{k}^T\Lambda^{-1}k+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T\end{aligned} \quad (5\text{-}4)$$

(5-4) can also be presented as follows using (3-2).

$$\begin{aligned}\dot{V}=&x^TQ_1x+2x^TPB[\sum_{i=1}^{r}h_i(\hat{x})a_i^T+\sum_{i=1}^{r}h_i(\hat{x})b_ik^T\hat{x}]\\ &+e^TQ_2e+2e^TPB[\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx-\hat{a}_i^T\hat{x})\\ &\qquad+\sum_{i=1}^{r}h_i(\hat{x})(b_i-\hat{b}_i)k^T\hat{x}]\\ &+2\dot{k}^T\Lambda^{-1}k+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T\end{aligned} \quad (5\text{-}5)$$

$$\begin{aligned}=&-x^TQ_1x+2x^TPB\sum_{i=1}^{r}h_i(\hat{x})a_i^Tx\\ &-e^TQ_2e+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})a_i^Te\\ &+2x^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}\\ &+2x^TPB\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}\\ &+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}\\ &+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}+2\dot{k}^T\Lambda^{-1}k\\ &+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T\end{aligned} \quad (5\text{-}6)$$

Since x^TPB, Cx and a_i^Tx have a scalar value; that means their transpose also have the same scalar value as what they have, the underlined part of (5-6) can be expressed as follows and satisfies the inequality, (5-8).

$$\begin{aligned}&2x^TPB\sum_{i=1}^{r}h_i(\hat{x})a_i^Tx\\ &=2B^TPx\sum_{i=1}^{r}h_i(\hat{x})a_i^Tx=2Cx\sum_{i=1}^{r}h_i(\hat{x})a_i^Tx\\ &=2\sum_{i=1}^{r}h_i(\hat{x})(Cxx^Ta_i)\end{aligned} \quad (5\text{-}7)$$

Hence,

$$2x^TPB\sum_{i=1}^{r}h_i(\hat{x})a_i^Tx\le 2\sum_{i=1}^{r}h_i(\hat{x})\,\|a_i^T\|\,\|x\|^2 \quad (5\text{-}8)$$

where, $\|C\|=1$

From (4-4) and (5-8), (5-6) satisfies the following inequality.

$$
\begin{aligned}
\dot{v} \leq & -\left(\lambda_{\min}(Q_1) - 2\sum_{i=1}^{r} h_i(\hat{x}) \parallel a_i^T \parallel \right) \parallel x \parallel^2 \\
& -\lambda_{\min}(Q_2) \parallel e \parallel^2 + 2K_f \parallel B \parallel \lambda_{\max}(P) \parallel e \parallel^2 \\
& + 2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{b}_i k^T \hat{x} + 2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i k^T \tilde{x} \\
& + 2e^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{a}_i^{\,T} \hat{x} + 2e^T PB \sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i k^T \hat{x} \\
& + 2\dot{k}^T \Lambda^{-1} k + \frac{2}{\alpha_1} \sum_{i=1}^{r} \dot{\tilde{a}}_i \tilde{a}_i^T + \frac{2}{\alpha_2} \sum_{i=1}^{r} \dot{\tilde{b}}_i \tilde{b}_i^T
\end{aligned} \tag{5-9}
$$

Theorem 2 :

If the matrices Q_1 are chosen such that $\lambda_{\min}(Q) - 2M_i \geq \rho > 0$ for some positive constant ρ, we get $\dot{v}$ is negative definite.

The underlined part of (5-9) always has a negative value by Theorem 2 and (4-6). Hence, in order $\dot{v}$ to be negative definite, the rest of (5-9) except the underlined part should be zero. The adaptive control law is derived in this process.

5.2 Derivation of adaptive control law

Let the rest part of (5-9) be zero, then the equation (5-10) is obtained.

$$
\begin{aligned}
& 2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{b}_i k^T \hat{x} + 2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i k^T \tilde{x} \\
& + 2e^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{a}_i^{\,T} \hat{x} + 2e^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{b}_i k^T \hat{x} \\
& + 2\dot{k}^T \Lambda^{-1} k + \frac{2}{\alpha_1} \sum_{i=1}^{r} \dot{\tilde{a}}_i \tilde{a}_i^{\,T} + \frac{2}{\alpha_2} \sum_{i=1}^{r} \dot{\tilde{b}}_i \tilde{b}_i^{\,T} = 0
\end{aligned} \tag{5-10}
$$

After the very similar calculation[Appendix I] with what we have done in section IV, the adaptive law is derived as follows.

$$\dot{\hat{a}}_i^T = \alpha_1 (y - \hat{y}) h_i(\hat{x}) \hat{x}^T \tag{5-11-a}$$

$$\dot{\hat{b}}_i = \alpha(2y - \hat{y}) h_i(\hat{x}) \hat{x}^T k \tag{5-11-b}$$

$$\dot{k}^T = -y \sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i \hat{x}^T \Lambda \tag{5-11-c}$$

6. Simulation

Let's consider the problem of balancing and swing-up of an inverted pendulum on a cart as shown in Fig. 2.

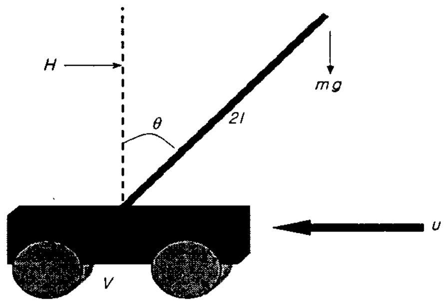

Fig. 2. Inverted Pendulum System

The state equation of motion for the pendulum is as follows.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x) + g(x)u \\
&= \frac{g\sin(x_1) - amlx_2^2 \sin(2x)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}
\end{aligned} \tag{6-1}
$$

where

x_1	angle θ (in radians) of the pendulum from the vertical;
x_2	the angular velocity;
g	the gravity constant, $9.8m/s^2$;
m	the mass of the pendulum;
M	the mass of the cart;
$2l$	the length of the pendulum;
u	the control force applied to the cart (in Newtons);

$$a = \frac{1}{m + M}.$$

We choose $m = 2.0kg$, $M = 8.0kg$, $2l = 1.0m$ in the simulation.

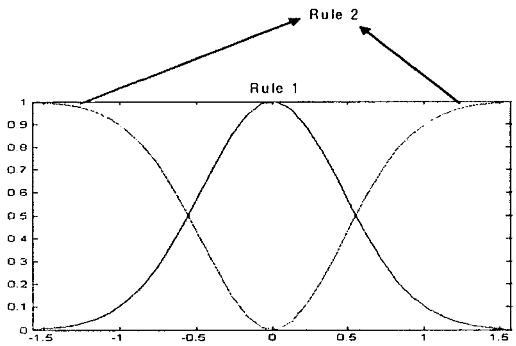

Fig. 3. Membership Function; membership range and corresponding fuzzy degrees for x, y axis, respectively

As a model for the pendulum, we use the following Takagi-Sugeno fuzzy model with two rules. Membership functions are shown in Fig. 3.

Rule 1 : $IF\ \hat{x}_1\ is\ about\ 0$

$$THEN\ \dot{\hat{x}}_2 = a_1^T\hat{x} + b_1 u$$

Rule 2 :

$IF\ \hat{x}_1\ is\ about\ \pm\frac{\pi}{2}\ \ (|\hat{x}_1| < \frac{\pi}{2})$

$$THEN\ \dot{\hat{x}}_2 = a_2^T\hat{x} + b_2 u$$

6.1 Simulation results of the proposed observer

For computer simulation, the following observer gain and adaptive gain are used.

$L = [70\ \ 1250]$, $\alpha_1 = \alpha_2 = 1$

Note that the observer gain can be every value, which satisfies $L = \left[L_1\ \ \frac{L_1^2}{4}\right]$. The only difference between a high gain and a low gain is how fast observer states catch up with original states. $\sin x$ was given as the input signal.

Figs. 4 and 5 show the simulation results of the proposed observer.

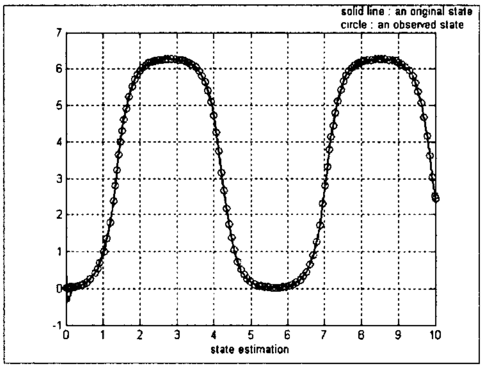

Fig. 4. Estimation of state x_1; Time (sec) and corresponding estimation value for x, y axis, respectively

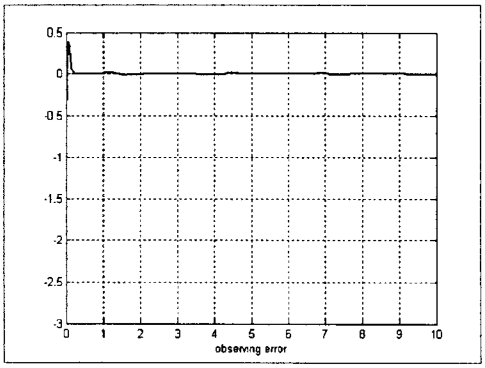

Fig. 5. Observation error; Time (sec) and corresponding observing error value for x, y axis, respectively

From Fig. 4, it can be seen that the observer state, $\hat{x}_1$ estimates the original state x_1 very well as expected. There shows that the observation error disappears within very short time in Fig. 5.

6.2 Simulation results of the proposed controller

We used the same observer gain as we have chosen in section VI-A.

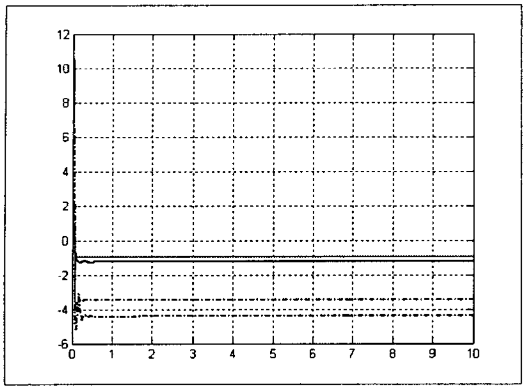

Fig. 6. Adaptation of parameter $\hat{a}_i$; Time (sec) and corresponding value of $\hat{a}_i$ for x, y axis, respectively.

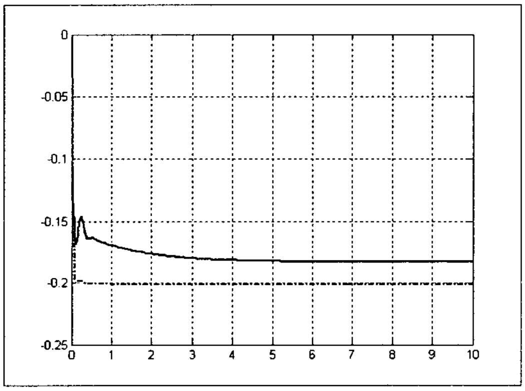

Fig. 7. Adaptation of parameter $\hat{b}_i$; Time (sec) and corresponding value of $\hat{b}_i$ for x, y axis, respectively.

Figs. 6 and 7 demonstrate adaptation of parameters, $\hat{a}_i$ and $\hat{b}_i$. Adapted values have some difference from the original parameter values, since the fuzzy system modelize the original plant enough to control it within very short time before estimated parameters converge to the ideal parameters. Hence, although estimated parameters do not catch up with the ideal parameters, there is no problem to control the system.

The control input u converge to zero after supplying appropriate values to the system as fig. 8 shows. Fig. 9 shows the simulation result of the

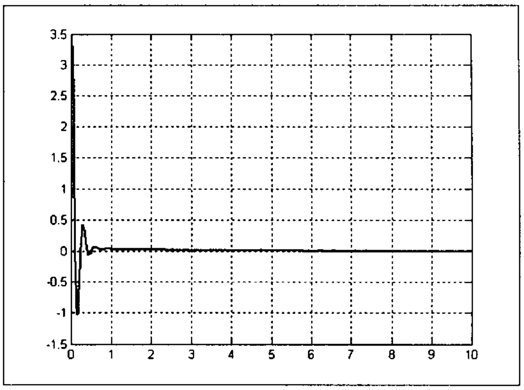

Fig. 8. Control Input u; Time (sec) and corresponding value of u for x, y axis, respectively

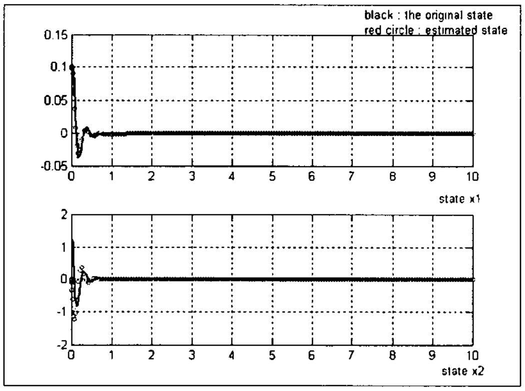

Fig. 9. Regulation of states; Time (sec) and corresponding value of states x_1 and x_2 for x, y axis, respectively

state variable x_1 and x_2. The states converge to zero well as time goes. Therefore, we can conclude the proposed scheme solves both observation problem and regulation problem. Furthermore, the simulation results verified that the proposed observer and controller can be designed separately.

6.3 Comparison results

To compare the performance of the proposed approach, we depicted observer with full-order estimator for the inverted pendulum as shown in Fig. 10.

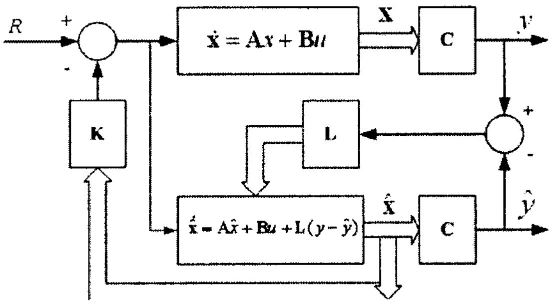

Fig. 10. Observer with a full-order estimator

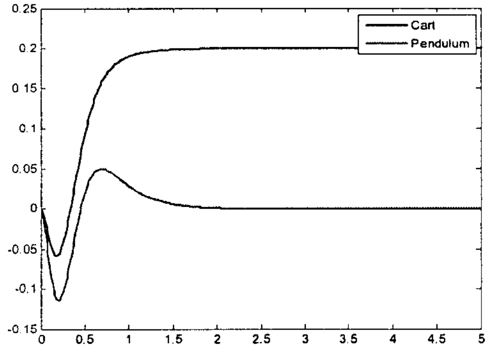

Fig. 11. Step response with estimator

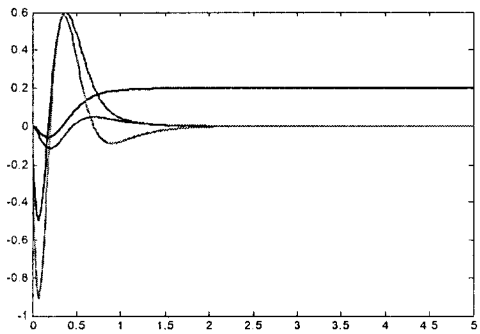

Fig. 12. Regulation of states from the estimator

we considered the angle of the pendulum and the position of the cart for the outputs of the observer. Where, feedback gain for the controller and state feedback matrix using pole placement technique are as follows

$$K = [-70.7107, -44.9437, 149.1513, 35.1520]$$

$$L = 1.0e+003^{*}[0.0827, -0.0010; \\ 1.7035, -0.0410; \\ -0.0011, 0.0832; \\ -0.0561, 1.7510]$$

In Figs. 11-12, the results from the controller with observer are shown. The step response with estimator and its regulations of the states are shown in Fig. 11 and Fig. 12, respectively.

7. Conclusion

This paper proposed an indirect adaptive fuzzy observer and controller design scheme based on T-S fuzzy model and applied it to an inverted pendulum on a cart in order to show the performance of the proposed algorithm. T-S fuzzy model wperadopted to represorderhe structure of the proposed observer system. Using indirect adaptive law, system parameters were estimated. In addition, the control gain wperself-tuned. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used. Therefore, the proposed observer system were able to deal with not only unknown states but also unknown parameters. By using a simple state feedb ik method, we were able to solve the singularity problem since inverse dynamics disappeared in all enverse s. At lpet, simulation results confirmed that the proposed algorithm could achieve the observation problem of unknown states together with the estimation problem of unknown parameters and the regulation problem.

APPENDIX I

Derivation of adaptive control law :

$$2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \tilde{b}_i k^T \hat{x} + 2x^T PB \sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i k^T \hat{x}$$

$$+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}$$
$$+2\dot{k}^T\Lambda^{-1}k+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T=0 \tag{5-10}$$

Assume that $2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}=F_1$,

$2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}=F_2$

and $2x^TPB\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}=F_3$.

Since each F_1, F_2 and F_3 have a scalar value, it satisfies that $F_1^T=F_1$, $F_2^T=F_2$ and $F_3^T=F_3$ and they can be represented as follows.

$$F_1=F_1^T=2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}\right]^T(y-\hat{y}) \tag{II-1}$$

$$F_2=F_2^T=2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}\right]^T(y-\hat{y}) \tag{II-2}$$

$$F_3=F_3^T=2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}\right]^T y \tag{II-3}$$

where, $e:=x-\hat{x}$, $B^TP=C$
and $C(x-\hat{x})=y-\hat{y}$

After substituting (II-1), (II-2) and (II-3) into (5-6), we divide (5-10) into three parts with respect to a_i, b_i and k in order to obtain adaptive control law.

With respect to a_i,

$$2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}\right]^T(y-\hat{y})+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T=0$$

$$\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T=-2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{a}_i^T\hat{x}\right]^T(y-\hat{y})$$

$$\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T=-\alpha_1(y-\hat{y})\sum_{i=1}^{r}h_i(\hat{x})\hat{x}^T\tilde{a}_i$$

$$\dot{\tilde{a}}_i^T=-\alpha_1(y-\hat{y})h_i(\hat{x})\hat{x}^T \tag{II-4}$$

With respect to b_i,

$$2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}\right]^T y+2\left[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}\right]^T(y-\hat{y})$$
$$+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T=0$$

$$\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T=-2[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}]^Ty$$
$$-2[\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_ik^T\hat{x}]^T(y-\hat{y})$$

$$\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T=-\alpha(2y-\hat{y})\sum_{i=1}^{r}h_i(\hat{x})\hat{x}^Tk\tilde{b}_i^T$$

$$\dot{\tilde{b}}_i=-\alpha(2y-\hat{y})h_i(\hat{x})\hat{x}^Tk \tag{II-5}$$

With respect to k,

$$2x^TPB\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}+2\dot{k}^T\Lambda^{-1}k=0$$
$$2\dot{k}^T\Lambda^{-1}k=-2x^TPB\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}$$
$$=-2\left[\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}\right]^TB^TPx$$
$$=-2\left[\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}\right]^Ty$$

$$\dot{k}^T\Lambda^{-1}k=-y\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_ik^T\hat{x}$$
$$=-y\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_i\hat{x}^Tk$$

$$\dot{k}^T=-y\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_i\hat{x}^T\Lambda \tag{II-6}$$

Since from (4-1-b) $\dot{\tilde{a}}_i=-\dot{\hat{a}}_i$ and $\dot{\tilde{b}}_i=-\dot{\hat{b}}_i$, and only $\hat{a}_i$ and $\hat{b}_i$ are available, (II-4), (II-5) and (II-6) should be modified into the following equations and they are the adaptive control law.

$$\dot{\hat{a}}_i^T=\alpha_1(y-\hat{y})h_i(\hat{x})\hat{x}^T \tag{5-11-a}$$

$$\dot{\hat{b}}_i=\alpha(2y-\hat{y})h_i(\hat{x})\hat{x}^Tk \tag{5-11-b}$$

$$\dot{k}^T=-y\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_i\hat{x}^T\Lambda \tag{5-11-c}$$

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Biography

Chang-Hwan Ahn

Chang-Hwan Ahn was born in Jeon-buk in Korea, on November 4, 1959. He received the B.Eng. degree from Wonkwang University in 1983 and M.Eng., Ph.D. degrees from the Inha University, Korea in 1991, 1999, respectively. From 1987 to 2005, he has been worked in KEPCO. Since 2005, he has been a Professor at Department of Digital Electronics, Inha Technical College. His research interests are in the area of Fuzzy Observer and Controller, lightning discharge phenomena, EMI/EMC.