

# Mooring Cost Sensitivity Study Based on Cost-Optimum Mooring Design

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**KEY WORDS:** Offshore mooring design, FPSO turret mooring, Harmony search, Optimization

**ABSTRACT:** *The paper describes results of a sensitivity study on an optimum mooring cost as a function of safety factor and allowable maximum offset of the offshore floating structure by finding the anchor leg component size and the declination angle. A harmony search (HS) based mooring optimization program was developed to conduct the study. This mooring optimization model was integrated with a frequency-domain global motion analysis program to assess both cost and design constraints of the mooring system. To find a trend of anchor leg system cost for the proposed sensitivity study, optimum costs after a certain number of improvisation were found and compared. For a case study a turret-moored FPSO with 3 × 3 anchor leg system was considered. To better guide search for the optimum cost, three different penalty functions were applied. The results show that the presented HS-based cost-optimum offshore mooring design tool can be used to find optimum mooring design values such as declination angle and horizontal end point separation as well as a cost-optimum mooring system in case either the allowable maximum offset or factor of safety varies.*

## 1. Introduction

The offshore mooring design requires significant amount of design cycles with the satisfaction of the complex design constraints in order to achieve an economically competitive solution (Ryu et al., 2007). Ryu et al. (2007) presented the anchor leg cost optimization via the harmony search (HS) and concluded that the HS was able to find the cost optimum solution. Fylling (1997) and Fylling and Kleiven (2000) addressed an application of mooring optimization of deepwater mooring and riser risers. A single point mooring of an FPSO was selected for a case study.

This study focuses on mooring cost sensitivity of the anchor leg system optimized by using the HS algorithm. Parameters that change for this sensitivity study include safety factor and allowable maximum offset of the floating platform.

This paper addresses HS-based mooring optimization. The main objective of the present mooring optimization includes (i) determination of length and diameter of each mooring component and (ii) search for the optimum declination angle of the anchor leg and horizontal end point separation between the fairlead and the anchor point. The validity of a mooring design was checked against safety factors of the mooring line top tension, allowable maximum platform offset, and bottom chain length. In this paper the HS algorithm is summarized first. Secondly, applied penalty functions are presented.

Lastly, mooring cost sensitivity results for a deepwater

FPSO moored by a 3 × 3 anchor leg system are summarized.

## 2. Harmony Search Algorithm

Compared to other simulation-based meta-heuristic optimization algorithms such as simulated annealing, tabu search, and generic algorithm (Simpson et al., 1994; Cunha and Sousa, 1999; Lippai et al., 1999), HS was adopted from musical process of finding ‘pleasant harmonies.’ For instance, when several notes from different musical instruments are played simultaneously on a random basis and this process is repeated, there is a possibility to find better harmonies. In HS, these better harmonies are saved in a certain size of memory by replacing the worst harmony in the memory until the pre-defined maximum number of improvisation, generating a new harmony, is reached.

The HS algorithm overcomes the drawbacks of conventional gradient methods: (i) it does not require complex calculus, (ii) it does not require starting value assumption, and (iii) it can consider discrete value as well as continuous value.

The HS has a relatively simple structure and does not require complex mathematical gradients for the functions in optimization process. Thus, it is free from mathematical divergence in the middle of computation (Geem, 2006b). The HS also does not require starting values for design variables. Thus, it has better chance to find global optimum rather than to be entrapped in local optima (Geem et al., 2001). By

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nature, HS can efficiently handle discrete variables as well as continuous variables (Lee and Geem, 2005). However, conventional gradient methods handle continuous variables only, which cause a problem when design variables have discrete values, such as commercial sizes made in a factory. To round continuous values up to discrete values may cause the final design worse (Geem, 2006a).

When compared with genetic algorithm (GA) as another popular soft-computing method, HS overcame the drawback of GA's major mechanism by explicitly considering the relationship among design variables (Geem, 2006c).

Fundamental five steps of an HS for the cost-optimum mooring system design are shown in Fig. 1, and they are summarized as follows:

- Step 1: Design variable / algorithm parameter initialization;
- Step 2: Harmony memory initialization;
- Step 3: Generation of a new harmony;
- Step 4: Harmony memory update if needed; and
- Step 5: Improvisation stopping criterion check.

### Step 1: Design variable / algorithm parameter initialization

The optimization is expressed as follows:

$$\text{Minimize } f(x) \quad (1)$$

$$\text{Subject to } x_i \in X_i, I = 1, 2, 3, \dots, n \quad (2)$$

where  $f(x)$  is an objective function;  $x$  is the vector of each design variable  $x_i$ ;  $X_i$  is the set of the possible values of each design variable which is bounded by the pre-defined range of the design variable;  $n$  is the total number of design variables.

In the mooring system design, the objective function is the mooring system cost which includes main anchor leg components, connecting components, installation equipments, certificates, etc. To simplify the present problem, only the material cost of the main anchor leg components was considered in this study. Therefore, the diameters and lengths of each mooring component are the design variables.

Initialization includes four HS algorithm parameters: harmony memory size (HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), and maximum number of improvisations (NI). All harmony vectors are stored in the harmony memory (HM) which has a form of  $(HMS) \times (n + 1)$  matrix. Columns one through  $n$  store the values of the design variables, and the last column contains the costs.

### Step 2: Initialization of HM

The initial HM consists of different solution vectors of HMS. Each solution vector has diameter and length values for each mooring component and the total cost of the

mooring system. In this study, there are three different anchor leg components (top chain, wire, and bottom chain), and the HM memory is shown in Eq. (3).

$$HM = \begin{bmatrix} l_1^1 & l_2^1 & l_3^1 & d_1^1 & d_2^1 & d_3^1 \\ l_1^2 & l_2^2 & l_3^2 & d_1^2 & d_2^2 & d_3^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l_1^{HMS} & l_2^{HMS} & l_3^{HMS} & d_1^{HMS} & d_2^{HMS} & d_3^{HMS} \end{bmatrix} \Rightarrow \begin{Bmatrix} c^1 \\ c^2 \\ \vdots \\ c^{HMS} \end{Bmatrix} \quad (3)$$

where  $l$  is length of each anchor leg component,  $d$  diameter, and  $c$  cost for each solution vector.

### Step 3: Improvisation of a new harmony

Improvisation of a new harmony is performed based on three rules: (i) memory consideration, (ii) pitch adjustment, and (iii) random selection.

### Step 4: Replacement of the worst harmony in the HM

If a new cost is better than the worst cost in the HM, the worst harmony vector is replaced by the new harmony vector. For the mooring design optimization problem, the global motion analysis is carried out between Steps 3 and 4 to check the mooring design requirements, for instance, allowable maximum offset, top tension, and the bottom chain length.

### Step 5: Stopping criterion

A conditional statement is applied to judge whether this harmony search loop needs to repeat or stop.

A typical mooring design cycle is summarized in Fig. 2. A damaged mooring condition needs to be assessed as a part of the mooring design, but in this present study only the intact case was considered to simplify the design process.

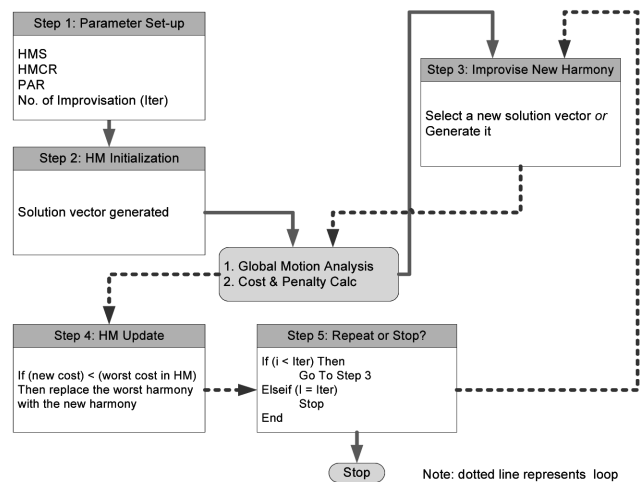


Fig. 1 Flow chart of HS algorithm for designing a cost-optimal mooring system (Ryu et al., 2007)

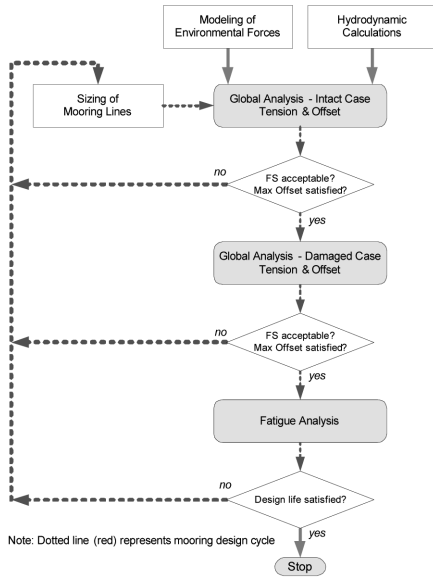


Fig. 2 Flowchart of typical mooring design cycle

### 3. Penalty Functions

Three design constraints implemented include (a) factor of safety (FS) for the top tension of the intact case, (b) no uplift of the bottom chain, and (c) allowable maximum platform offset. To better guide search for a cost optimum solution, following three different types of the penalty function were employed for the above mentioned three design constraints, respectively:

$$p_a = c \times (T_{max}/MBL)^2 \quad (4)$$

$$p_b = C_{average} \quad (5)$$

$$p_c = c \times (r_{max}/r_{max, design})^2 \quad (6)$$

where  $p_a$ ,  $p_b$ , and  $p_c$  represent the cost penalties for the three different design constraints mentioned above,  $c$  calculated material cost,  $C_{average}$  average cost of the saved harmony memory,  $T_{max}$  calculated maximum top tension,  $MBL$  minimum break load,  $r_{max}$  calculated maximum offset, and  $r_{max, design}$  allowable maximum design offset.

### 4. Case Study: FPSO Mooring Design

As a case study an FPSO turret-moored by  $3 \times 3$  chain-wire-chain anchor leg system is considered. The FPSO with a permanent turret mooring system in water depth (WD) of 800m was defined as the base case. The FPSO vessel particulars used are detailed in Ryu et al. (2007). Followed is the summary of the design constraints for the base case:

(a) FS for intact case = 1.67;

(b) no uplift at the anchor position; and

(c) allowable maximum FPSO offset = 7.5% of water depth.

Each anchor leg is made up of a combination of studless chain and spiral strand wire rope. The objective function, i.e. the total sum cost of the main mooring components, can be expressed as follows,

$$C = \sum_{j=1}^M \sum_{i=1}^N f(l_i, d_i) \quad (7)$$

where  $C$  represents the total sum cost of the main mooring components,  $f(l_i, d_i)$  the individual cost of mooring component  $i$  with length  $l_i$  and diameter  $d_i$ ,  $M$  the total number of anchor legs, and  $N$  the number of mooring components in the mooring system. Equation (7) also means that the mooring component cost is a function of mooring components' length and diameter which are directly related to the material weight.

The sign conventions utilized for the analysis of motions and loads in earth-fixed and vessel-fixed local coordinate systems are summarized in Ryu et al. (2007). The nine identical anchor legs for the FPSO vessel are arranged in three groups of 3 anchor legs each. One group of anchor legs is oriented 15 degrees CCW from North; the other two groups are arranged 120 degrees apart from the Northern group as shown in Fig. 3.

The anchor leg fairleads are separated by 13 degrees in each group on the turret, and arranged on an 8-meter radius. The anchor legs depart the turret at 5-degree spacing between adjacent legs in a group. The anchor radius is variable to be determined as a result of the optimization process. The ballast load draft (8.55m) was applied.

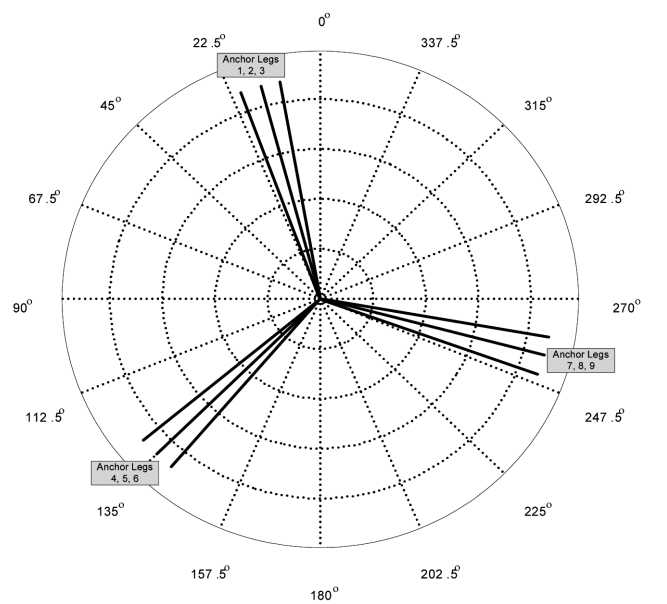


Fig. 3 FPSO  $3 \times 3$  anchor leg pattern

**Table 1** Summary of design environmental condition

	Value	Heading [deg]
Wind	39knots	185
Current	2.1knots	185
Wave	Hs = 10.9m Tp = 13.7s $\gamma = 2.12$	215
Swell	Hs = 1.0m Tp = 13.0s $\gamma = 2.77$	215

#### 4.1 Design environmental condition

The design environmental condition is summarized in Table 1. A JONSWAP spectrum was used to model both the wind waves and the swell, while the NPD spectrum was used to model the gustiness of the wind.

#### 4.2 Estimate of mooring line properties and cost

Expressions for mooring line properties such as minimum break load (MBL), axial stiffness (EA), and weight ( $W$ ) were derived using catalogue data supplied by manufacturers. For wire rope the properties can be described as a function of diameter  $d$  using the formula:  $y = A \times d^B$ , where  $d$  is the diameter of the line in millimeters, and  $A$  and  $B$  are constant. Table 2 provides the coefficients for the various properties of the rope.

For chain, offshore mooring chain Grade R4 was chosen for the study and the following approximate formulas (Ramnas Bruk) were applied for its properties:

$$\begin{aligned} MBL &= 0.0274 \times d^2 \times 44 - 0.08 \times d / 9.81 \\ EA &= (11.86 \times d^2 - 0.042 \times d^3) \times 10^4 / 9810 \\ W &= A \times d^2 \end{aligned} \quad (8)$$

where  $d$  is chain diameter in mm and  $A$  is weight coefficient (0.0170 for wet weight; 0.0195 for dry weight). As expressed in Eq. (7), the main anchor leg component cost is a function of its diameter and length so the cost estimate of the studless chain Grade R4 and wire rope can be expressed as follows:

$$C_{chain} = 0.06320 \times d^2 \times l \quad (9)$$

$$C_{wire} = 0.03415 \times d^2 \times l \quad (10)$$

**Table 2** Coefficients for estimation of wire rope properties

	Unit	A	B
MBL	MT	0.1025	1.9927
Axial Stiffness	MN	0.1512	1.9010
Wet Weight	kg/m	0.0045	1.9871
Dry Weight	kg/m	0.0065	1.9582

**Table 3** Input bound for fairlead declination angle

	Lower (deg)	Upper (deg)
Declination	20	70

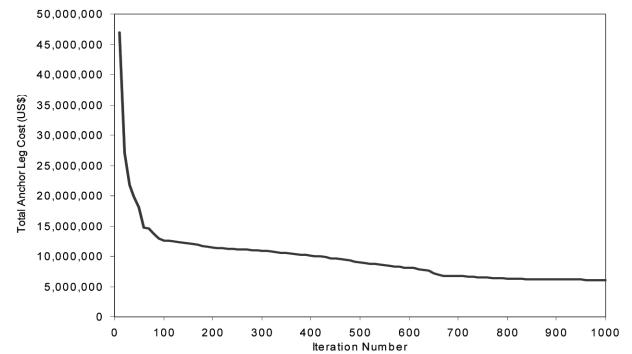
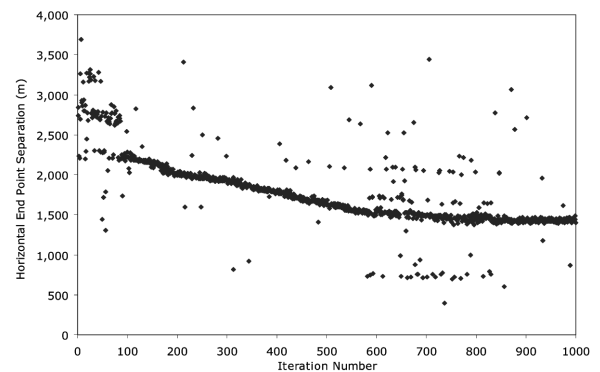
where  $C$  is cost in USD (\$),  $d$  the diameter in mm, and  $l$  the length in meter.

#### 4.3 Harmony search computation

In this case study, the applied values of the HS parameters are: HMS = 10, HMCR = 0.95, PAR = 0.8, and NI = 1,000. The input lower and upper bounds for each design parameter, i.e. declination angle, lengths and diameters of top chain, wire, and bottom chain are summarized in Table 3.

A total of 1,000 improvisations (i.e. iterations) were performed for each case to find cost-optimal mooring designs, and Fig. 4 shows the trend of average total anchor leg cost in the harmony memory. Since HMS = 10 was applied, there were ten different anchor leg designs in the harmony memory. Within approximately first 100 iterations, the average total cost dropped rapidly with a good cost convergence trend. After that, the cost search shows gradual convergence.

The horizontal end point separation (HEPS) between fairlead and anchor is presented in Fig. 5. Initial HEPS

**Fig. 4** Average total anchor leg cost in HM (base case)**Fig. 5** Optimum horizontal end point separation (base case)

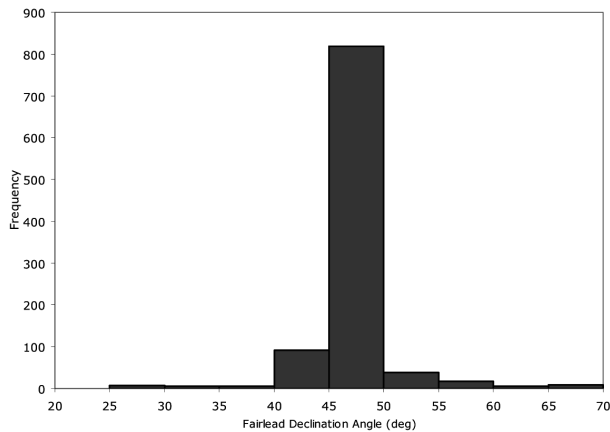


Fig. 6 Distribution of top chain declination angle (base case)

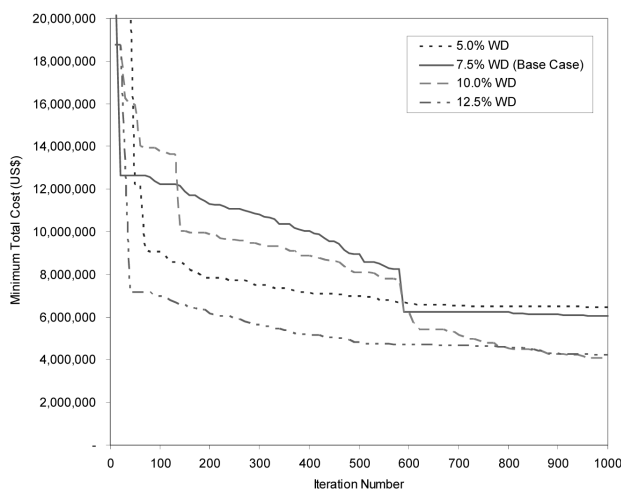


Fig. 7 Minimum total anchor leg cost with various required maximum offsets

ranges from 2,200m to 3,700m. However, as iteration increases, it converges to 1,500m which is an optimum HEPS. As the harmony search progresses, it is also found that relatively either small or large HEPS, i.e. "tight" or "loose" mooring systems, are saved in the harmony memory as a mooring leg design with low cost.

Figure 6 presents a distribution of the top chain declination angles measured from the horizontal plane. This result indicates that an optimum mooring line declination angle can be between 45 and 50 degrees for this case.

Figure 7 summarizes the trend of the minimum total anchor leg cost in the harmony memory as a function of the number of iteration and the required maximum offset. Four different required maximum offsets (5%, 7.5%, 10%, and 12.5% of the water depth) were applied to investigate in what trend the harmony search finds low anchor leg costs. After 900 iterations, all four cases show a good convergence in the total anchor leg cost.

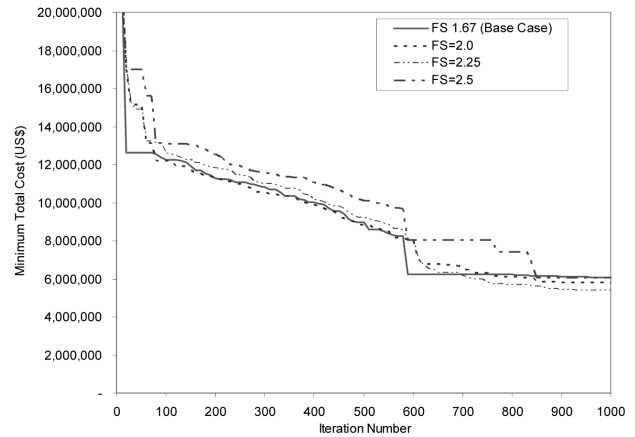


Fig. 8 Minimum total anchor leg cost with various safety factors

As expected, the smaller required maximum offset results in higher anchor leg cost since a tighter mooring system is required to keep the floater within a smaller range. Also, it is observed that the rapid cost reduction occurs within approximately first 50 iterations. The final optimum cost ranges from \$4M to \$6.5M.

Results of the anchor leg cost sensitivity with various safety factors are shown in Fig. 8. Four different factors of safety (1.67, 2.0, 2.25, and 2.5) were investigated. It is interesting to observe that the total minimum anchor leg cost is not significantly sensitive to the factor of safety. The results show that all four total minimum anchor leg costs at 1,000th iteration were within 15%.

## 5. Conclusions

A mooring optimization design tool using the harmony search algorithm and a frequency domain global analysis tool was developed to conduct a study on the cost sensitivity as a function of allowable maximum offset and safety factor of the mooring system. The trend of cost-optimum mooring design search over 1,000 iterations was compared.

A case study on a permanent turret mooring system for an FPSO in deepwater was performed. Judging from the trend of the cost-optimum search in the mooring design as also shown in Figs. 7 and 8, the variation in the safety factor does not significantly affect the final optimum mooring system cost (Fig. 8) while the allowable maximum offset shows a larger impact on the final optimized mooring cost (Fig. 7).

As shown in the iteration trend of the anchor leg declination angle at the fairlead and horizontal end point separation, it is also shown that harmony search is capable of finding optimum design values of these two parameters as well as cost-optimum mooring system.

To better find a feasible, cost optimum mooring system,

three different penalty functions were applied for the design constraints considered. Influence of these penalty functions in the HS-based cost optimum mooring design is one of the future research areas.

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2008년 6월 5일 원고 접수

2009년 1월 21일 최종 수정본 채택