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BURST-ERROR-CORRECTING BLOCK CODE USING FIBONACCI CODE

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Abstract. The errors take place in the communication channel but they are often burst-error types. From properties of the Fibonacci code, it is not difficult to detect the burst-errors accompanying with this code. Fibonacci codes for correcting the double-burst-error patterns are presented. Given the channel length with the double-burst-error type, Fibonacci code correcting these errors is constructed by a simple formula.

1. Introduction

The Fibonacci sequence is a sequence of positive integers arranged in nondecreasing order, which is very useful tool in various areas. In [4], authors suggested useful identities in combinatorial counting numbers via Fibonacci numbers.

Let $U = (u_1, u_2, \cdots)$ be a sequence of positive integers arranged in nondecreasing order. We define the sequence U to be *complete* if every positive integer n is the sum of some subsequence of U, that is,

$$n = \sum_{i=1}^{\infty} a_i u_i \text{ where } a_i = 0 \text{ or } 1.$$

Theorem 1.1. *Let $U = (u_1, u_2, \ldots)$ be a sequence. If $u_1 = 1$ and $u_{n+1} \leq 2u_n$, then the sequence U is complete.*

From the above theorem 1.1, it can be easily shown that, the well known fact, the Fibonacci sequence is complete(see [1]).

Since every positive integer is the sum of its subsequence, we can construct a Fibonacci code, introduced in §II of [2].

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Let f_α, $\alpha \geq 2$, be defined by

$$\cdots b_{\alpha+1} b_\alpha b_{\alpha-1} \cdots b_3 b_2$$

where $b_\alpha = 1$ and $b_\beta = 0$ for any $\alpha \neq \beta \geq 2$. Each α-th Fibonacci number F_α is expressed as f_α in the binary numbers. $\mathbf{0}$ is defined to satisfy $b_\beta = 0$ for each $\beta \geq 2$. We define the operation $+_2$ or $\sum$ componentwise as the exclusive or. Let $(V, +_2, \mathbf{0})$ be the algebra generated by the set $\{f_\alpha\}_{\alpha \geq 2}$. Each element $v \in V$ has an expression $v = \sum_{\alpha \geq 2} \hat{b}_\alpha f_\alpha$ where $\hat{b}_\alpha \in \{0, 1\}$. Since Fibonacci sequence is complete, every positive number can be expressed as a sum of some elements in the set $\{F_\alpha\}$. Because of the property

$$\forall\, 3 \leq \alpha \in \mathbb{N}, F_\alpha = F_{\alpha-1} + F_{\alpha-2},$$

the addition of the two consecutive Fibonacci numbers is excluded. When we define $0 = \sum_{\alpha \geq 2} b_\alpha F_\alpha$ where $b_\alpha = 0$ for each $\alpha \geq 2$, every element in the monoid $(\mathbb{N}, +, 0)$ has a binary expression in $\{F_\alpha\}$ without the consecutive ones. Then we can find the injection

$$\varphi : (\mathbb{N}, +, 0) \to (V, +_2, \mathbf{0}); n = \sum\nolimits_{\alpha \geq 2} b_\alpha F_\alpha \mapsto \sum\nolimits_{\alpha \geq 2} b_\alpha f_\alpha$$

The element in the set $\mathcal{F} = \{\varphi(n) | n \in \mathbb{N}\}$ is called the *Fibonacci code.*

We define the partial order $\sqsubseteq$ in the set V. By a unique expression $v \in V$, let

$$v = \sum_{\alpha \geq 2} b_\alpha f_\alpha \quad \text{and} \quad \hat{v} = \sum_{\alpha \geq 2} \hat{b}_\alpha f_\alpha \in V$$

with $b_\alpha, \hat{b}_\alpha \in \{0, 1\}$. $v \sqsubseteq \hat{v}$ is defined to satisfy

$$\forall \alpha \geq 2, \ \ b_\alpha \ \leq \ \hat{b}_\alpha$$

The *subordinate set* $W_\sqsubseteq$ of $W \subset V$ is defined by the set $\{v \in V \,|\, \exists w \in W$ satisfying $v \sqsubseteq w\}$. When $W = W_\sqsubseteq$, W is called *self-subordinate.* An *error pattern* is defined by a self-subordinate subset of $(V, \sqsubseteq)$ containing $\{f_\alpha\}_{\alpha \geq 2}$.

We can give the ordering for the error patterns by the containment. Because of the self-subordinate, codes to correct the error pattern W have to do for any error pattern U contained in W. ([3], §3)

Example 1.2. *Here is the single-error pattern.*

$$S_1 = \ \{\delta f_\alpha |\ \alpha \geq 2;\ \delta \in \{0, 1\}\}$$

The double-burst-error patterns are described as follows :

$$B_2 = \{\delta_1 f_\alpha +_2 \delta_2 f_{\alpha+1} |\ \alpha \geq 2;\ \delta_j \in \{0,1\} \textit{ for } j = 1,2\}$$

$$\begin{aligned} \ell B_2 = \{\sum_{i=1}^{\ell} (\delta_{\alpha_i} f_{\alpha_i} +_2 \delta_{\alpha_i+1} f_{\alpha_i+1}) \mid 2 \leq \alpha_1 \leq \alpha_1 + 1 \leq \alpha_2 \\ \leq \cdots \leq \alpha_\ell \leq \alpha_\ell + 1, \quad |\alpha_i - \alpha_{(i+1)}| \geq 3; \\ \delta_t \in \{0,1\} \textit{ for } t \in \{\alpha_1, \ldots, \alpha_\ell + 1\}\ \} \end{aligned}$$

The burst error pattern $\mathcal{E}$ (B_2 or ℓB_2) in Example 1.2 is not contained in $\mathcal{F}$. Thus $\mathcal{F} +_2 \mathcal{E} = \{f +_2 \varepsilon | f \in \mathcal{F} \text{ and } \varepsilon \in \mathcal{E}\}$ is the subset of the algebra $(V, +_2, 0)$ containing $\mathcal{F}$.

Example 1.3. *Let $C_1 \subset V$ be a subset containing $f_2 +_2 f_4 +_2 f_6$ and $f_4 +_2 f_6 +_2 f_8$. This can not be a single-error-correcting code because*

$$(f_2 +_2 f_4 +_2 f_6) +_2 (f_2) = f_4 +_2 f_6 = (f_4 +_2 f_6 +_2 f_8) +_2 (f_8)$$

The subset $C_2 \subset V$ containing $f_2 +_2 f_5 +_2 f_7$ and $f_3 +_2 f_5 +_2 f_8$ can not be a B_2-error-correcting code because

$$(f_2 +_2 f_5 +_2 f_7) +_2 (f_2 +_2 f_3) = f_3 +_2 f_5 +_2 f_7 = (f_3 +_2 f_5 +_2 f_8) +_2 (f_7 +_2 f_8)$$

Let $I = (b_{n-1}, \ldots, b_3, b_2)$ and $J = (c_{n-1}, \ldots, c_3, c_2)$ denote two (arbitrary) binary numbers. The *Hamming distance* between I and J, denoted by $H(I, J)$, is the number of bits where the two binary numbers differ. It is straightforward to define the Fibonacci cube based on the Fibonacci codes.

Let N denote an integer, where $1 \leq N \leq F_n$ for some n. Let I_F and J_F denote the Fibonacci code of i and j, where $0 \leq i, j \leq N - 1$. The *Fibonacci cube of size* N is a graph $(V(N), E(N))$, where $V(N) = \{0, 1, \ldots, N-1\}$ and $(i, j) \in E(N)$ if and only if $H(I_F, J_F) = 1$.

Let I_B and J_B denote the ordinary binary representation of integers i and j. A *Boolean cube of dimension* n, denoted by B_n, is a graph (V_n, E_n), where $V_n = \{0, 1, 2, \ldots, 2^n - 1\}$ and $(i, j) \in E_n$ if and only if $H(I_B, J_B) = 1$.

For a positive integer $n \geq 2$, a Fibonacci cube of order n, denoted by Γ_n, is a graph consisting recursively of two disjoint subgraphs Γ_{n-1} and Γ_{n-2}, which are of unequal sizes in general. Each node in Γ_{n-2} is then connected to a counterpart node in Γ_{n-1}. As the basis, Γ_0 is an empty graph and Γ_1 is a graph with a single node. It is well known that the Fibonacci cube contains about $\frac{1}{5}$ fewer edges than the Boolean cube for the same number of nodes.

2. Fibonacci double-burst-error-correcting code

It is presented Fibonacci double-burst-error-correcting block codes with respect to some double-burst-error patterns.

DEFINITION 2.1. *For any $k \geq 1$ with $\alpha \geq 4(k+1)$, we define*

$$F_k(\alpha) = \varphi(F_\alpha - F_{\alpha-(4k+1)}) = \sum_{i=1}^{2k+1} f_{\alpha_i}$$

where $\alpha_1 = \alpha-(4k+2)$ and $\alpha_i = \alpha_1+3+2(i-2)$ for each $2k+1 \geq i \geq 2$. We will use the notation $\{F_k(\alpha)\} = \{\alpha_1, \alpha_2, \ldots, \alpha_{2k+1}\}$.

EXAMPLE 2.2. *Here are some Fibonacci codes which we are interested in.*

$$F_1(8) = \varphi(F_8 - F_3) = f_2 +_2 f_5 +_2 f_7 = 0000000101001$$
$$F_1(9) = \varphi(F_9 - F_4) = f_3 +_2 f_6 +_2 f_8 = 0000001010010$$
$$F_1(10) = \varphi(F_{10} - F_5) = f_4 +_2 f_7 +_2 f_9 = 0000010100100$$
$$F_2(12) = \varphi(F_{12} - F_3) = f_2 +_2 f_5 +_2 f_7 +_2 f_9 +_2 f_{11} = 0001010101001$$
$$F_2(13) = \varphi(F_{13} - F_4) = f_3 +_2 f_6 +_2 f_8 +_2 f_{10} +_2 f_{12} = 0010101010010$$
$$F_2(14) = \varphi(F_{14} - F_5) = f_4 +_2 f_7 +_2 f_9 +_2 f_{11} +_2 f_{13} = 0101010100100$$

From the property of $+_2$, it is clear that

$$\{F_k(\alpha) +_2 F_k(\beta)\} = \{\{F_k(\alpha)\} \cup \{F_k(\beta)\}\} \setminus \{\{F_k(\alpha)\} \cap \{F_k(\beta)\}\}$$

LEMMA 2.3. *Assume that $\alpha \leq \beta$. Then we have*

$$|\{F_k(\alpha) +_2 F_k(\beta)\}| = \begin{cases} 0 & \text{if } \alpha = \beta, \\ \geq 2 \cdot 2k = 4k & \text{if } |\alpha-\beta| \text{ is odd}, \\ 2 \cdot \{1 + \frac{|\alpha-\beta|}{2}\} \geq 4 & \text{otherwise.} \end{cases}$$

Proof. We need to investigate the case $\{F_k(\alpha)\} \cap \{F_k(\beta)\} \neq \emptyset$. Let $I_{2k+1} = \{i \in \mathbb{N} | 1 \leq i \leq 2k+1\}$. When $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| \neq 0$, there is at least one pair of numbers $(i,j), 1 \leq j \leq i \leq 2k+1$, in $I_{2k+1} \times I_{2k+1}$ satisfying $\alpha_i = \beta_j$. When $i = j$, it is clear that $\alpha = \beta$. In case of $\alpha_i = \beta_j$ with $1 = j < i$, $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| = 1$ because, for each $2 \leq \ell \leq 2k+1$, $\beta_\ell = \alpha_i + 3 + 2(\ell-2) \neq \alpha_t$ with $1 \leq t \leq 2k+1$. If $\alpha_i = \beta_j$ with $2 \leq j < i$, then $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| = 2k-(i-j)$ because, for each $2 \leq \ell \leq 2k-(i-j)$, $\beta_\ell = \beta_2 + 2(\ell-2) = \alpha_{i-(j-2)} + 2(\ell-2) = \alpha_{i-j+\ell}$.

Consider the number $|\alpha-\beta|$. For each $i \in I_{2k+1}$, $\beta_i - \alpha_i = |\alpha-\beta|$. Let $|\alpha-\beta|$ be odd. If $|\alpha-\beta| = 3+2(i-2)$ for $2 \leq i \leq 2k+1$, then

$\beta_1 = \alpha_1 + |\alpha - \beta| = \alpha_i$. Otherwise $\{F_k(\alpha)\} \cap \{F_k(\beta)\} = \emptyset$. Thus $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| \leq 1$ when $|\alpha - \beta|$ is odd. If $|\alpha - \beta|$ is even, then it never happens that $\beta_1 = \alpha_i$ with some i, $2 \leq i \leq 2k+1$, because $\beta_1 - \alpha_1 = 3 + 2(i-2) = |\alpha - \beta|$. When $|\alpha - \beta|$ is even with $0 < |\alpha - \beta| \leq 4k-2$, $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| \neq 0$ because

$$\beta_2 = \alpha_2 + |\alpha - \beta| = \alpha_{2+(|\alpha-\beta|/2)}$$

Thus $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| = 2k - \frac{|\alpha-\beta|}{2}$. If $|\alpha - \beta|$, which is greater than $4k$ -2, is even, then $\{F_k(\alpha)\} \cap \{F_k(\beta)\} = \emptyset$.

From the formula

$$|\{F_k(\alpha) +_2 F_k(\beta)\}| = 2 \cdot \{(2k+1) - |\{F_k(\alpha)\} \cap \{F_k(\beta)\}|\},$$

the result follows. □

THEOREM 2.4. *For any fixed integer* $k \geq 1$, $\mathcal{F}_k^1 = \{\mathbf{0}, F_k(\alpha) | \alpha \geq 4(k+1)\}$ *is a* B_2*-error-correcting block code.*

Proof. Assume there are at least one pair $(F_k(\alpha), F_k(\beta))$, $\alpha \neq \beta$, satisfying

$$F_k(\alpha) +_2 \varepsilon_1 = F_k(\beta) +_2 \varepsilon_2 \text{ for some } \varepsilon_1, \varepsilon_2 \text{ in } B_2$$

There exist $i, j \in \mathbb{N} - \{1\}$ and $\delta_m, \sigma_m \in \{0,1\}$, for $m \in \{1,2\}$, such that

$$\begin{aligned} F_k(\alpha) +_2 \varepsilon_1 &= F_k(\alpha) +_2 \delta_1 f_i +_2 \delta_2 f_{i+1} \\ &= F_k(\beta) +_2 \sigma_1 f_j +_2 \sigma_2 f_{j+1} = F_k(\beta) +_2 \varepsilon_2 \end{aligned}$$

Thus $F_k(\alpha) +_2 F_k(\beta) = \varepsilon_1 +_2 \varepsilon_2 = \delta_1 f_i +_2 \delta_2 f_{i+1} +_2 \sigma_1 f_j +_2 \sigma_2 f_{j+1}$. Since $\alpha \neq \beta$, $|\alpha - \beta| \neq 0$.

Case 1. Let $|\alpha - \beta|$ be odd. By Lemma 2.3, we need to show only for $k = 1$. Because $|\alpha - \beta|$ is odd, $|\{F_1(\alpha)\} \cap \{F_1(\beta)\}| \leq 1$ so that $\beta_1 = \alpha_i$ for $1 < i \leq 3$. Thus $\{F_1(\alpha) +_2 F_1(\beta)\} = \{\alpha_1, \alpha_2(\text{ or } \alpha_3), \beta_2, \beta_3\}$. Then $F_1(\alpha) +_2 F_1(\beta)$ is not the type of $\varepsilon_1 +_2 \varepsilon_2$ because $(\alpha_2, \beta_2, \beta_3) = (\alpha_1 + 3, \alpha_1 + 8, \alpha_1 + 10)$ and $(\alpha_3, \beta_2, \beta_3) = (\alpha_1 + 5, \alpha_1 + 6, \alpha_1 + 8)$.

Case 2. Let $|\alpha - \beta| \neq 0$ be even. By Lemma 2.3, we need only to show for $|\alpha - \beta| = 2$. i.e. $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| = 2k - \frac{|\alpha-\beta|}{2} = 2k - 1$ and, for $2 \leq i \leq 2k$, $\beta_i - \alpha_i = |\alpha - \beta| = 2$ so that $\beta_i = \alpha_i + 2 = \alpha_{i+1}$.

Thus, for any $k \geq 1$, $\{F_k(\alpha) +_2 F_k(\beta)\} = \{\alpha_1, \beta_1, \alpha_2, \beta_{2k+1}\}$ because $\beta_i = \alpha_{i+1}$ with $2 \leq i \leq 2k$. $\{F_k(\alpha) +_2 F_k(\beta)\}$ is not in the form of $\varepsilon_1 +_2 \varepsilon_2$ since $(\beta_1, \alpha_2, \beta_{2k+1}) = (\alpha_1 + 2, \alpha_1 + 3, \alpha_1 + (4k+3))$.

It is easy to show that there is no $\alpha \geq 4(k+1)$ with $k \geq 1$ such that $\mathbf{0} +_2 \varepsilon_1 = F_k(\alpha) +_2 \varepsilon_2$ for some $\varepsilon_1, \varepsilon_2$ in B_2, because $\{F_k(\alpha)\} = \{\alpha_1, \ldots, \alpha_{2k+1}\}$ and $F_k(\alpha)$ can not be in $B_2 +_2 B_2 = \{\varepsilon_1 +_2 \varepsilon_2 | \varepsilon_1, \varepsilon_2 \in B_2\}$. □

Corollary 2.5. *For any fixed integer* $k \geq 1$, $\mathcal{S}_k^1 = \mathcal{F}_k^1 \cup \{f_2 +_2 f_4 +_2 f_6\}$ *is also a single-error-correcting block code.*

Proof. It is easy to show that $\mathbf{0}+_2(f_2+_2f_4+_2f_6)$ can not be in $S_1+_2S_1$. If we can show that $\{(f_2 +_2 f_4 +_2 f_6) +_2 S_1\} \cap \{F_k(\alpha) +_2 S_1\} = \emptyset$ for any $F_k(\alpha) \in \mathcal{F}_k^1$, then $\mathcal{S}_k^1$ is a single-error-correcting block code because also is $\mathcal{F}_k^1$ since $S_1 \subset B_2$.

Assume that there exists some $F_k(\alpha) \in \mathcal{F}_k^1$ satisfying

$$(f_2 +_2 f_4 +_2 f_6) +_2 F_k(\alpha) \in S_1 +_2 S_1$$

Since an element of $S_1 +_2 S_1$ has at most Hamming distance 2, $(f_2 +_2 f_4 +_2 f_6) +_2 F_k(\alpha)$ also does. By Definition 2.1, each $\alpha_i - \alpha_1$ for $i \geq 2$, in $\{F_k(\alpha)\}$, is $3 + 2(i-2)$. When $\emptyset \neq \{F_k(\alpha)\} \cap \{2, 4, 6\}$, it has only order 1. Thus $(f_2 +_2 f_4 +_2 f_6) +_2 F_k(\alpha)$ has at least Hamming distance 4. This is a contradiction. □

From Theorem 2.4, we pursue a conclusion with ℓB_2 error pattern for $\ell \geq 2$.

Theorem 2.6. *For any fixed integer* $k \geq 2$ *with* $2 \leq \ell \leq k$, $\mathcal{F}_k^\ell = \{\mathbf{0}, F_k(\alpha) | \alpha = 4(k+1)+2\ell(n-1)$ *or* $4(k+1)+2\ell(n-1)+1$ *for* $1 \leq n \in \mathbb{N}\}$ *is a* ℓB_2*-error-correcting block code.*

Proof. Assume there is at least one pair $(F_k(\alpha), F_k(\beta))$, $\alpha \neq \beta$, satisfying

$$F_k(\alpha) +_2 \varepsilon_1 = F_k(\beta) +_2 \varepsilon_2 \text{ for some } \varepsilon_1, \varepsilon_2 \text{ in } \ell B_2$$

Thus $F_k(\alpha)+_2F_k(\beta) = \varepsilon_1+_2\varepsilon_2 \in \ell B_2+_2\ell B_2$. Since $\alpha \neq \beta$, $|\alpha-\beta| \neq 0$.

Case 1. Let $|\alpha - \beta|$ be odd with $\alpha \leq \beta$. By Lemma 2.3, we need to show only for $|\{F_k(\alpha)\} \cap \{F_k(\beta)\}| = 1$. For each $i, 1 \leq i \leq 2k+1$, $\beta_i - \alpha_i = |\alpha-\beta| = 2\ell m+1$ for some integer $m > 0$. $\beta_1 = \alpha_1 + 2\ell m + 1 = \alpha_{\ell m+1}$. When $\{F_k(\alpha)\} \cap \{F_k(\beta)\}$ is $\beta_1 = \alpha_{\ell m+1}$,

$$\begin{aligned} &\{F_k(\alpha) +_2 F_k(\beta)\} \\ =&\{\alpha_1, \ldots, \alpha_{\ell m}, \alpha_{\ell m+2}, \beta_2, \ldots, \alpha_{2k+1}, \beta_{2k-\ell m+1}, \beta_{2k-\ell m+2}, \ldots, \beta_{2k+1}\} \end{aligned}$$

where $\beta_j = \alpha_{k+j} + 1$ for each $2 \leq j \leq 2k - \ell m + 1$. Thus $F_k(\alpha) +_2 F_k(\beta)$ can not be in $\ell B_2 +_2 \ell B_2$.

Case 2. Let $|\alpha - \beta| = 2\ell m$ for $m \geq 1$. If $|\alpha - \beta| > 4k - 2$, then $|\{F_k(\alpha) +_2 F_k(\beta)\}| = 4k + 2$ so that $F_k(\alpha) +_2 F_k(\beta)$ can not be in $\ell B_2 +_2 \ell B_2$. If $|\alpha - \beta| \leq 4k - 2$, then

$$\{F_k(\alpha) +_2 F_k(\beta)\} = \{\alpha_1, \ldots, \alpha_{\ell m}, \beta_1, \alpha_{\ell m+1}, \beta_{2k-\ell m+2}, \ldots, \beta_{2k+1}\}$$

Thus $F_k(\alpha) +_2 F_k(\beta)$ can not be in $\ell B_2 +_2 \ell B_2$.

Since any $F_k(\alpha) \in \mathcal{F}_k^\ell$ is not in $\ell B_2 +_2 \ell B_2 = \{\varepsilon_1 +_2 \varepsilon_2 | \varepsilon_1, \varepsilon_2 \in \ell B_2\}$, it is not possible that $\mathbf{0} +_2 \varepsilon_1 = F_k(\alpha) +_2 \varepsilon_2$ for some $\varepsilon_1, \varepsilon_2$ in ℓB_2. $\square$

It is necessary to find the size of $\mathcal{F}_k^\ell$ with $1 \le \ell \le k$ for a given channel length $n \ge 4k+3$. Let $\lfloor r \rfloor, r \in \mathbb{R}$, be the largest integer less than or equal to r.

Corollary 2.7. *Given ℓ, k, and the channel length n with $1 \le \ell \le k$,*

$$|\mathcal{F}_k^\ell| = \lfloor \frac{n-(4k+2)}{2\ell} \rfloor + \lfloor \frac{n-(4k+3)}{2\ell} \rfloor + 2$$

Proof. The position of $f_{\alpha_{2k+1}}$ for each codeword $F_k(\alpha)$ and the condition α in Theorem 2.6 give the maximum number of the codewords for the given channel length. $\square$

3. Conclusion

Fibonacci burst-error-correcting codes are presented for the various double-burst-error type. Also their sizes depending on the channel length is found. From the formulas in the above theorems, it is easy to design the corresponding codes to any given double-burst-error pattern and detect these errors occuring in the communication.

References

[1] Ross Honsberger, *Mathematical Gems III*, The Mathematical Association of America, 1985.

[2] W-J Hsu, *Fibonacci Cubes - A New Interconnection Topology*, IEEE Transactions on Parallel and Distriuted Systens **4** (1993), no. 1, 3-12.

[3] J. D. H. Smith, *Loop transversals to linear codes*, J. Comb., Info. and Syst. Sci. **17** (1992), 1-8.

[4] G-Y Lee, J-S Kim and S-H Cho, *Some combinatorial identities via Fibonacci numbers*, Discrete Appied Math., **130** (2003), 527-534.

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