Performance Analysis of Selection Combining Technique for *M*PSK over Independent But Non-Identically Distributed Rayleigh Fading Channels

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ABSTRACT

This paper provides new exact-closed form expressions for average SER and average BER as well as outage probability for *M*-PSK signaling with selection combining over independent but non-identically distributed Rayleigh fading paths. The validity of these expressions is verified by the Monte-Carlo simulations. All of numerical results are in excellent agreement with simulation results.

Key Words : Selection Combining (SC), independent but non-identically distributed Rayleigh fading (i.n.d.), *M*-PSK, BER, SER, and Outage Probability

Ⅰ. Introduction

Diversity is an effective technique used in wireless communication systems to combat the performance degradation caused by fading. It can alleviate the deleterious effect of fading by means of multiple reception of the same information bearing signals. Receiving several replicas of the same signal requires some kind of combining techniques in order to obtain a single representation of the desired symbol. There are three common types of diversity combining techniques: selection combining (SC) which selects the signal with the largest instantaneous SNR from those diversity paths; equal-gain combining (EGC) which coherently combines all available paths weighting each with an equal gain; and maximal-ratio combining (MRC) which also coherently combines all available paths but weighs each with the respective gain of the path. Among them, SC gives the most inferior performance, MRC gives the best and the optimum performance, and EGC has a performance quality in between the others. SC and MRC are the two extremes of complexity quality tradeoff. Although optimum performance is highly desirable, practical systems often sacrifice some performance in order to reduce their complexity. Instead of using MRC which requires exact knowledge of the channel state information, a system may use SC which simply requires SNR measurements. This leads to a simple receiver structure that is hardware-feasible and cheaper to implement. Another benefit of using SC as opposed to MRC is reduced power consumption at the receiver.

In the early analyses of SC, the assumption was made that the fading on the *N* received paths is Rayleigh distributed and both independent and identically distributed (i.i.d.) from path to path. Under this assumption, the average symbol error rate (SER) has been investigated [1] as well as the average bit error rate (BER) for *M*-PSK modulation. However, in certain environments, it may be more appropriate to consider independent but non-identically distributed (i.n.d.) channels. There are two reasons for interest in the case of

※ 이 논문은 2007년도 정부(과학기술부)의 재원으로 한국과학재단의 지원을 받아 수행된 연구임(No. R01-2008-000-20400-0)
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논문번호 : KICS2008-10-476 접수일자 : 2008년 10월 11일, 최종게재논문통보일자 : 2008년 11월 06일

i.n.d. diversity channels. In the first place, the diversity branches in a practical system are frequently unbalanced because of differing noise figures, different feeder-line lengths, etc. In the second place, in cooperative communication systems, it is more appropriate if these systems are investigated under dissimilar channels [2-4]. To the best of our knowledge, there is no closed form of outage probability, average SER and average BER of SC for the case of i.n.d. diversity branches. Hence in this paper, we derive compact expressions for the probability density function (pdf) and the moment-generating function (MGF) of the selective combiner output which are then used to derive a single integral expression for SER of M-PSK. In addition, BER and the outage probability of the SC system are also evaluated in closed forms by exploiting the methods discussed in [1],[5].

The rest of this paper is organized as follows. In section II, we introduce the model under study. Section III shows the formulas allowing for evaluation of the average SNR, MGF, outage probability, average SER, and average BER. Section IV, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in Section V.

Ⅱ. System Model

We assume that there are N available diversity branches experiencing slow and frequency-non selective fading. Then the received baseband signal of the k-th diversity branch, $r_k(t)$, is given by:

$$r_k(t) = h_k e^{-j\phi_k} s(t) + n_k(t), k = 1,\ldots,N \tag{1}$$

where $s(t)$ is the complex baseband transmitted signal with average signal energy per symbol $2E_S$. h_k and ϕ_k are the amplitude and phase of the channel gain for the k-th diversity branch, respectively, and $n_k(t)$ is a zero-mean complex white Gaussian noise process with two-sided power spectral density (PSD) $2N_0$ [W/H]. It is assumed that the channel gains and the noise process are independent.

Let γ_k denote the instantaneous SNR per bit of k-th diversity branch defined by:

$$\gamma_k = h_k^2 E_b/N_0 \text{ and } \overline{\gamma_k} = \lambda_k E_b/N_0 \tag{2}$$

where $\overline{\gamma_k}$ is the mean SNR per bit of the k-th diversity branch, $\lambda_k = E[h_k^2]$ and $E_s/N_0 = \log_2(M) E_b/N_0$.

Ⅲ. Performance Analysis

3.1 pdf

With selection combining, the branch with the largest bit energy-to-noise ratio is always selected so the instantaneous bit energy-to-noise ratio at the output of selective combiner is

$$\gamma = \max\{\gamma_1, \cdots, \gamma_j, \cdots, \gamma_N\} \tag{3}$$

where N is the number of branches. If the branches are independently faded then order statistics gives the cumulative distribution function (CDF).

$$F_\gamma(\gamma) = \Pr(\gamma_1 \le \gamma, \cdots, \gamma_j \le \gamma, \cdots, \gamma_N \le \gamma) = \prod_{j=1}^{N} F_{\gamma_j}(\gamma) \tag{4}$$

where $F_{\gamma_j}(\gamma) = \Pr(\gamma_j \le \gamma)$ is the corresponding CDF of γ_j.

We know that when the strongest diversity branch is selected from a total N available i.n.d. diversity branches, the joint pdf of γ for N-branch SC is given by differentiating (4).

$$f_\gamma(\gamma) = \frac{\partial}{\partial\gamma} \prod_{j=1}^{N} F_{\gamma_j}(\gamma) \tag{5}$$

From (5), we have

$$f_\gamma(\gamma) = \sum_{j=1}^{N} f_{\gamma_j}(\gamma) \prod_{\substack{k=1 \\ k \ne j}}^{N} \Pr(\gamma_k \le \gamma) \tag{6}$$

where, for the Rayleigh fading channel case:

$$f_{\gamma_j}(\gamma) = \frac{1}{\overline{\gamma_j}} e^{-\gamma/\overline{\gamma_j}}, \quad \Pr(\gamma_k \le \gamma) = 1 - \exp(-\gamma/\overline{\gamma_k}) \tag{7}$$

Substituting (7) into (6), we obtain:

$$\begin{aligned} f_\gamma(\gamma) &= \sum_{j=1}^{N} \frac{1}{\overline{\gamma_j}} e^{-\gamma/\overline{\gamma_j}} \prod_{\substack{k=1 \\ k \ne j}}^{N} \left(1 - e^{-\gamma/\overline{\gamma_k}}\right) \\ &= \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} C_j e^{-C_j \gamma} \right] \end{aligned} \tag{8}$$

where $C_j = \sum_{k=1}^{j} \overline{\gamma_{i_k}^{-1}}$. The detail derivation is presented in [Appendix A].

3.2 MGF of output SNR

The MGF of γ is defined by [6]

$$M_\gamma(s) = E\{e^{s\gamma}\} = \int_0^\infty f_\gamma(\gamma) e^{s\gamma} d\gamma \tag{9}$$

Substituting (8) into (9), we get

$$M_\gamma(s) = \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} (1 - s C_j^{-1})^{-1} \right] \tag{10}$$

3.3 Average Output SNR

The average SNR per bit of the combiner output can be easily obtained from the first derivative of $M_\gamma(s)$ evaluated at $s=0$. Differentiating (10) with respect to s and evaluating the result at $s=0$, we obtain

$$\begin{aligned} \overline{\gamma_{SC}} &= \frac{dM_\gamma(s)}{ds}\Big|_{s=0} \\ &= \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} C_j^{-1} \right] \\ &= \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} \overline{\gamma_{i_k}^{-1}} \right] \end{aligned} \tag{11}$$

3.4 Outage Probability

The outage probability P_{out} is defined as the probability that the SC output SNR falls below a certain predetermined threshold SNR γ_{th} and hence can be obtained by integrating the pdf of γ.

$$P_{out} = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma \tag{12}$$

Solving (12) gives:

$$P_{out} = \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} (1 - e^{-C_j \gamma_{th}}) \right] \tag{13}$$

Note that for $\overline{\gamma_1} = \overline{\gamma_2} = \cdots = \overline{\gamma_N} = \overline{\gamma}$, (13) becomes:

$$P_{out} = (1 - e^{-\gamma_{th}/\overline{\gamma}})^N \tag{14}$$

which is in agreement with the previous known result [7, p. 326, eq. (6.58)] as expected.

3.5 Average Symbol Error Rate

For the case of coherently detected M-PSK, to evaluate the average SER with SC on i.n.d. Rayleigh fading, we merely replace γ_s with $\gamma \log_2 M$ in [8, eq. (5)], and use MGF-based approach which analogous to [5, p. 322, eq. (9.15)], namely

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\gamma \left(-\frac{g_{MPSK} \log_2 M}{\sin^2\theta} \right) d\theta \tag{15}$$

where $g_{MPSK} = \sin^2(\pi/M)$. Finally, substituting (10) in (15) gives the desired result:

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} \frac{\sin^2\theta}{\sin^2\theta + a_j} \right] d\theta \tag{16}$$

and it is proven in [Appendix B] where $a_j = g_{MPSK} C_j^{-1} \log_2 M$.

$$P_s = \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} \left(\frac{M-1}{M} \right) \times \left[1 - \sqrt{\frac{a_j}{1+a_j}} \left(\frac{M}{(M-1)\pi} \right) \times \left[\frac{\pi}{2} + \tan^{-1} \left(\sqrt{\frac{a_j}{1+a_j}} \cot\frac{\pi}{M} \right) \right] \right] \right] \tag{17}$$

In addition, P_s can be obtained by another approach [1], [see Appendix C].

$$\begin{aligned} P_s &= 1 - \Pr(\theta \in \Theta_C) \\ &= 1 - \sum_{j=1}^{N} \left[(-1)^{-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} I\left(\frac{\pi}{M}, -\frac{\pi}{M}; \frac{1}{C_j} \right) \right] \\ &= 1 - \sum_{j=1}^{N} \left[(-1)^{-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} 2I\left(\frac{\pi}{M}, 0; \frac{1}{C_j} \right) \right] \end{aligned} \tag{18}$$

where $\Pr(\theta \in \Theta_C) = \Pr(\theta \in [-\pi/M, \pi/M])$ is the probability of the phase angle θ lying in the decision region Θ_C corresponding to correct reception and

$$I\left(\frac{\pi}{M}, 0; \frac{1}{C_j} \right) = \frac{1}{2M} + \frac{1}{2}\beta_j \left[\frac{1}{2} + \frac{\tan^{-1}(\alpha_j)}{\pi} \right] \tag{19a}$$

$$\mu_j = \sqrt{\log_2(M)/C_j} \sin(\pi/M) \tag{19b}$$

$$\alpha_j = \sqrt{\log_2 M/C_j} \cos(\pi/M) / \sqrt{(\mu_j)^2 + 1} \tag{19c}$$

$$\beta_j = \mu_j / \sqrt{(\mu_j)^2 + 1} \tag{19d}$$

As a check, consider the dual diversity case with BPSK, i.e., $N=2$ and $M=2$. Then, from (17) & (18) we have

$$P_s = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma_1}}{1+\overline{\gamma_1}}} - \sqrt{\frac{\overline{\gamma_2}}{1+\overline{\gamma_2}}} + \sqrt{\frac{\overline{\gamma_1}\,\overline{\gamma_2}}{\overline{\gamma_1}+\overline{\gamma_2}+\overline{\gamma_1}\,\overline{\gamma_2}}}\right) \tag{20}$$

which agrees with [5, p. 413, eq. (9.265)].

3.6 Average Bit Error Rate

It is assumed that the bit-symbol mappings follow a Gray code. To obtain the BER of M-PSK with SC on i.n.d. Rayleigh fading channel, we proceed analogous to [1].

$$P_b = \frac{1}{\log_2 M}\sum_{m=1}^{M} e_m \Pr\{\theta \in \Theta_m\} \tag{21}$$

where $\Theta_m = [\theta_L^m, \theta_U^m] = [(2m-3)\pi/M, (2m-1)\pi/M]$ for $m = 1, \cdots, M$ and e_m is the number of bit errors in the decision region Θ_m [see Appendix D]. The probability $\Pr\{\theta \in \Theta_m\}$ is

$$\Pr\{\theta \in \Theta_m\} = \sum_{j=1}^{N}\left[(-1)^{j-1} \sum_{\substack{i_1, i_2, \cdots, i_j = 1 \\ i_1 < i_2 < \cdots < i_j}}^{N} I(\theta_U^m, \theta_L^m; C_j^{-1})\right] \tag{22}$$

where

$$I(\theta_U^m, \theta_L^m; C_j^{-1}) = \frac{\theta_U^m - \theta_L^m}{2\pi} + \frac{1}{2}\beta_U^m\left(\frac{1}{2} + \frac{\tan^{-1}(\alpha_U^m)}{\pi}\right) - \frac{1}{2}\beta_L^m\left(\frac{1}{2} + \frac{\tan^{-1}(\alpha_L^m)}{\pi}\right) \tag{23a}$$

and

$$\mu_U^m = \sqrt{\log_2 (M)/C_j}\,\sin(\theta_U^m) \tag{23a}$$

$$\mu_L^m = \sqrt{\log_2 (M)/C_j}\,\sin(\theta_L^m) \tag{23b}$$

$$\alpha_U^m = \sqrt{\log_2 M/C_j}\,\cos(\theta_U^m)/\sqrt{(\mu_U^m)^2 + 1} \tag{23c}$$

$$\alpha_L^m = \sqrt{\log_2 M/C_j}\,\cos(\theta_L^m)/\sqrt{(\mu_L^m)^2 + 1} \tag{23d}$$

$$\beta_U^m = \mu_U^m/\sqrt{(\mu_U^m)^2 + 1} \tag{23e}$$

$$\beta_L^m = \mu_L^m/\sqrt{(\mu_L^m)^2 + 1} \tag{23f}$$

The detail derivation is presented in [see Appendix C].

Ⅳ. Numerical Results

In this section, some examples of the average SER, BER and outage probability of SC for the case i.n.d. diversity branches in Rayleigh fading channel are given. Results computed using our theoretical analysis and Monte Carlo simulation are compared.

Fig. 1 shows the outage probability of SC versus the average branch SNR per bit with diversity branches of 3 for different values of thresholds $\gamma_{th} = 0, 5, 10$ dB. Under the assumption that the total average SNR for i.i.d. channels is equal to that for i.n.d. channels, i.e., $\lambda_1 = \lambda_2 = \lambda_3 = 1$ for i.i.d. channels and $\lambda_1 = 0.5$, $\lambda_2 = 1$, $\lambda_3 = 1.5$ for i.n.d. channels, we can see that the system under i.i.d case slightly outperforms that under i.n.d. case, and the performance difference between them is very small. The results exacted from Fig. 1 leads us to the conclusion that as similar to MRC, the SC is optimum in the sense of minimizing its performance (outage probability) if and only if the average channel powers that SC receives from each path are the same, namely $\lambda_1 = \lambda_2 = \cdots = \lambda_N$.. In addition, it is obvious that our analytical results and the simulation results are in excellent agreement.

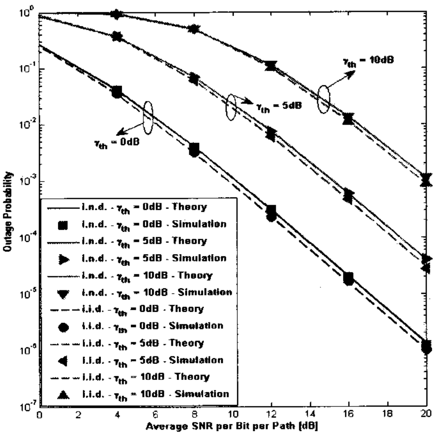

Fig. 1. Outage probability of SC with different thresholds $\gamma_{th} = 0, 5, 10$ dB and $N=3$.

From Fig. 2 to Fig. 7, for ease of analysis, it is assumed that λ_j with $j=1,\cdots,N$ are uniformly distributed between 0 and 1. Fig. 2 shows the outage probability versus the average branch SNR per bit with different diversity branches when the predetermined threshold γ_{th} is fixed at 0 dB. Clearly, these curses show that for a fixed value of *N*, a significant decrease in the outage probability is obtained as the predetermined threshold increases.

From Fig. 3 and Fig. 6, we study the average SER, BER performance for different levels of *M*-PSK modulation and different number of diversity branches. Note that with Gray code used for bit-symbol mappings, average BER of BPSK is same with that of QPSK. In addition, in Fig. 4 we compare two expressions for average SER derived by two approaches (eq. 17 and eq. 18). As expected, the results from (17) and (18) as well as from simulation are in excellent agreement.

In Fig. 7, the performance of BPSK with difference diversity combining techniques at the destination are illustrated. The BER curves confirm that, as expected, under same channel conditions, the performance of a system employing MRC receiver is always better as compared to an equivalent system using EGC or SC by around 1-3 dB. Moreover, increasing number of paths will

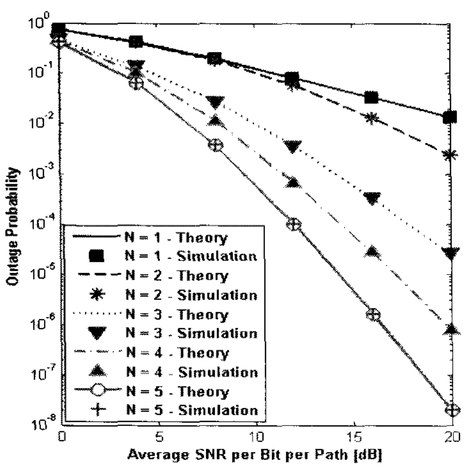

Fig. 2. Outage probability for different diversity branches with $\gamma_{th}=0$ dB

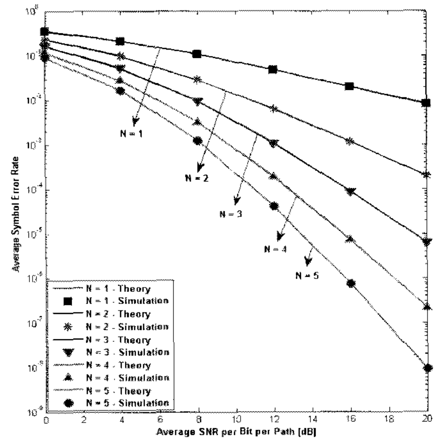

Fig. 3. Average SER of QPSK versus the average SNR per Bit per Path with different diversity branches.

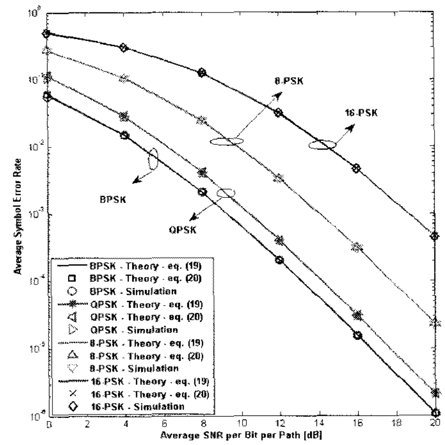

Fig. 4. Average SER of SC versus the average SNR per Bit per Path with different levels of *M*-PSK and N = 3.

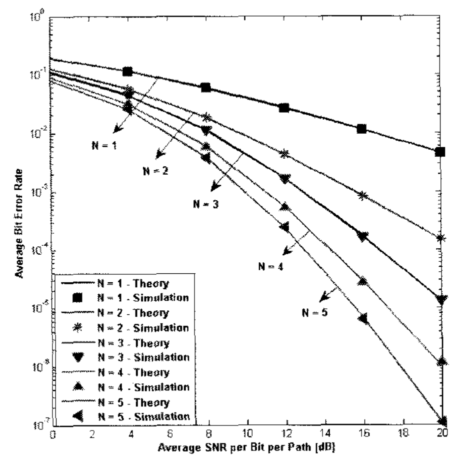

Fig. 5. Average BER of 8-PSK versus the average SN R per Bit per Path with different diversity branches.

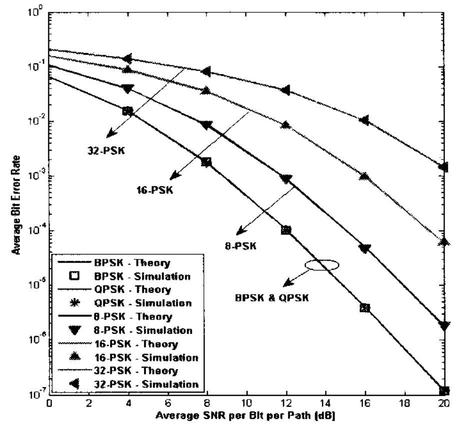

Fig. 6. Average BER of SC versus the average SNR per Bit per Path with different levels of M-PSK and N = 4.

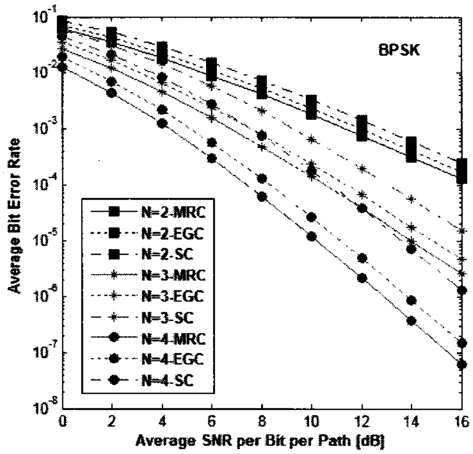

Fig. 7. Average BER of BPSK versus the average SNR per Bit per Path with different combining techniques.

increase the gaps between them. Then, SC can be viewed as a combining technique that trades performance for hardware-complexity reducion.

V. Conclusion

In this paper, the average BER, SER and outage probability of SC over slow and frequency - non selective fading channels of the case of i.n.d. diversity branches were analyzed. Simulations results are in excellent agreement with the derived expressions. The agreement with some known results validates the analysis. The expressions are general and offer a convenient way to evaluate any system which exploits SC technique.

Appendix A

PDF of SNR for the case i.n.d. diversity branches in Rayleigh fading channel.

$$\begin{aligned} f_\gamma(\gamma) &= \sum_{j=1}^{N} \frac{1}{\overline{\gamma_j}} e^{-\gamma/\overline{\gamma_j}} \prod_{\substack{k=1 \\ k\neq j}}^{N} \left(1-e^{-\gamma/\overline{\gamma_k}}\right) \\ &= \sum_{i_1=1}^{N} \overline{\gamma_{i_1}^{-1}} e^{-\gamma\overline{\gamma_{i_1}^{-1}}} + \cdots \\ &+ (-1)^{j-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} \left[\left(\sum_{k=1}^{j}\overline{\gamma_{i_k}^{-1}}\right) e^{-\gamma\left(\sum_{k=1}^{j}\overline{\gamma_{i_k}^{-1}}\right)}\right] + \cdots \\ &+ (-1)^{N-1} \sum_{\substack{i_1,i_2,\cdots,i_N=1 \\ i_1<i_2<\cdots<i_N}}^{N} \left[\left(\sum_{k=1}^{N}\overline{\gamma_{i_k}^{-1}}\right) e^{-\gamma\left(\sum_{k=1}^{N}\overline{\gamma_{i_k}^{-1}}\right)}\right] \end{aligned} \tag{A-1}$$

Let us define $C_j = \sum_{k=1}^{j} \overline{\gamma_{i_k}^{-1}}$, we can rewrite (A-1) as the following form.

$$f_\gamma(\gamma) = \sum_{j=1}^{N} \left[(-1)^{j-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} C_j e^{-C_j\gamma}\right] \tag{A-2}$$

For example with $N=2$:

$$\begin{aligned} f_\gamma(\gamma) &= \frac{1}{\overline{\gamma_1}} e^{-\gamma/\overline{\gamma_1}}\left(1-e^{-\gamma/\overline{\gamma_2}}\right) + \frac{1}{\overline{\gamma_2}} e^{-\gamma/\overline{\gamma_2}}\left(1-e^{-\gamma/\overline{\gamma_1}}\right) \\ &= \frac{1}{\overline{\gamma_1}} e^{-\gamma/\overline{\gamma_1}} + \frac{1}{\overline{\gamma_2}} e^{-\gamma/\overline{\gamma_2}} - \left(\frac{1}{\overline{\gamma_1}}+\frac{1}{\overline{\gamma_2}}\right) e^{-\gamma\left(\frac{1}{\overline{\gamma_1}}+\frac{1}{\overline{\gamma_2}}\right)} \\ &= \sum_{j=1}^{2} \left[(-1)^{j-1} \sum_{\substack{i_1,\cdots,i_j=1 \\ i_1<\cdots<i_j}}^{2} C_j e^{-\gamma C_j}\right] \end{aligned}$$

where: $C_1|_{i_1=1} = \overline{\gamma_1^{-1}}$ $C_1|_{i_1=2} = \overline{\gamma_2^{-1}}$ and $C_2|_{i_1=1, i_2=2} = \overline{\gamma_1^{-1}} + \overline{\gamma_2^{-1}}$.

Appendix B

$$\begin{aligned} P_s &= \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sum_{j=1}^{N}\left[(-1)^{-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} \frac{\sin^2\theta}{\sin^2\theta + a_j}\right] d\theta \\ &= \sum_{j=1}^{N}\left[(-1)^{-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} K_1\right] \end{aligned} \tag{B-1}$$

where K_1 is defined as follows:

$$K_1 = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{\sin^2\theta}{\sin^2\theta + a_j} d\theta \quad \text{(B-2)}$$
$$= \frac{1}{\pi}\int_0^{(M-1)\pi/M} \left(1 - \frac{a_j}{\sin^2\theta + a_j}\right) d\theta$$
$$= \frac{M-1}{M} - \frac{1}{\pi}K_2$$

where K_2 is written as

$$K_2 = \int_0^{(M-1)\pi/M} \frac{a_j}{a_j + \sin^2\theta} d\theta \quad \text{(B-3)}$$
$$= \int_0^{(M-1)\pi/M} \frac{a_j(1+\tan^2\theta)}{a_j(1+\tan^2\theta) + \sin^2\theta} d\theta$$

Changing the variable: $t = \tan\theta \Rightarrow d\theta = dt/(1+t^2)$ and rewriting (B-3), we get

$$K_2 = \frac{a_j}{1+a_j}\int_0^{\tan[(M-1)\pi/M]} \frac{dt}{t^2 + a_j/(1+a_j)} \quad \text{(B-4)}$$
$$= \sqrt{\frac{a_j}{1+a_j}} \tan^{-1}\left(\sqrt{\frac{1+a_j}{a_j}} \tan\frac{(M-1)\pi}{M}\right)$$
$$= \sqrt{\frac{a_j}{1+a_j}} \left[\frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{a_j}{1+a_j}} \cot\frac{(M-1)\pi}{M}\right)\right]$$
$$= \sqrt{\frac{a_j}{1+a_j}} \left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{a_j}{1+a_j}} \cot\frac{\pi}{M}\right)\right]$$

Substituting (B-4) into (B-2), and then (B-2) into (B-1), we obtain (17).

Appendix C

In this part we find the probability of the phase angle θ lying in region Θ. Let the angles θ_U, θ_L define an arbitrary decision region Θ. With no loss of generality, it is assumed that $\phi = 0$.

$$\Pr\{\theta \in \Theta\} = \int_{\theta_L}^{\theta_U}\int_0^{\infty} f_\theta(\theta|\phi,\gamma) f_\gamma(\gamma) d\gamma d\theta$$
$$= \int_{\theta_L}^{\theta_U}\int_0^{\infty} f_\theta(\theta|\phi,\gamma) \sum_{j=1}^{N}\left[(-1)^{j-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} C_j e^{-C_j\gamma}\right] d\gamma d\theta$$
$$= \sum_{j=1}^{N}\left[(-1)^{j-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} \int_{\theta_L}^{\theta_U}\int_0^{\infty} f_\theta(\theta|\phi,\gamma) C_j e^{-C_j\gamma} d\gamma d\theta\right] \quad \text{(C-1)}$$

where $f_\theta(\theta|\phi,\gamma)$ is defined by [1, eq. (9b)]. By using [1, eq. (10) & eq. (18)] & (8), we get:

$$\Pr\{\theta \in \Theta\} = \sum_{j=1}^{N}\left[(-1)^{j-1} \sum_{\substack{i_1,i_2,\cdots,i_j=1 \\ i_1<i_2<\cdots<i_j}}^{N} I(\theta_U, \theta_L, C_j^{-1})\right] \quad \text{(C-2)}$$

where

$$I(\theta_U, \theta_L, C_j^{-1}) = \frac{\theta_U - \theta_L}{2\pi} + \frac{1}{2}\beta_U\left(\frac{1}{2} + \frac{\tan^{-1}(\alpha_U)}{\pi}\right) - \frac{1}{2}\beta_L\left(\frac{1}{2} + \frac{\tan^{-1}(\alpha_L)}{\pi}\right) \quad \text{(C-3a)}$$

and

$$\mu_U^m = \sqrt{\log_2(M)/C_j}\,\sin(\theta_U^m) \quad \text{(C-3b)}$$
$$\mu_L^m = \sqrt{\log_2(M)/C_j}\,\sin(\theta_L^m) \quad \text{(C-3c)}$$
$$\alpha_U^m = \sqrt{\log_2 M/C_j}\,\cos(\theta_U^m)/\sqrt{(\mu_U^m)^2+1} \quad \text{(C-3d)}$$
$$\alpha_L^m = \sqrt{\log_2 M/C_j}\,\cos(\theta_L^m)/\sqrt{(\mu_L^m)^2+1} \quad \text{C-3e)}$$
$$\beta_U^m = \mu_U^m/\sqrt{(\mu_U^m)^2+1} \quad \text{(C-3f)}$$
$$\beta_L^m = \mu_L^m/\sqrt{(\mu_L^m)^2+1} \quad \text{(C-3g)}$$

Reference

[1] S. Chennakeshu and J. B. Anderson, "Error Rates for Rayleigh Fading Multichannel Reception of MPSK," IEEE Transactions on Communications, Vol.43, pp.338-346, February/March/April, 1995.

[2] N. C. Beaulieu and J. Hu, "A Closed-Form Expression for the Outage Probability of Decode-and-Forward Relaying in Dissimilar Rayleigh Fading Channels," IEEE Communications Letters, Vol.10, pp.813-815, December, 2006.

[3] J. Hu and N. C. Beaulieu, "Performance Analysis of Decode-and-Forward Relaying with Selection Combining," IEEE Communications Letters, Vol.11, pp.489-491, June, 2007.

[4] I. H. Lee and D. Kim, "BER Analysis for Decode-and-Forward Relaying in Dissimilar Rayleigh Fading Channels," IEEE Communications Letters, Vol.11, pp.52-54, 2007.

[5] M. K. Simon and M.-S. Alouini, Digital communication over fading channels, 2nd ed. Hoboken, N.J.: John Wiley & Sons, 2005.

[6] A. Papoulis and S. U. Pillai, Probability, random variables, and stochastic processes, 4th ed. Boston: McGraw-Hill, 2002.

[7] T. S. Rappaport, Wireless communications: principles and practice. Upper Saddle River, N.J.: Prentice Hall PTR, 1996.

[8] J. W. Craig, "A New, Simple and Exact Result for Calculating The Probability of Error For Two-Dimensional Signal Constellations," in MILCOM, 1991.

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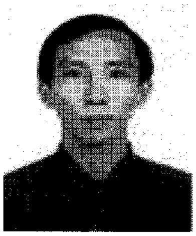

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