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Robustness and Actuator Bandwidth of MRP-Based Sliding Mode Control for Spacecraft Attitude Control Problems

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Abstract

Nonlinear sliding surface design in variable structure systems for spacecraft attitude control problems is studied. A robustness analysis is performed for regular form of system, and calculation of actuator bandwidth is presented by reviewing sliding surface dynamics. To achieve non-singular attitude description and minimal parameterization, spacecraft attitude control problems are considered based on modified Rodrigues parameters (MRP). It is shown that the derived controller ensures the sliding motion in pre-determined region irrespective of unmodeled effects and disturbances.

Keywords: variable structure control, sliding surface, nonlinear control, modified Rodrigues parameters

1. Introduction

The control of large-angle spacecraft maneuvers generally poses highly nonlinear characteristics, which govern dynamic equations, control rate saturation and its limits, and incomplete state knowledge due to sensor measurement noise or failure. The use of variable structure control for attitude maneuvers has been proposed for the adequate treatment of such problems. An optimal control approach for the sliding surface synthesis problem using quaternions was presented by Vadali (Vadali 1986). Differential geometric approach to single-axis maneuver using Cayley-Rodrigues attitude parameter was formulated by Sira-Ramirez and Dwyer (Dwyer & Ramirez 1988, Ramirez 1988). Nonlinear optimal control theory and feedback linearization methods also have been applied to the spacecraft maneuver problem (Dwyer 1984, Damaren & D'Eleuterio 1989).

In this paper, variable structure control for spacecraft attitude maneuvers is considered. The robustness analysis is taken for nonlinear system with parameter variations and disturbances, and the analysis result is applied to regular form of variable structure systems(VSS). Chattering is one of main topics on VSS and a constraint of control signal bandwidth. By studying sliding surface dynamics, calculation of actuator bandwidth is present in the function of control gain factors. To take advantage of the modified Rodrigues parameters:

(1) minimal parameterization of attitude representation,

(2) singularity avoidance of rotations

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a sliding mode controller is derived based on MRP.

It is shown that the derived sliding mode controller ensures *bound-layer reachability* (Dwyer & Ramirez 1988) irrespective of unmodeled effects and disturbances.

2. Theoretical Background

In this section, a review of the variable structure control and dynamic equations of motions for a three-axis stabilized spacecraft is presented.

2.1 Equations of Motion

The attitude of spacecraft is assumed to be presented by quaternions. The quaternion representation is defined as

$$\beta_0 = \cos\frac{\Phi}{2}, \quad \beta_i = e_i \sin\frac{\Phi}{2}, \quad i = 1,2,3 \tag{1}$$

where e_i is a unit vector corresponding to the axis of rotation and Φ is the angle of rotation. For minimal parameterization of attitude, we use modified Rodrigues parameters(MRP). The transformation from quaternions to the MRP vectorσ is given by (Schaub 1998, Shuster 1993)

$$\sigma_i = \frac{\beta_i}{\beta_0 + 1}, \quad i = 1,2,3 \tag{2}$$

Using Eq. (1), the modified Rodrigues parameters are written as

$$\sigma = \tan\frac{\Phi}{4}\hat{e} \tag{3}$$

Studying Eq. (3) it is evident that the MRP have a geometric singularity at $\Phi = \pm 360$ degrees. But the non-uniqueness of the MRP allows one to avoid their singularities (Schaub 1998). The MRP kinematic differential equation in vector form by using the spacecraft's body angular velocity vector(ω), is

$$\dot{\sigma} = \frac{1}{4}\left[(1-\sigma^2)I + 2[\tilde{\sigma}] + 2\sigma\sigma^T\right]\omega = B(\sigma)\omega \tag{4}$$

where the notation $\sigma^2 = (\sigma^T\sigma)$ is used, and *I* is the 3×3 identity matrix. The tilde matrix operator $[\tilde{\sigma}]$ is defined as

$$[\tilde{\sigma}] = \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix} \tag{5}$$

The inverse transformation of B(σ) in Eq. (4) is in explicit vector form (Schaub 1998)

$$B^{-1}(\sigma) = \frac{4}{(1+\sigma^2)^2}\left[(1-\sigma^2)I - 2[\tilde{\sigma}] + \sigma\sigma^T\right] \tag{6}$$

Let *J* be the rigid body inertia matrix and *u* be control torque vector. Euler's rotational equations of motions for a rigid body are given by

$$\dot{\omega} = f(\omega) + J^{-1}u \tag{7}$$

where

$$f(\omega) = J^{-1}[\tilde{J\omega}]\omega \tag{8}$$

2.2 Variable Structure Control

The system presented as

$$\dot{x_1} = f_1(x_1, x_2), \tag{9}$$

$$\dot{x_2} = f_2(x_1, x_2) + G(x_1, x_2)u \tag{10}$$

is referred to as a regular form in VSC literature, where $x_1 \in R^n, x_2 \in R^m, u \in R^m$, and *G* is m×m matrix. It is possible to specify the sliding manifold *s* = 0 as (Utkin 1992)

$$s = x_2 - s_0(x_1) = 0 \tag{11}$$

where $s_0(x_1)$ is the solution of equation s=0 regarding to x_2. The equation of motion along the manifold s=0 in Eq. (11), or $x_2 = s_0(x_1)$ will be of the form

$$\dot{x_1} = f_1(x_1, s_0(x_1)) \tag{12}$$

As a result, we face a design problem for the system with an m–dimensional control $s_0(x_1)$ rather than (m+n)–dimensional control. Now we consider the disturbance and uncertainty to see the robustness characteristic of VSC. We consider the system with disturbance and uncertainty as the following:

$$\dot{x} = f + \Delta f + (G + \Delta G)u(t) + d(t) \tag{13}$$

where $x \in R^m$, and $f, \Delta f, G, \Delta G$ depend on x and f and Δf are nonlinear driving term and their uncertainties, *G* and ΔG are the control gain and its uncertainty, and $d(t)$ and $u(t)$ represent external disturbance and control signals. Futhermore $\Delta f, \Delta G$ and d are assumed to be bounded by known, continuous functions, i.e.

$$\left.\begin{array}{r} |\Delta f| \leqq F(x,t) \\ |\Delta G| \leqq \Gamma(x,t) \\ |d| \leqq D(x,t) \end{array}\right\} \tag{14}$$

The attractivity of the sliding surface is assured by enforcing the *sliding mode existence condition*, which for a single input case are given by the following relations:

$$\begin{aligned} \lim_{s \longrightarrow 0^+} \frac{d}{dt} s < 0 \\ \lim_{s \longrightarrow 0^-} \frac{d}{dt} s > 0 \end{aligned} \tag{15}$$

For general vector–valued situations conditions, Eq. (15) is replaced by the Lyapunov-type condition below:

$$\frac{d}{dt}||s||^2 = 2s^T \frac{ds}{dt} < 0 \tag{16}$$

This can be guaranteed by a control law which forces the system to comply the condition. In this paper the following form of ds/dt , which satisfies the *sliding mode existence condition*, is considered:

$$\frac{ds}{dt} \equiv \dot{s} = -Ps - Ksign(s) \tag{17}$$

where $P > 0$ and $K > 0$. P and K assure the two conditions for sliding. P is effective for large s, and K is dominant for small s. Due to uncertainties, Eq. (17) needs to be modified to

$$\dot{s} = -Ps - Ksign(s) + \Delta f + d + \Delta G u \tag{18}$$

The following selection is sufficient to ensure Eq. (16):

$$K = F + \Gamma|u| + D >= |\Delta f + d + \Delta G u| \tag{19}$$

From the relation Eq. (19), K is disturbance control factor.

3. Controller Design

In this section a sliding mode controller is derived. The measurement of both the spacecraft attitude and angular rate are assumed to be available. The nonlinear model for spacecraft motion can be characterized by Eq. (4) and Eq. (7) and is of regular form as Eq. (9), (10) with $x_1 = \sigma, x_2 = \omega$ and $G = J^{-1}$.

The sliding surface is defined as Eq. (11) with $s_0(\sigma) = B^{-1}(\sigma)\rho(\sigma)$.

$$\begin{aligned} s &= \omega - s_0(\sigma) \\ &= \omega - B^{-1}(\sigma)\rho(\sigma) \end{aligned} \tag{20}$$

The $\rho(\sigma)$ determines the desired form of evolution σ, given by

$$\rho(\sigma) = \Pi(\sigma - \sigma_d) \tag{21}$$

where σ_d is the desired final value of the attitude parameter, and Π is a diagonal matrix with negative elements. The dynamics for the attitude parameter would be a system with exponential rate of decay. So diagonal elements in Π can be thought of time constant in attitude parameter dynamics. Considering Eq. (16), the sliding mode controller which satisfies the *sliding mode existence condition* of Eq. (16) is obtained by (Dwyer & Ramirez 1988)

$$u = -J\left\{ f(\omega) - \frac{\partial s_0}{\partial \sigma}[B(\sigma)s_0(\sigma) + B(\sigma)s] + Ps + Ksat(s,\epsilon) \right\} \tag{22}$$

The saturation function, $sat(s,\epsilon)$ used to reduce chattering in the control torques, is defined by

$$sat(s_i,\epsilon) = \begin{cases} sign(s_i) & if|s_i| > \epsilon \\ \frac{s_i}{\epsilon} & if|s_i| \leqq \epsilon \end{cases} \quad i = 1,2,3 \tag{23}$$

where $\epsilon > 0$ is the"boundary layer thickness."

3.1 Regulation Problem

The regulation problem requires that the final attitude of spacecraft be zero, i.e. the desired evolution of attitude parameters $\sigma_d = 0$ in Eq. (21). To simplify the problem, we assume that Π matrix in Eq. (21) is as form of $\Pi = \gamma I$ with scalar value γ, which yields $s_0(\sigma)$ to

$$s_0(\sigma) = 4\gamma(1+\sigma^2)^{-2}\left\{(1-\sigma^2)I - 2[\tilde{\sigma}] + 2\sigma\sigma^T\right\}\sigma \tag{24}$$

which can be further simplified using the $[\tilde{\sigma}]\sigma = 0$ and the identity $[\tilde{\sigma}]^2 = \sigma\sigma^T - \sigma^2 I$ to

$$s_0(\sigma) = 4\gamma(1+\sigma^2)^{-1}\sigma \tag{25}$$

The partial derivative$\partial s_0/\partial\sigma$ is given by

$$\frac{\partial s_0}{\partial \sigma} = 4\gamma(1+\sigma^2)^{-1}\left\{I - 2(1+\sigma^2)^{-1}\sigma\sigma^T\right\} \tag{26}$$

3.2 Tracking Problem

The tracking problem requires that the attitude of spacecraft follow a desired form of attitude evolution. For tracking, $s_0(\sigma)$ is derived to

$$\begin{aligned} s_0(\sigma) &= 4\gamma(1+\sigma^2)^{-1}\sigma \\ &\quad -4\gamma(1+\sigma^2)^{-2}\left\{(1-\sigma^2)I - 2[\tilde{\sigma}] + 2\sigma\sigma^T\right\}\sigma_d \end{aligned} \tag{27}$$

The partial derivative $\partial s_0/\partial\sigma$ of Eq. (27) is given by

$$\begin{aligned} \frac{\partial s_0}{\partial\sigma} &= 4\gamma(1+\sigma^2)^{-1}\left\{I - 2(1+\sigma^2)^{-1}\sigma\sigma^T\right\} \\ &\quad -8\gamma(1+\sigma^2)^{-2}\left\{\sigma\sigma_d^T - \sigma_d\sigma^T + [\tilde{\sigma_d}] + (\sigma_d^T\sigma)I\right\} \\ &\quad +16\gamma(1+\sigma^2)^{-3}\left\{(1-\sigma^2)I - 2[\tilde{\sigma}] + 2\sigma\sigma^T\right\}\sigma_d\sigma^T \end{aligned} \tag{28}$$

3.3 Actuator Bandwidth

By replacing $sign(s)$ with $sat(s,\epsilon)$, Eq. (18) becomes

$$\dot{x} = -Ps - K\frac{s}{\epsilon} + \Delta f + \Delta Gu + d \tag{29}$$

when $|s| < \epsilon$, i.e. sliding phase. From Eq. (18), we can see that the *sliding mode existence condition* is satisfied when $|s| \geqq \epsilon$. It is desirable to keep ϵ small, so that near ideal sliding mode is obtained. Eq. (29) may be rewritten as

$$\dot{s} + (P + \frac{K}{\epsilon})s = \Delta f + \Delta Gu + d \tag{30}$$

which is very similar to a low–pass filter between s and $(\Delta f + \Delta Gu + d)$. If $\lambda \equiv P + K/\epsilon$ is defined. We can find that smaller ϵ gives larger λ, i.e. higher actuator bandwidth. This gives a lower bound for ϵ based on actuator limitations. If we set $\lambda = 2\omega/3$, yielding:

$$\epsilon = \frac{3K}{2\omega - 3P} \tag{31}$$

Note that ω in Eq. (31) is the response bandwidth of the actuator. It is very important to note that design parameters P and K assure the sliding condition, and gives actuator bandwidth.

4. Simulation

An example of a three–axis rest–to–rest maneuver is presented and the simultaneous reorientation of all axes is considered here, with variable control torques and the following inertia matrix taken from (Dwyer & Ramirez 1988)

$$J = diag[114 \quad 86 \quad 87] \ \text{kg} - \text{m}^2$$

To show the robustness of the controller, disturbance torques are assumed to be

$$d = [0.005\sin(\omega t) \quad 0.003\cos(\omega t) \quad -0.005\sin(\omega t)]^T \ \text{N} - \text{m}$$

where ω is the disturbance torque frequency, assumed to be 1 rad/s. And the other initial conditions and design parameters are the following:

$$\sigma(t_0) = [-0.1 \ \ 0.5 \ \ 1.0]^T, \ \ \omega = [0 \ \ 0 \ \ 0]^T, \ \ K = 0.02I, \ \ Q = 0.02I$$

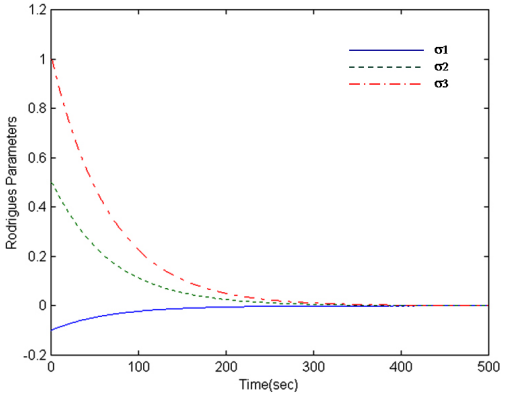

Figure 1. Plot of modified Rodrigues parameters trajectories.

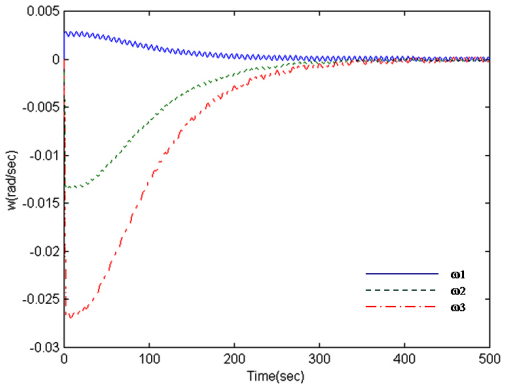

Figure 2. Plot of angular velocity trajectories.

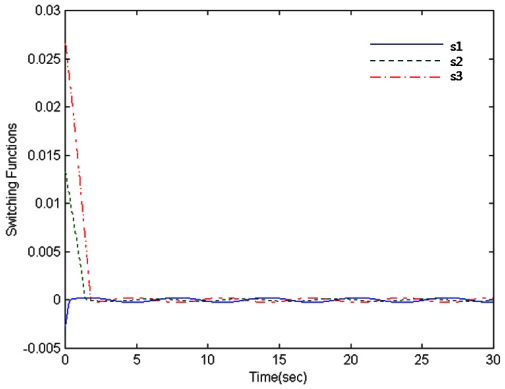

Figure 3. Plot of switching functions.

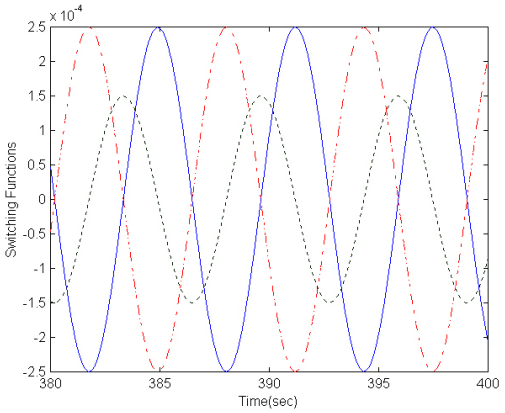

Figure 4. Switching functions during the terminal maneuver.

The regulation problem is considered. The constant $\gamma = -0.015\text{sec}^{-1}$, and the boundary layer thickness is set to $\epsilon = 0.001$. Also, the control torques are limited to 1.0 N-m. Figure 1 depicts the time trajectories of modified Rodrigues parameters. Figure 2 shows the angular velocity trajectories. Figure 3 and 4 show the switching functions trajectories. Figure 4 shows the switching functions during the terminal maneuver, from the figure one can see that the state trajectories remains well inside switching region specified bound layer thickness. The control torques are applied to accomodate disturbance torques as shown in Figures 5∼7.

5. Conclusions

Nonlinear sliding surface design for large-angle spacecraft maneuvers is studied. The sliding

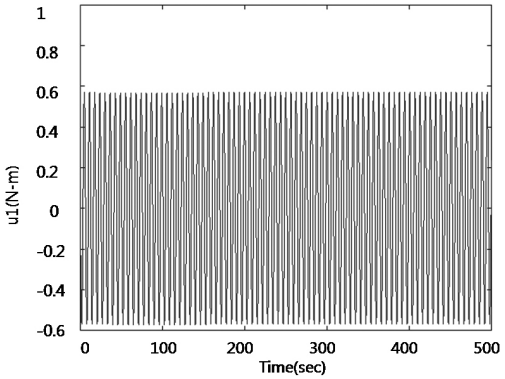

Figure 5. Control Torque u_1.

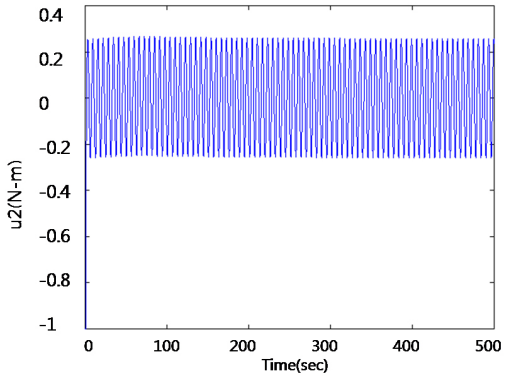

Figure 6. Control Torque u_2.

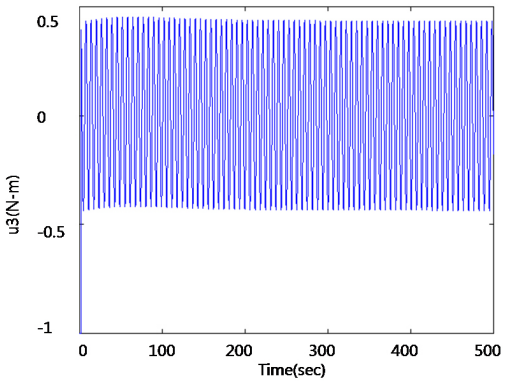

Figure 7. Control Torque u_3.

mode controller is derived based on modified Rodrigues parameter, and analysis of robustness is peformed for the derived control scheme in the presence of unmodeled effects and disturbances. The actuator bandwidth is considered using bound layer thickness and sliding mode dynamics. Simulation results show that the control system performs acceptably in the presence of external disturbances.

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