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Computer Aided Teaching of Structural Engineering Using Adaptive Schemes in the Finite Element Method

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Abstract

A simple outline for teaching adaptive scheme based finite element method for planar problems as a part of computer aided teaching of structural engineering curriculum is presented. Displacement based finite element formulation for planar problems and representative strain value based adaptive scheme for mesh generation are considered. As examples, a cantilever beam with a concentrated load treated as a planar problem and stretching of a plate with a circular hole are analyzed with displacement based finite element method with adaptive meshes. The examples and outlines show how adaptive based finite element method may become an essential part of computer aided teaching of structural engineering.

Key words **:** Computer aided teaching, Structural engineering, Adaptive schemes, Finite element method

요 지

본 연구는 적응적 요소망을 사용한 유한요소법으로 해석된 2차원 문제를 전산을 활용한 구조공학 교육의 중요한 부분으로 활용하는 대요를 제시한다. 적응적 요소망 형성방법에서는 비교적 간단한 요소의 대표 변형률 값을 사용하였고 유한요소법으로는 광범위하게 활용되고 있는 변위법을 사용하였다. 적용한 예제는 기본 역학의 대표적인 문제인 집중 하중을 받는 캔틸레버보와 중앙에 원형 구멍을 갖은 평판 인장 문제이다. 교육 대요와 예제는 적응적 요소망을 사용한 유한요소법이 전산을 활용한 구조공학 교육에 중요한 부분이 될 수 있는 것을 보여준다.

핵심용어 : 전산 활용 교육, 구조 공학, 적응적 전략, 유한요소법

1. Introduction

Computers are now an essential tool in structural engineering practice and they are also used widely in structural engineering education. In practice, various aspects of computers are utilized, from the initial conception and throughout the analysis, design, construction and maintenance of structures during the entire life cycle. This paper deals with using computers in the analysis of structures, and specifically in the use of computers in the structural engineering education. Historically, structural engineering has been taught as two separate subjects, structural analysis and structural design. But now the most important principle behind structural analysis education is the understanding the process of solution; for this, analysis should not be a separate subject but be part of an integrated program where in smaller scale it involves structural analysis and design, and in larger scale it is part of many aspects of engineering related to the life cycle of structures starting from the initial conception, design, construction and throughout the maintenance during the service time. Analysis in some part plays a major role, while in others, it may be only minor. Education of structural analysis as a part of civil engineering curriculum must consider this (Biggs, 1999; MacLeod, 2002; Rafiq and Easterbrook, 2005).

The finite element method is practically the most widely used computer based method of structural analysis (Belytschko et al., 1996; Logan, 1992; Zienkiewicz et al., 2005). Although comprehensively understanding the method before it can be properly used may be beyond the scope of undergraduate engineering program, a condensed introduction must be integrated into an undergraduate civil engineering curriculum. A set of outlines and studies for displacement based planar finite element formulation for stress analysis with a simple adaptive mesh generation scheme is appropriate for this. Essential concepts about the finite element method for this purpose are describe in this paper.

2. Structural analysis in structural Engineering Education

Learning about computers and use of computers in engineering are now an essential part of engineering education in an undergraduate curriculum. For civil engineering majors, a set of courses in structural analysis is at the core of structural

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engineering part of the program, where a well designed program integrates analysis with other parts such as design, materials, and project management.

Although practical computer based structural analysis generally involves analysis of continuum based on the finite element method, undergraduate students are often not even introduced to the method mainly because there are no credit hours available for the subject and also because of the tendency to think that many credit hours are needed to properly introduce the subject. There are many other legitimate demands for credit hours in an engineering curriculum so if basic introduction of the finite element method is to be included, an effective but condensed program of study that can be covered in less than 3 credit hours during a semester need to designed. In the earlier part of the structural analysis education, students are generally taught mechanics of material, structural mechanics, and analysis of structures dealing with beams, trusses and frames. In the computer aided structural analysis of trusses and frames, structures are modeled as discrete member node systems and the direct stiffness method is covered(Gere, 2001; Logan, 1992; Norris et al., 1991; Zienkiewicz et al., 2005). Most structural analysis portion of civil engineering curriculum covers up to this. Teaching of the finite element method is either deferred to graduate school or only peripherally introduced because of the presumption that proper introduction of the method needs many credit hours and the presumption that a diverse set of prerequisite subjects such as numerical methods, computer basics and traditional structural analysis subjects are required.

However, practicing engineers mostly do not have graduate degrees and most are self taught or learn the subject after undergraduate education. Thus, condensed finite element structural analysis education to be integrated into a general structural engineering part of undergraduate civil engineering curriculum is needed. The outline described in the next section dealing with displacement based finite element analysis for planar problems with adaptive mesh schemes is to meet this purpose.

3. Finite element analyses with adaptive meshes for planar problems

The finite element method is now probably the most widely used computer based method of analysis commonly applied to simple and complex problems in stress analysis, heat flow, fluid dynamics, electromagnetic field analysis, and others. The popularity of the method is due to the ease of use and to the many reliable results the method yields for fairly large class of engineering problems (Belytschko et al., 1996; Logan, 1992; Zienkiewicz et al., 2005). A typical structural engineer's use of the finite element method generally does not involve programming anymore but usually involves running a completed finite element program for analysis of structures and interpretation of the results. In the following subsections, an outline for what needs to be taught about the finite element method as a part of civil engineering curriculum is described.

3.1 Learning displacement based finite element analysis

Proper use of a finite element program requires the knowledge about the finite element algorithm used in the program but not necessarily the details of the coding. The general knowledge here includes the following:

(1) mathematics behind finite element formulations;
(2) basics in numerical computations that cause error; and
(3) other fundamentals of computer use.

For structural engineers, the most commonly used finite element analysis is displacement based formulations for stress analysis. For this, students need to thoroughly understand the following basic aspects of the finite element method:

(1) shape functions that approximate actual variance of variables;
(2) element stiffness; and
(3) the direct stiffness method.

Input data for a typical finite element program include the following:

(1) geometric, material, and loading descriptions of the problem;
(2) choice of element type; and
(3) finite element mesh of the finite element model of structure.

Other than the mesh, preparing input data for finite element analysis does not require much knowledge about the finite element method and this has in many cases resulted in improper use and sometimes lead to the misconception that the method maybe used without the basic knowledge about the finite element method. An inappropriate mesh may render inaccurate and unreliable results. Thus a robust mesh is essential for a good finite element analysis; because of many possibilities of error, fine mesh everywhere is not a general solution. Generation of a good mesh requires basic understanding of the finite element method, theoretical and empirical experience in the field of application, and the comprehension of the limitations of the finite element code used. Generally an adaptive mesh generating scheme described in the next section alleviates this problem. Introduction of this to students has many benefits. Students learn the fundamentals of the finite element formulations and the basic concepts about error estimations. Error estimation methods generally deal in some form with energy in structural models and understanding of this is also a basic part of structural engineering education.

3.2 Adaptive mesh generation based on representative strain values

There are following types of adaptive mesh generation schemes:

(1) R type;
(2) H type;
(3) P type; and
(4) combinations of these.

The R type optimizes the positions of the nodes, the H type subdivides the original elements where the original boundaries of elements are kept, and the P type changes order of the degree of polynomial in the shape functions used in the element formulation. Practically, combinations of R and H type are efficient in optimizing meshes and programming for this is rather simple. This paper only considers the H type adaptive scheme for simplicity and for clarity in visualization; in addition, H type alone can essentially show the characteristics of error estimation and mesh refinement. The problem class considered is a plane stress problem, and the element type used is a four node bilinear isoparametric element with displacement based formulation (de Las Casas, 1988; Heesom and Mahdjoubi, 2001; Zienkiewcz and Zhu, 1987; Zhu et al., 1991).

A general adaptive scheme starts with an initial mesh, evaluates error using finite element analysis results, refines mesh with some strategy using error estimates, and iterates with convergence criteria. The initial mesh may be from an expert system or may be generated roughly with some common domain knowledge. Note that in H type, the boundaries of the initial mesh remain till the end. The types of error in finite element analysis results include the following: round off error from limited number of significant digits in numerical computations, truncation error from the exclusion of higher order terms in the derivation of governing differential equations for computer coding, modeling error related to the validity of assumptions in the models, inherited error from the previous computations in an algorithm, data error in the input, and discretization error from finite element models. Many of these errors are unavoidable in a finite element analysis, thus at best. finite element analysis results are approximations. Accurate error estimation is inherently a difficult problem but in an adaptive scheme, accurate error evaluation is actually not needed; only good criterion for mesh refinement is required. For this, simple representative strains in elements often sufficiently serve this purpose (Choi and Yu. 1998; Ladeveze and Oden, 1998; McFee and Giannacopoulos, 2001; Ohnimus et al., 2001).

The adaptive mesh generation described uses representative strain values in each element as a basis for mesh refinement in the iterative process. These values are not even intended to evaluate error in the elements. In some elements, it indicates error, in others, it has nothing to do with error. In all cases, it is intended to provide a relative criterion for mesh refinement. The representative values are simple manipulations of strain values at the Gauss points in each element. These values are always computed in a displacement based finite element analysis (Jeong and Yoon, 2003; Jeong et al., 2003; Yoon and Jeong, 2005).

To compare the errors in various elements, norms of vector quantities are often used. The following shows a general energy norm of error in a finite element:

$$\|E\| = \left[\int_{\Omega}(\varepsilon-\bar{\varepsilon})^T D(\varepsilon-\bar{\varepsilon})d\Omega\right]^{1/2} \tag{1}$$

where, ε is the exact strain vector, and $\bar{\varepsilon}$ is the computed strain vector. The Ω represents the domain and D is the stiffness matrix. For a typical problem, ε is not known, and if it is known, there is no need for finite element analysis. Thus ε can not be used in the expression to evaluate error.

If the representative strain value is used in place of , Eq. (1) yields

$$\|E^*\| = \left[\int_{\Omega_i}(\varepsilon^*-\bar{\varepsilon})^T D(\varepsilon^*-\bar{\varepsilon})d\Omega\right]^{1/2} \tag{2}$$

Since the entire domain is the sum of all the subdomains of the finite elements, the error for the entire domain may be expressed as follows:

$$\|E\|^2 = \sum_{i=1}^{m}\|E^*\|_i^2 \tag{3}$$

where, m is the number of elements and i is the element number.

The expression in Eq. (3) may be used as the representative value for each element and this is used as the basis for mesh refinement. The advantage of using this expression is that practically no additional computation is necessary and manipulation of this expression can include many of the criteria for refinement such as strain gradient in each element.

For each element, the representative strain values are computed using the following equations.

$$\|e\|_{ix} = \frac{\sum_{i=1}^{n_{ex}}[(\varepsilon_{ix}-\varepsilon_x^*)^2]^{\frac{1}{2}}}{A_i} \tag{4}$$

$$\|e\|_{ix} = \frac{\sum_{i=1}^{n_{ex}}[(\varepsilon_{ix}-\varepsilon_x^*)^2]^{\frac{1}{2}}}{A_i} \tag{5}$$

$$\|e\|_{ix} = \frac{\sum_{i=1}^{n_{ex}}[(\varepsilon_{ix}-\varepsilon_x^*)^2]^{\frac{1}{2}}}{A_i} \tag{6}$$

where, $\|e\|_{ix}$ represent axial strain in the x direction, n_{gx} is number of Gauss points in local x direction, ε_{jx} is the x directional strain at j'th Gauss point, ε_x^* is the average x directional axial strain, and A_i is the area of the i'th element.

The symbols $\|e\|_{iy}$, n_{gy}, ε_{jy} and ε_y^* in Eq. (5) represent the similarly defined values in the y direction. The symbols $\|e\|_{ixy}$, n_{gxy}, ε_{jxy} and γ_{xy}^* in Eq. (6) represent the similarly defined shear strain values.

4. Case studies

Two classic and basic educational mechanics problems are described: a cantilever beam with a concentrated load and a uniform stretching of a plate with a hole problem. The two cases are analyzed using the educational outline of finite element analysis with representative strain based adaptive schemes described in the previous sections. The element is displacement based four node plane stress isoparametric element. In the evaluation of the element stiffness, four interior Gauss points are used. The strain values computed at these Gauss points are used to compute the representative strain values in the adaptive schemes for mesh refinement.

4.1 Cantilever beam case

Fig. 1 shows the cantilever beam case with a concentrated load at the free end. The modulus of elasticity is 2.1 × 10^6 kgf/cm^2, the Poisson's ratio is 0.3 and the thickness of the beam is 1 mm. The value of the concentrated load is 1 kgf. The initial mesh is shown in Fig. 2(a) and the representative strain values from finite element analysis results using the initial mesh are shown in Fig. 2(b). As shown, elements 1 and 2 have large representative strain values and these are further divided. Based on this, Fig. 3 shows the first and second progressively adaptive refined meshes. The successive adaptive meshes will show that the refined meshes converge to an optimal and desired mesh where more elements are concentrated at the fixed end where moment variance is high. In addition, analysis results including displacements, strains and stresses converge to analytical results commonly given in elasticity texts (Timoshenko and Goodier, 1970; Gere, 2001).

4.2 Plate with a circular hole case

Fig. 4 shows the plate with a circular hole case with a uniformly distributed load on each side. The modulus of elasticity is 2.1 × 10^6 kgf/cm^2, the Poisson's ratio is 0.3, and the thickness of the plate is 1 mm. The value of the uniformly distributed load is 1 kgf/cm. The initial mesh is shown in Fig. 5(a) where only the upper right quarter is modeled. The representative strain values from finite element analysis results using the initial mesh are shown in Fig. 5(b). As shown, elements 1 and 2 have large representative strain values and these are more finely divided. Based on this, Fig. 6 shows the first and second progressively adaptive refined meshes. Further refine-

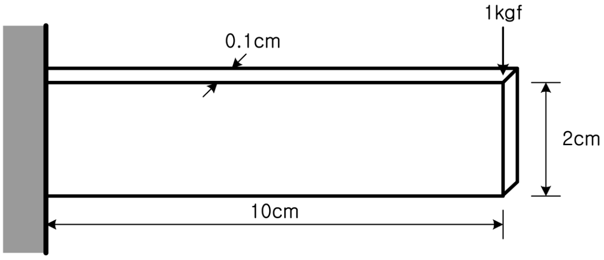
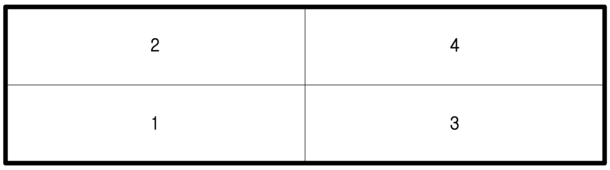
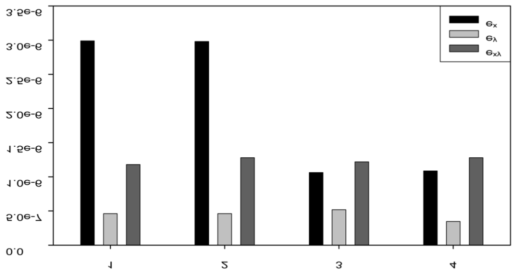


Fig 1. Cantilever beam case

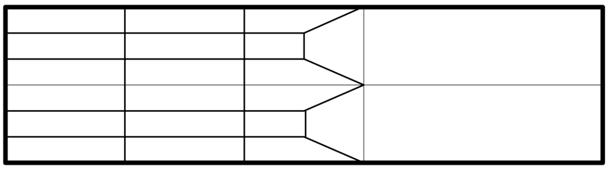


(a) Initial mesh

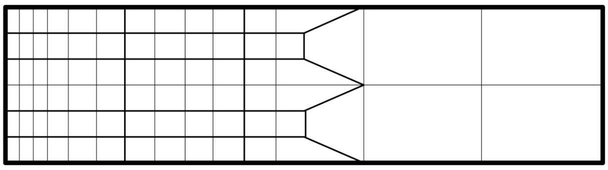


(b) Representative strains for each element in the initial mesh

Fig 2. Initial mesh and representative strains for cantilever beam case



(a) First adaptive mesh



(a) Second adaptive mesh

Fig 3. Progress of adaptive meshes for cantilever beam case

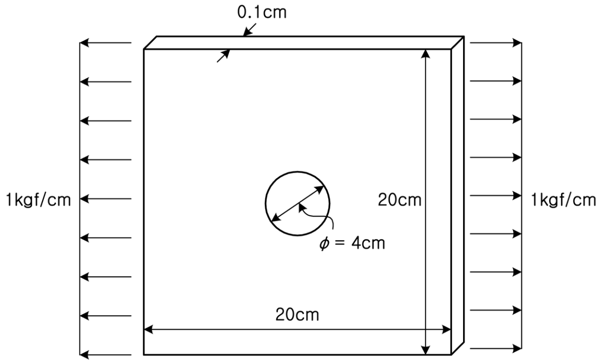
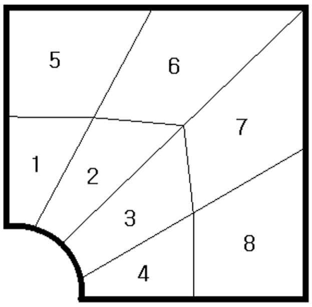


Fig 4. Plate with a hole case



(a) Initial mesh

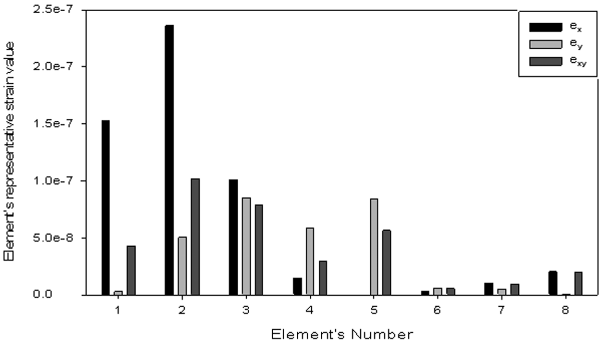
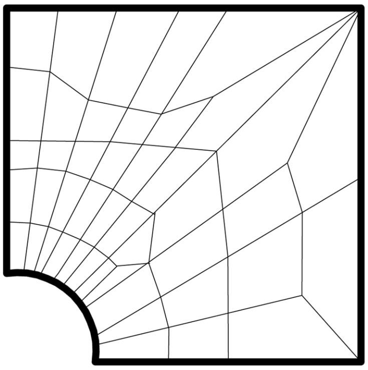
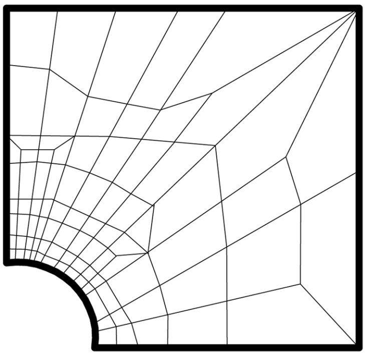


Fig. 5 Initial mesh and representative strains for plate with a hole case



(a) First adaptive mesh



(b) Second adaptive mesh

Fig. 6 Progress of adaptive meshes for plate with a hole case

ment will show convergence to an optimal and desired final mesh where analysis results including displacements, strains and stresses converges to analytical results commonly given in elasticity texts (Timoshenko and Goodier, 1970; Gere, 2001).

5. Conclusions

Structural engineering practice expects undergraduate civil engineering graduates to have basics in computer aided analysis of structures. In the core of computer aided analysis of structures is the finite element analysis. Generally industry does not have formal training program for this and the universities have been negligent in properly introducing to the students the basics of finite element analysis. The outlined guide and the two cases on the displacement based finite element analysis of planar problems with adaptive mesh generation schemes show how a condensed program of study can be formulated into a civil engineering curriculum. The outline covers the basic background knowledge for finite element analyses of structures. Students learn the basic finite element analysis procedure, behavior of analysis results, and how the results can change due to different meshes for the same problem. In addition, students learn fundamental applications of energy concepts and estimations of error, two concepts generally considered essential but difficult for novice structural engineers.

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