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Assessing Cognitive Attributes in the 8th grade Geometry

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This study identified what cognitive attributes are required of eighth graders to solve geometrical problems such as 'Recall,' 'Analyze,' 'Justify,' 'Synthesize/Integrate,' and 'Solve Non-routine Problems' by using the cognitive diagnostic theory. The five attributes are proved as the skills for solving the geometric problems. Many students have not fully mastered the attributes of 'Justify' and 'Synthesize/Integrate'. There was high correlation between these attributes. 'Analyze' best predicted the changes in the geometric achievement. And while students with high levels of geometrical achievement have mastered all the five attributes, those in the mid- and low-level range of performance have mastered fewer attributes.

I. Introduction

Van Hiele (1986) argued that there are different levels through which students learn geometry. Students usually start by visually cognizing shapes. They then go on to analyze and explain the attributes of these shapes. Next, they recognize the relations among different shapes, and come to infer the attributes of shapes deductively. Korean curriculum evolves around this theory of different levels of geometrical learning and organizes the process of learning from the elementary through the middle schools in a hierarchical manner accordingly. Geometry at the deductive level, however, is quite difficult for students to grasp. There is still an ongoing dispute as to whether it is appropriate to introduce eighth graders to deductive geometry.

New Korean curriculum in 2007, therefore, now seeks to reduce the burden of proof on students, sufficing it to understand mere attributes of shapes in eighth grade. Aside from those required to carrying out the task of proving, what other cognitive abilities and skills do students need to learn geometry?

The stated objective of learning geometry in the eighth grade under the 7th curriculum is this: "Students must be able to prove simple attributes of triangles and quadrangles using the conditions of congruence applied to triangles." Not everything that students learn at school, however, involves the task of proving. Textbooks and instructions involved checking whether students fully understand and incorporate into their knowledge the attributes of triangles and quadrangles and evaluating whether students are able to solve problems within mathematical situations. In other

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words, middle-school geometry required students not only to prove, but also to recall their knowledge, analyze and synthesize the conditions given in questions, and solve non-routine problems.

It is true that what is assessed and how it is assessed provides the clearest indication of what is valued about formal education (Hamilton, 2003). Nevertheless, we do not have an accurate grasp of what abilities and skills other than those required for proving are being taught and evaluated in the eighth grade. Using various test items, this study therefore seeks to identify what skills and abilities Korean curriculum in geometry requires to students and to apply the cognitive diagnostic theory to measuring how well students master the attributes of geometry. Kim, Kim, and Song (2008) suggested that the cognitive diagnostic theory may prove to be useful in evaluating students' performance in mathematics. Their study, however, merely hinted at how the findings relating to the cognitive diagnostic theory might be put to use. Kim, Song and Kim (2008) explored the evolution of the Q-Matrix using the fusion model. But this study seeks to identify what cognitive attributes that school tests assess and how students perform in the area of geometry only. The objectives of this study can be stated in the following three questions:

1) What cognitive attributes have students mastered in the area of geometry? What correlations exist among these attributes?
2) What is the relationship between students' cognitive attributes and their performance?
3) How do the cognitive attributes that students have mastered differ by achievement level?

Ⅱ. Theoretical Background

1. Cognitive Abilities and Skills in Geometry

Tall, Gray, Ali, Crowley, DeMarois, McGowen, Pitta, Pinto, Thomas and Yuso (2001) argue that, the higher the student's cognitive level, the more he is likely to attempt to prove an ideal or abstract object than a visible, physical object. And Hershkowitz (1990) argues that geometry can be taught in two ways: one, as consisting in exploration into space; the other, as consisting of logical structures. In other words, to study geometry requires an ability to recognize visual-spatial objects as well as an ability to infer logically the relationship among such objects. Therefore, geometry requires an ability to analyze the characteristics of shapes and relations among them; an ability to synthesize the information gained from such analysis; an ability to recognize the shape in question and to recall relevant information; and an ability to justify logically inferences drawn.

NCTM(2000) says that instructional programs from prekindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. In grades 6 - 8 all students should -

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;
- create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

Students explore characteristics of shapes, infer and justify their inferences deductively and have to solve these problems within mathematical situations. For them to be able to do this, they must recall whatever relevant information they have gained since the elementary school. They must also analyze the relationship among the objects in question, analyze the shapes, and make valid inferences from them. They must draw connections among related mathematical concepts, determine the truth or falsity of the given claim, and solve mathematical problems in unfamiliar contexts.

These cognitive skills and abilities are measured not only in class, but by formal evaluations as well. This study will assess students' cognitive abilities and skills using questions for evaluation.

2. Cognitive Diagnostic Theory

Cognitive Diagnostic assessment has an advantage of evaluating examinees and test items cognitively (Hartz, Roussos, & Stout, 2002). Cognitive Diagnosis evaluates examinees with respect to their level of competence in each attribute. This evaluation can be done by giving individual feedback to examinee using the attributes measured by the assessment. An attribute is identified as a "task, subtask, cognitive process, or skill" involved in the assessment (Tatsuoka, 1995, p.330).

In Cognitive Diagnosis model, within one item, there can be several attributes measured. These attributes are described on Q-matrix, a $K \times n$ matrix containing ones and zeros, where K indicates the number of attributes we wish to assess and n indicates the number of items on the test(Tatsuoka, 1983, 1990, 1995).

Among the models of the Cognitive Diagnosis approach, the Fusion Model, developed by Hartz, Roussos, & Stout (2002), is considered to be successful because it satisfies three conditions for an effective model. Three conditions are like below.

- The model should give an estimation of examinee attributes.
- The model should relate items to attributes.
- The model should provide statistical identification of the model's parameters.

Many of the Cognitive Diagnosis models have a problem of statistical identifiability, however, the Fusion Model solves the problem of statistical identifiability by reducing the number of parameters involved in the modeling (Hartz et. al, 2002).

The Cognitive Diagnosis models based on IRT define the probability of observing the response of examinee j to item i given the examinee's ability parameters and item parameters. The equation of Fusion model is like this:

$$P(X_{ij} = x \mid \alpha_j, \eta_j) = \pi_i^* \prod_{k=1}^{K} (r_{ik}^{*\,(1-\alpha_{jk})q_{ik}}) P_{c_i}(\eta_j)$$

In above equation, q_{ik} is the attribute k which is measured by item i. If α_{jk}=1, it indicates that examinee j has mastered attribute k, and 0 otherwise. The symbol $X_{ij} = x$ indicates the response of examinee j to item i, where x = 1 indicates a correct response and x = 0 indicates an incorrect response. The parameter π_i^* is the probability of correctly applying all item i required attributes, given α_{jk}=1. It can be explained as the probability of an examinee who has mastered all attributes for item i can correctly apply those attributes when solving for item i. It can be interpreted as the Q-based item i

difficulty (Hartz et al., 2002). The parameter r^*_{ik} is the proportional parameter representing the ratio of the likelihood of a correct answer given mastery versus non-mastery (Hartz et.al, 2002). It can be interpreted as the item i discrimination parameter for attribute k (Hartz, 2002; Hartz et al., 2002). $P_{c_i}(\eta_j)$ is the probability of applying the skills correctly, especially the skills not specified by the Q-matrix.

Ⅲ. Method

1. test instrument

The test items chosen for this study are examinations written by eighth graders at M Middle School in Seoul. The sample is the normal student in Korea, and the examination at the school is ordinary test. As this study is for the use in school, I selected this sample. Of the questions on the mid-term and final examinations given in the second semester in M middle school, this study analyzed students' answers to the seventeen multiple-choice questions involving geometry. While these tests combined multiple-choice and constructed-response items, this study focused on the multiple-choice items only since the level of cognitive skills and abilities each question requires could be differentiated (Thissen, Wainer & Wang, 1993). The items included in the test have been developed and reviewed by three mathematics teachers.

2. Identifying Attributes of items

Each question included as a test item contains attributes necessary to solve it. The types of behavior that are necessary to solve mathematical problems and also mathematically meaningful were identified as attributes to be analyzed. To this end, this study selected sub-level factors from the cognitive domain of assessment framework of TIMSS 2007 (Mullis et al., 2005) as the attributes to be analyzed. The TIMSS mathematics framework contains specific factors of cognitive skills and abilities, enabling the identification of attributes. The framework indicates the area of cognitive skills and abilities to be considered in designing questions and can also be used as the basis to support which attributes each question has. The cognitive skills and abilities defined by the TIMSS consists of three areas: knowing, applying, and reasoning. Each area, in turn, consists of sub-level factors, as shown in <Table III-1>, that can be identified and tested. This study refers to these sub-level factors to identify and analyze attributes that are commonly tested by the questions on the tests.

<Table Ⅲ-1> TIMSS 2007 cognitive framework

Cognitive domain	sub-level factors
Knowing	Recall Recognize Compute Retrieve Measure Classify/ Order
Applying	Select Represent Model Implement Solve routine problems
Reasoning	Analyze Generalize Synthesize/integrate Justify Solve non-routine problems

Of the sub-level factors in the cognitive domains shown by <Table III-1>, this study chose 'Recall,' 'Analyze,' 'Justify,' 'Synthesize/Integrate,' and 'Solve Non-routine Problems' as the attributes tested by the questions included in the test items. I assigned 16 sub-level factors to the items in test instrument of this study plausibly, and then reduced the number of attributes so that it satisfied to the condition of Fusion Model. At last I decided the 5 attributes which is confirmed by mathematics teachers. Each attribute is described within the TIMSS framework as follows:

- Recall: Recall definitions, terminology, number properties, geometric properties, and notation.
- Analyze: Determine and describe or use relationships between variables or objects in mathematical situations; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three dimensional figures; and make valid inferences from given information.
- Justify: Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.
- Synthesize/integrate: Combine (various) mathematical procedures to establish results, and combine results to produce a further result. Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.
- Solve non-routine problems: Solve problems set in mathematical or real life contexts where target students are unlikely to have encountered closely similar items, and apply mathematical procedures in unfamiliar or complex contexts. Use geometric properties to solve non-routine problems.

3. Creating Q-Matrix

In order to use the fusion model, it is necessary to create Q-Matrix to connect each question to the attributes to be explained. To create the Q-Matrix, I first ascribed attributes to the questions. The questions at first had the 'Solve Routine Problems' and 'Recognize' attributes, but not the 'Justify' attribute. The fusion model, however, showed that these questions did not adequately reflect the six chosen attributes. After consulting the mathematics teachers, we excluded the 'Solve Routine Problems' and 'Recognize' attributes from the questions, adding the 'Justify' attribute instead and modifying the connections between questions and the attributes.

The examples of attributes connected to the test items are as follows.

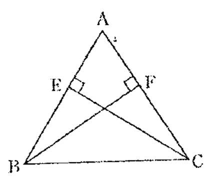
1. 다음 주어진 명제 중 그 역이 참인 명제는?

① 정삼각형은 이등변삼각형이다.
② 12의 약수는 4의 약수이다.
③ $x=3$이면 $2x-6>0$이다.
④ $a>b$이면 $ac>bc$이다.
⑤ $a=b$이면 $ac=bc$이다.

[Figure Ⅲ-1] Item 1

Item 1, for example, is a question that requires the attributes of 'Recall' and 'Justify.' Students are asked to justify which of the reverse statement is true. In doing so, they must recall the definition and characteristics of each object.

다음 그림과 같이 「$\overline{AB}=\overline{AC}$인 △ABC의 두 점 B, C에서 각각 $\overline{AC}$, $\overline{AB}$에 내린 수선의 길이는 같다.」임을 증명할 때, 다음 중 필요하지 않은 것은?

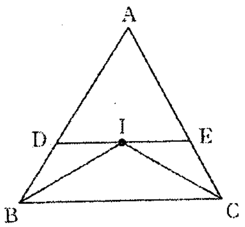


① $\overline{AB}=\overline{AC}$　② $\angle AFB=\angle AEC$
③ $\overline{AF}=\overline{AE}$　④ $\angle A$는 공통
⑤ $\triangle ABF\equiv\triangle ACE$

[Figure Ⅲ-2] Item 2

Item 2 calls for the attributes of 'Justify' and 'Synthesize/Integrate.' In the given figure of Triangle ABC, students should be able to find Triangles ABF and ACE. Next, reflecting on the conditions of triangular congruence, they must infer from the given conditions that the two given triangles are in fact congruent. In other words, they must be able to find and integrate multiple bits and pieces of information to obtain an answer. This process is to prove that Line BF and Line CE are of the same length, and hence, should be understood in the context of justification.

다음 그림에서 점 I는 $\overline{AB}=\overline{AC}$인 $\triangle ABC$의 내심이다. $\overline{DE}//\overline{BC}$이고, $\triangle ADE$의 둘레의 길이가 12cm일 때, $\overline{AB}$의 길이는?



① 3cm ② 4cm ③ 5cm ④ 6cm ⑤ 7cm

[Figure III-3] Item 3

Item 3 requires the attribute of 'Solve Non-routine Problems.' In addition, students must also be able to 'analyze' DI=DB.

<Table III-2> shows the Q-Matrix created to connect the 17 questions to 5 cognitive attributes.

4. Performing the Test

The eighth graders at M Middle School in Seoul participated in the tests. Of the 466 students in 14 classes, 239 were male and 227 were female. Part of the tests was conducted in October, 2008, as part of the mid-term examination; the rest were conducted in December, 2008, as part of the final examination.

5. Method of Analysis

The item parameters and student parameters of Fusion Model were estimated by using MCMC (Markov Chain Monte Carlo) estimation method of Arpeggio program(Hartz, Roussos, & Stout, 2002).

A frequency analysis was conducted to determine what attributes students have mastered. The inter-attribute correlations were determined using Pearson's correlation coefficient as applied to ppm, representing the probability each student had in mastering each given attribute.

To determine the relationship between each attribute and students' performance, each attribute was treated as an independent variable and the overall score as a dependent variable. A regression analysis involving these variables was run then.

To determine how students differed from one another in terms of performance, the proportions of attributes mastered by the high-level, mid-level, and low-level students were calculated respectively.

IV. Results

1. Cognitive attributes

Here we analyzed how the cognitive attributes and the students' cognitive skills are distributed.

A. Distribution of Masters

The average score for the 17 questions was 12.64, with a standard deviation of 3.75. <Table IV-1> shows the levels of cognitive skills mastered by students and the corresponding numbers and ratios of students. 76.6% of the students have mastered the 'Recall' attribute; 77.5%, the 'Analyze' attribute; 41.4%, the 'Synthesize/Integrate' attribute; 54.5%, the 'Justify' attribute; and 72.1%, the 'Solve Non-routine Problems' attribute. In the descending order of how well each attribute is mastered by students: 'Analyze,' 'Recall,' 'Solve Non-routine Problems,' 'Synthesize/Integrate,' and 'Justify'.

<Table Ⅳ-1>
Number of Students that mastered each attribute

attribute	number of masters (%)
Recall	357 (76.6)
Analyze	361 (77.5)
Synthesize/integrate	193 (41.4)
Justify	254 (54.5)
Solve non-problems	336 (72.1)

<Table Ⅲ-2> Q-matrix

item	Recall	analyze	justify	synthesize /integrate	solve non-routine problems	# of attributes
1	1		1			2
2			1	1		2
3		1			1	2
4		1	1			3
5		1		1		2
6		1		1	1	3
7	1	1		1		3
8	1			1		2
9		1	1		1	3
10	1	1				2
11		1	1			2
12		1	1		1	3
13	1	1		1		3
14		1	1			2
15		1	1			2
16		1	1			2
17		1		1		2
Total	5	14	9	7	4	40

<Table IV-2> shows the parameter of each question in the fusion model involving the 5 attributes. In the MCMC model, to decide whether or not a test item should be included requires the autocorrelation of the estimated parameter. Because the item parameter of each question chosen for this study and the student parameter of each student showed an 0.2 or less, the questions were included as test items. The relatively high values of the estimated item parameters π_i^*(ranging between 0.79 and 0.99) indicate that each question strongly calls for the mathematical attributes indicated by the Q-Matrix. Most of the estimated student parameters r_{ik}^* ranged from 0.34 to 0.99, showing that each question was clearly linked to each attribute to be represented by it. In other words, estimating the parameter of each question shows that the questions included in this study require and well differentiate the six attributes.

B. Correlation of attributes

<Table IV-3> shows the correlations among the cognitive attributes required for geometry. All the five cognitive attributes were correlated to one another, within the significance level of .01.

<Table Ⅳ-3> Correlation between attributes

	Recall	Analyze	Justify	Synthesize /integrate
Analyze	.785**	1		
Synthesize /integrate	.601**	.590**	1	
Justify	.670**	.650**	.799**	1
Solve non-routine problems	.766**	.821**	.787**	.743**

* p<.05, ** p<.01, *** p<.001

<Table Ⅳ-2> estimate value of item parameter and student parameter

item	π_i^*	r_{ik}^*				
		Recall	analyze	justify	synthesize /integrate	solve non-problems
1	0.89	0.68		0.36		
2	0.81			0.57	0.70	
3	0.98		0.67			0.80
4	0.98		0.67	0.94		
5	0.86		0.43		0.31	
6	0.98		0.45		0.82	0.49
7	0.97	0.49	0.77		0.91	
8	0.99	0.73			0.99	
9	0.99		0.89	0.99		0.97
10	0.98	0.36	0.47			
11	0.99		0.72	0.98		
12	0.99		0.83	0.94		0.91
13	0.97	0.89	0.49		0.92	
14	0.79		0.47	0.57		
15	0.88		0.62	0.66		
16	0.99		0.46	0.68		
17	0.79		0.41		0.34	

The two attributes with the strongest correlation were 'Analyze' and 'Solve Non-routine Problems' (.836). The ability to decompose geometric figures and make valid inferences was closely related to the ability to find answers to non-routine problems. In other words, both attributes belonged to the 'reasoning' area within the TIMSS 2007 framework. The two attributes with low correlation were 'Recall' and 'Synthesize/Integrate'(.601). The ability to recall information from past memories was not so much relatively related to the ability to synthesize and integrate multiple pieces of information.

2. Mathematical Achievement and Cognitive Attributes

In this section we seek to determine the relationship between the cognitive attributes required by geometry and the overall mathematical achievement. In order to do so, this study investigated how having each attribute could predict the overall mathematical achievement, especially in the area of geometry. To this end, a regression analysis was run to see how knowing the values on the independent variable can help predict the changes in the values on the dependent variable. In this case, the dependent variable is the sum of the scores (representing mathematical achievement) and the independent variable is the probability of mastering (ppm) the five cognitive attributes. By stepwise method five models were yielded to be analyzed. The fifth model with the strongest explicatory power contains all the five cognitive attributes. In this model, there was 95.7% chance of explaining the changes in mathematical performance with the coefficient of determination.

According to <Table IV-4>, for the 'Analyze' attribute, the coefficient of regression is 4.105; the F-value is 22.222; and the significance probability is .000. In other words, the 'Analyze' attribute, at the significance level of .001, has a significant effect on mathematical (geometrical) achievement. For the 'Justify' attribute, the coefficient of regression is 2.598; the F-value is 14.901; and the significance probability is .000. The attribute, at the significance level of .01, also affects mathematical (geometrical) achievement. For the 'Synthesize/Integrate' attribute, the coefficient of regression is 2.404; the F-value is 15.846; and the significance probability is .000. This attribute, at the significance level of .001, also has a significant effect on mathematical (geometrical) performance. For the 'Recall' attribute, the coefficient of regression is 1.334; the F-value is 7.174; and the significance probability is .000. At the significance level of .001, 'Recall' also significantly affects mathematical (geometrical) performance. For the 'Solve Non-routine Problems' attribute, the coefficient of regression is .813; the F-value is 2.974; and the probability significance is .000. At the significance level of .001, the attribute also significantly affects mathematical (geometrical) achievement. For the standardized coefficient of

<Table IV-4> Regression results

모형	B	standard error	β	F	p	R^2
(constant)	4.543	.097		47.046	.000	.967
Analyze	3.375	.124	.371	27.172	.000	
Recognize	3.310	.176	.291	18.829	.000	
Synthesize /integrate	1.410	.127	.151	11.135	.000	
Recall	1.146	.125	.121	9.154	.000	
Justify	1.386	.228	.109	6.074	.000	
Solve non routine problems	1.222	.212	.099	5.774	.000	

regression of the ppm of each cognitive attribute: 'Analyze' .405; 'Justify' .273; 'Synthesize/Integrate' .253; 'Recall' .120; and 'Solve Non-routine Problems' .064. In other words, the higher the standardized coefficient of regression of a given attribute, the larger the effect of the attribute on mathematical achievement.

3. Cognitive Skills and Abilities by Level of Geometrical Achievement

This study also analyzed what cognitive skills and abilities students have at different levels of geometrical performance. This process helps identify what cognitive skills and abilities are lacking in students with low-level geometrical performance. Because achievement tends to be measured by how well students do or score on given tests, this study used students' scores on the tests as the measure of their performance levels. The top 33% of students are classified as the 'high-level performance' group, while the bottom 33% of students are classified as the 'low-level performance' group. 29.4% of the total students, or the 137 students who scored 16 points or higher on the tests were assigned to the high-level performance group; the next 42.5% of the total students, or the 198 students who scored between 11 and 15 points, were assigned to the mid-level performance group; and the next 28.1% of the total students, or the 131 students who scored 10 points or lower were assigned to the low-level performance group.

<Table IV-5> lists the number of students by achievement level who have mastered the cognitive attributes.

<Table Ⅳ-5>
Distribution of masters by achievement level

attribute	High (%)	Mid (%)	Low (%)	Total (%)
Recall	137(100.0)	187(91.9)	38(29.0)	357(76.6)
Analyze	137(100.0)	192(97.0)	32(24.4)	361(77.5)
Justify	137(100.0)	114(57.6)	3(2.3)	193(41.4)
Synthesize/integrate	137(100.0)	56(28.3)	0(0.0)	254(54.5)
Solve non routine problems	137(100.0)	176(88.9)	23(17.6)	336(72.1)

The high-level students have mastered all the five cognitive attributes.

In the mid-level group, 57.6% of students have mastered the 'Justify' attribute; 91.9%, the 'Recall' attribute; 97.0%, the 'Analyze' attribute; and 28.3%, the 'Synthesize/Integrate' attribute. In other words, they lag significantly behind the high-level students in terms of the attributes of 'Justify' and 'Synthesize/Integrate.' In particular, the 'Synthesize/Integrate' attribute tends to affect achievement most. Since they have not sufficiently mastered this attribute, these students fall into the mid-level performance group.

Low-level students have mastered 2.3% of the 'Justify' attribute; 17.6% of the 'Solve Non-routine Problems' attributes; 29.0% of the 'Recall' attribute; 24.4% of the 'Analyze' attribute; and 0% of the 'Synthesize/Integrate' attribute. These students have never mastered the 'Synthesize/Integrate'.

<Table Ⅳ-6>
The number of attribute of each achievement level

# of attributes	High (%)	Mid (%)	Low (%)	Total (%)
0		1(0.5)	71(54.2)	72(15.5)
1		3(1.5)	31(23.7)	34(7.3)
2		24(12.1)	22(16.8)	46(9.9)
3		58(29.3)	7(5.3)	65(13.9)
4		65(32.8)	0(0.0)	65(13.9)
5	137(100.0)	47(23.7)	0(0.0)	184(39.5)

The numbers of attributes each group of students has mastered is listed in Table <IV-6>.

While the high-level students have mastered all the five attributes, the mid-level students differed widely, from those who have mastered none to those who have mastered all the five. The low-level students also differed widely in terms of distribution, from 54.2% of them who have not mastered any single attribute to those few who have mastered three of the attributes.

V. Conclusion and Discussion

This study identified and analyzed what cognitive attributes are required of eighth graders to solve geometrical problems. Mathematics requires diverse thinking skills, such as generalization, analogy, specialization, and so forth. But there has not been any study so far that sought to identify, through test questions, what cognitive skills and abilities are required by geometry. By determining the cognitive attributes required by each question and confirming which of these attributes are required by each question using the cognitive diagnostic theory, this study helps determine how these cognitive attributes affect students' performances in geometry. The cognitive attributes that geometry calls for are the attributes of 'Recall,' 'Analyze,' 'Justify,' 'Synthesize/Integrate,' and 'Solve Non-routine Problems.' Most students have not fully mastered the attributes of 'Justify' and 'Synthesize/Integrate,' suggesting that these two are the most difficult of the attributes to master. Specially the attribute most crucial to successfully solving geometrical problems is that of 'Justify'. Given the fact that it is also one of the attributes that students have least mastered, students are likely to be disadvantaged by geometrical problems requiring this particular attribute. While students with high levels of geometrical achievement have mastered all the five attributes, those in the mid- and low-level range of performance have mastered fewer attributes.

In South Korea, a great emphasis is on educational objective and fair evaluation, while the question of post-assessment educational solutions tends to garner little attention in terms of both research and practice. Assessments may be conducted for their stated objectives, but not many teachers and researchers know how to interpret the results of assessments and how to apply those results to improving the quality of teaching and learning. The application of results of assessments according to the cognitive diagnostic models, such as the one used in this study, in the area of teaching geometry will help students improve their academic performances by helping them recognize and understand what cognitive attributes they need to improve. Moreover, these models can help teachers assess their students better, especially in the area of geometry, identifying what areas to emphasize and establishing better teaching plans. If teachers can understand what attributes, skills, or abilities each question on each test seeks to measure and guide students' learning accordingly, assessments will help improve the quality of education at schools. This study is significant not only because it represents the attempt to apply the cognitive diagnostic theory to mathematical contexts, but also because it shows how the theory may be applied to teaching geometry.

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중학교 2학년 기하에서의 인지 속성 평가

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본 연구는 중학교 2학년 기하 문제를 해결하는 데 필요한 속성이 무엇인지를 확인하고, 학생들이 그러한 속성을 얼마나 숙달하고 있는지를 분석하였다. 중학교 2학년 기하 영역의 선다형 문항은 회상하기, 분석하기, 정당화하기, 종합하기, 비정형 문제해결의 5가지 속성을 요구하고 있었으며, 이것은 수학 교사들의 내용적 판단뿐 아니라 인지진단이론의 모수에 의해서도 확인되었다.

학생들은 정당화하기와 종합하기의 속성을 많이 숙달하지 못한 편으로 나타났다. 5가지 속성은 서로 높은 상관관계가 있었으며, 회귀분석 결과 분석하기가 기하 성취도 변화를 가장 잘 예측하는 변수였다. 성취수준별로 숙달한 속성의 수는 달랐는데, 중 수준 학생들은 상 수준과 비교하여 정당화하기, 비정형 문제해결의 숙달 비율이 낮았으며, 하 수준 학생들은 종합하기나 정당화하기의 속성을 거의 숙달하지 못했고 회상하기, 분석하기, 비정형 문제해결의 속성 또한 30% 미만의 학생들이 숙달하고 있었다. 이 결과는 개인에 따라 다른 정보를 제공하고 학생 개개인의 강점과 약점을 산출해준다는 점에서 학생 평가에 유용하게 활용될 수 있다.

* **key words:** 기하(geometry), 인지 속성(cognitive attribute), 인지진단이론(cognitive diagnostic theory)

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