JOURNAL OF THE
CHUNGCHEONG MATHEMATICAL SOCIETY
Volume **22**, No. 2, June 2009

A FIXED POINT APPROACH FOR THE APPROXIMATION OF JORDAN TRIPLE LINEAR DERIVATIONS IN BANACH ALGEBRAS

Juri Lee*, Eunyoung Son**, and Ick-Soon Chang***

Abstract. We adopt the idea of Cădariu and Radu to prove the stability of Jordan triple linear derivations and we take account of the Jacobson radical range problem for linear derivations.

1. Introduction and preliminaries

Let $\mathcal{A}$ be a Banach algebra over the real or complex field $\mathbb{F}$. An additive mapping $\mu : \mathcal{A} \to \mathcal{A}$ is called a *ring derivation* (resp., *Jordan ring derivation*) if

$$\mu(xy) = \mu(x)y + x\mu(y) \text{ (resp.,} \mu(x^2) = \mu(x)x + x\mu(x))$$

for all $x, y \in \mathcal{A}$. An additive mapping $\mu : \mathcal{A} \to \mathcal{A}$ is called a *Jordan triple ring derivation* if $\mu(xyx) = \mu(x)yx + x\mu(y)x + xy\mu(x)$ for all $x, y \in \mathcal{A}$. Note that if $\mu(\lambda x) = \lambda\mu(x)$ for all $\lambda \in \mathbb{F}$ and all $x \in \mathcal{A}$, then we say that μ is *linear*.

The stability problem of functional equations has originally been formulated by Ulam [16] in 1940: *Under what condition does there exists a homomorphism near an approximate homomorphism*? As an answer to the problem of Ulam, Hyers has proved the stability of the additive functional equation [8] in 1941. A generalized version of the theorem of Hyers for approximately additive mappings was given by Aoki [1] in 1950 and for approximately linear mappings was presented by Rassias [12] in 1978. Since then, a great deal of work has been done by a number of authors. In particular, the stability result concerning derivations

Received April 20, 2009; Accepted May 26, 2009.
2000 Mathematics Subject Classification: Primary 39B52, 39B72.
Key words and phrases: stability, fixed point, Jordan triple linear derivation, radical range.
Correspondence should be addressed to Ick-Soon Chang, ischang@mokwon.ac.kr.
This work was supported by the Brain Korea 21 Project in 2009.

between operator algebras was first obtained by Šemrl [13]. Recently, Badora [2] gave a generalization of the Bourgin's result [4]. He also dealt with the Hyers-Ulam stability and the Bourgin-type superstability of ring derivations in [3].

In 1955, Singer and Wermer [14] obtained a fundamental result which started investigation into the ranges of linear derivations on Banach algebras. The result, which is called the Singer-Wermer theorem, states that any continuous linear derivation on a commutative Banach algebra maps into the Jacobson radical. They also made a very insightful conjecture, namely that the assumption of continuity is unnecessary. This was known as the Singer-Wermer conjecture and was proved in 1988 by Thomas [15]. The Singer-Wermer conjecture implies that any linear derivation on a commutative semisimple Banach algebra is identically zero which is the result of Johnson [9]. On the other hand, Hatori and Wada [7] showed that a zero operator is the only ring derivation on a commutative semisimple Banach algebra with the maximal ideal space without isolated points. Note that this differs from the above result of Johnson. Based on these facts and a private communication with Watanabe [11], Miura *et al.* proved the Hyers-Ulam-Rassias stability and Bourgin-type superstability of ring derivations on Banach algebras in [11].

The theorem [10], which is called the *alternative of fixed point*, play an important role in proving the stability problem. Recently, Cădariu and Radu [6] applied the fixed point method to the investigation of the Cauchy additive functional equation. In the present paper, we adopt the idea of Cădariu and Radu and establish the stability of Jordan triple ring derivations. In addition, we take account of the Jacobson radical range problem of linear derivations.

2. Main results

In this section, the element e of a Banach algebra $\mathcal{A}$ will be a unit. As a matter of convenience, for a given mapping $f : \mathcal{A} \to \mathcal{A}$, we use the following abbreviation:

$$\begin{aligned} D_{\alpha,\beta} f(x, y, z, w) &:= f(\alpha x + \beta y + zwz) - \alpha f(x) - \beta f(y) \\ &\quad - f(z)wz - zf(w)z - zwf(z) \end{aligned}$$

for all $x, y, z, w \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}$.

THEOREM 2.1. *Let $\mathcal{A}$ be a Banach algebra. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a function $\varphi : \mathcal{A}^4 \to [0, \infty)$ such that*

$$\lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 2^n z, 2^n w)}{2^n} = 0, \tag{2.1}$$

$$\|D_{\alpha,\beta} f(x, y, z, w)\| \le \varphi(x, y, z, w) \tag{2.2}$$

for all $x, y, z, w \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. If there exists a positive constant $L < 1$ such that

$$\varphi(2x, 2x, 2x, 0) \le 2L\varphi(x, x, x, 0) \tag{2.3}$$

for all $x \in \mathcal{A}$, then there exists a unique Jordan triple linear derivation $\mu : \mathcal{A} \to \mathcal{A}$ such that

$$\|f(x) - \mu(x)\| \le \frac{1}{2(1 - L)}\varphi(x, x, x, 0) \tag{2.4}$$

for all $x \in \mathcal{A}$.

Proof. We consider the set

$$\mathcal{X} := \{g \mid g : \mathcal{A} \to \mathcal{A},\ g(0) = 0\}$$

and the generalized metric on $\mathcal{X}$,

$$d(g, h) = \inf\{K \in [0, \infty] : \|g(x)-h(x)\| \le K\varphi(x, x, x, 0),\ \text{for all } x \in \mathcal{A}\}.$$

One can easily check that $(\mathcal{X}, d)$ is complete.

Next, let $T : \mathcal{X} \to \mathcal{X}$ be a mapping defined by $Tg(x) := \frac{g(2x)}{2}$ for all $x \in \mathcal{A}$.

We first verify that T is a strictly contractive on $\mathcal{X}$: Observe that for all $g, h \in \mathcal{X}$,

$$\begin{aligned} d(g, h) \le K \quad &\Longrightarrow \|g(x) - h(x)\| \le K\varphi(x, x, x, 0),\ x \in \mathcal{A} \\ &\Longrightarrow \Big\|\frac{1}{2}g(2x) - \frac{1}{2}h(2x)\Big\| \le LK\varphi(x, x, x, 0),\ x \in \mathcal{A} \\ &\Longrightarrow \big\|Tg(x) - Th(x)\big\| \le LK\varphi(x, x, x, 0),\ x \in \mathcal{A} \\ &\Longrightarrow d(Tg, Th) \le LK. \end{aligned}$$

Hence we see that $d(Tg, Th) \le Ld(g, h)$ for all $g, h \in \mathcal{X}$.

We now assert that $d(Tf, f) < \infty$: If we consider $\alpha = \beta = 1$, $y = z = x$, $w = 0$ in (2.2) and we divide both sides by 2, then we arrive at

$$\|Tf(x) - f(x)\| \le \frac{1}{2}\varphi(x, x, x, 0)$$

for all $x \in \mathcal{A}$, that is, $d(Tf, f) \le \frac{1}{2} < \infty$.

Therefore, by the alternative of fixed point, we can prove that there is a unique Jordan triple linear derivation $\mu : \mathcal{A} \to \mathcal{A}$ satisfying the inequality (2.4): Now, from the alternative of fixed point, it follows that there exists a fixed point μ of T such that $\lim_{n\to\infty} d(T^n f, \mu) = 0$, that is, $\mu(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ for all $x \in \mathcal{A}$.

Again, by use of the alternative of fixed point, we lead to the inequality

$$d(f, \mu) \leq \frac{1}{1-L} d(Tf, f) \leq \frac{1}{2(1-L)},$$

which yields (2.4).

In order to claim that the mapping $\mu : \mathcal{A} \to \mathcal{A}$ is a Jordan triple linear derivation, let us take $\alpha = \beta = 1 \in \mathbb{U}$, $z = w = 0$ in (2.2). Then it becomes

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y, 0, 0). \tag{2.5}$$

for all $x, y \in \mathcal{A}$. Replacing $2^n x$ and $2^n y$ instead of x and y in (2.5) and dividing by 2^n, we have by (2.1)

$$\lim_{n\to\infty} \left\| \frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n} \right\| \leq \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 0, 0)}{2^n} = 0.$$

This means that μ is additive. Letting $z = w = 0$ in (2.2), we have

$$\|f(\alpha x + \beta y) - \alpha f(x) - \beta f(y)\| \leq \varphi(x, y, 0, 0) \tag{2.6}$$

for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. If we also replace x and y with $2^n x$ and $2^n y$ in (2.6), respectively, and then divide both sides by 2^n, we see that

$$\begin{aligned} &\lim_{n\to\infty} \left\| \frac{f(2^n(\alpha x + \beta y))}{2^n} - \alpha \frac{f(2^n x)}{2^n} - \beta \frac{f(2^n y)}{2^n} \right\| \\ &\quad \leq \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 0, 0)}{2^n} = 0. \end{aligned}$$

So we get $\mu(\alpha x + \beta y) = \alpha\mu(x) + \beta\mu(y)$ for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. Let us now assume that λ is a nonzero complex number and that N is a positive integer greater than $|\lambda|$. Then by applying a geometric argument, there exist $\lambda_1, \lambda_2 \in \mathbb{U}$ such that $2\frac{\lambda}{N} = \lambda_1 + \lambda_2$. In particular, due to the additivity of μ, we obtain $\mu(\frac{1}{2}x) = \frac{1}{2}\mu(x)$ for all $x \in \mathcal{A}$. Thus we have that

$$\begin{aligned} \mu(\lambda x) &= \mu\left(\frac{N}{2} \cdot 2 \cdot \frac{\lambda}{N} x\right) = N\mu\left(\frac{1}{2} \cdot 2 \cdot \frac{\lambda}{N} x\right) = \frac{N}{2}\mu((\lambda_1 + \lambda_2)x) \\ &= \frac{N}{2}(\lambda_1 + \lambda_2)\mu(x) = \frac{N}{2} \cdot 2 \cdot \frac{\lambda}{N}\mu(x) = \lambda\mu(x) \end{aligned}$$

for all $x \in \mathcal{A}$. Also, it is obvious that $\mu(0x) = 0 = 0\mu(x)$ for all $x \in \mathcal{A}$, that is, μ is linear.

Let us now take $x = y = 0$ in (2.2). Then it follows that

$$\|f(zwz) - f(z)wz - zf(w)z - zwf(z)\| \leq \varphi(0,0,z,w). \tag{2.7}$$

for all $z, w \in \mathcal{A}$. Replacing $2^n z$ and $2^n w$ instead of z and w in (2.7) and dividing by 8^n, we have by (2.1)

$$\begin{aligned} &\lim_{n\to\infty} \left\| \frac{f(8^n zwz)}{8^n} - \frac{f(2^n z)}{2^n} wz - z\frac{f(2^n w)}{2^n} z - zw\frac{f(2^n z)}{2^n} \right\| \\ &\quad \leq \lim_{n\to\infty} \frac{\varphi(0,0,2^n z, 2^n w)}{2^n} = 0. \end{aligned}$$

Hence it implies that μ is a Jordan triple linear derivation.

Assume that there exists another Jordan triple linear derivation $\mu_1 : \mathcal{A} \to \mathcal{A}$ satisfying the inequality (2.4). Since μ_1 is linear, we get

$$\mu_1(x) = \frac{\mu_1(2x)}{2} = (T\mu_1)(x)$$

and so μ_1 is a fixed point of T. In view of (2.4) and the definition of d, we know that $d(f, \mu_1) \leq \frac{1}{2(1-L)} < \infty$, that is, $\mu_1 \in \Delta = \{g \in \mathcal{X} : d(f,g) < \infty\}$. Due to the alternative of fixed point, we find that $\mu = \mu_1$, which proves that μ is unique. $\square$

Corollary 2.2. *Let $\mathcal{A}$ be a Banach algebra. Assume that p is given with $0 < p < 1$. Suppose that a mapping $f : \mathcal{A} \to \mathcal{A}$ satisfies the inequality*

$$\|D_{\alpha,\beta} f(x,y,z,w)\| \leq \varepsilon(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p) \tag{2.8}$$

for all $x, y, z, w \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$ and for some $\varepsilon > 0$. Then there exists a unique a Jordan triple linear derivation $\mu : \mathcal{A} \to \mathcal{A}$ such that

$$\|f(x) - \mu(x)\| \leq \frac{3\varepsilon}{|2 - 2^p|}\|x\|^p$$

for all $x \in \mathcal{A}$.

We here remark that any Jordan triple ring derivation on 2-torsion free semiprime ring is a ring derivation [5].

Theorem 2.3. *Let $\mathcal{A}$ be a commutative semiprime Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a function $\varphi : \mathcal{A}^4 \to [0, \infty)$ satisfying*

$$\lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 2^n z, 2^n w)}{2^n} = 0, \ \lim_{n\to\infty} \frac{\varphi(0,0,2^n z, w)}{2^n} = 0 \tag{2.9}$$

and the inequality (2.2)*. If there exists a positive constant* $L < 1$ *satisfying* (2.3)*, then* f *is a linear derivation which maps* $\mathcal{A}$ *into its Jacobson radical.*

Proof. By Theorem 2.1, there exists a unique Jordan triple linear derivation $\mu : \mathcal{A} \to \mathcal{A}$ satisfying (2.4), where $\mu(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ for all $x \in \mathcal{A}$. Replacing z by $2^n z$ in (2.7) and dividing by 4^n, we have by the condition (2.9)

$$\begin{aligned} &\lim_{n\to\infty} \Big\| \frac{f(4^n zwz)}{4^n} - \frac{f(2^n z)}{2^n} wz - zf(w)z - 2^n zw \frac{f(2^n z)}{2^n} \Big\| \\ &\quad \le \lim_{n\to\infty} \frac{\varphi(0,0,2^n z, w)}{4^n} = 0, \end{aligned}$$

which means that $\mu(zwz) = \mu(z)wz + zf(w)z + zw\mu(z)$ for all $z, w \in \mathcal{A}$. Using the linearity of μ, this equation now can be rewritten as

$$\begin{aligned} &\mu(2^n z \cdot w \cdot 2^n z) = 4^n \mu(z)wz + 4^n zf(w)z + 4^n zw\mu(z), \\ &\mu(z \cdot 4^n w \cdot z) = 4^n \mu(z)wz + zf(4^n w)z + 4^n zw\mu(z). \end{aligned}$$

Hence $zf(w)z = z\frac{f(4^n w)}{4^n}z$ and then we obtain $zf(w)z = z\mu(w)z$ as $n \to \infty$. If $z = e$, we arrive at $f = \mu$. Thus we see that f is a Jordan triple linear derivation. Since $\mathcal{A}$ is a commutative semiprime, f is a linear derivation and so f maps $\mathcal{A}$ into its Jacobson radical. □

THEOREM 2.4. *Let* $\mathcal{A}$ *be a Banach algebra. Suppose that* $f : \mathcal{A} \to \mathcal{A}$ *is a mapping for which there exists a function* $\varphi : \mathcal{A}^4 \to [0,\infty)$ *satisfying*

$$\lim_{n\to\infty} 8^n \varphi\Big(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}, \frac{w}{2^n}\Big) = 0 \tag{2.10}$$

and the inequality (2.2)*. If there exists a positive constant* $L < 1$ *such that*

$$\varphi(x,x,x,0) \le \frac{L}{2}\varphi(2x,2x,2x,0) \tag{2.11}$$

for all $x \in \mathcal{A}$*, then there exists a unique Jordan triple linear derivation* $\mu : \mathcal{A} \to \mathcal{A}$ *such that*

$$\|f(x) - \mu(x)\| \le \frac{L}{2(1-L)}\varphi(x,x,x,0) \tag{2.12}$$

for all $x \in \mathcal{A}$*.*

Proof. First of all, if we take $x = 0$ in (2.11), then we see that $\varphi(0,0,0,0) = 0$, because of $0 < L < 1$. So letting $x = y = z = w = 0$ in (2.2), one obtains $f(0) = 0$.

We now use the definitions for $\mathcal{X}$ and d, the generalized metric on $\mathcal{X}$, as in the proof of Theorem 2.1. Then $(\mathcal{X}, d)$ is complete. We define a mapping $T : \mathcal{X} \to \mathcal{X}$ by $Tg(x) := 2g(\frac{x}{2})$ for all $x \in \mathcal{A}$. Using the same argument as in the proof of Theorem 2.1, T is a strictly contractive on $\mathcal{X}$ with the Lipschitz constant L. In addition, we prove that $d(Tf, f) \leq \frac{L}{2} < \infty$.

Now it follows from the alternative of fixed point that there exists a fixed point μ of T such that $\mu(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{A}$, because $\lim_{n\to\infty} d(T^n f, \mu) = 0$. In addition, we have the inequality $d(f, \mu) \leq \frac{1}{1-L} d(Tf, f) \leq \frac{L}{2(1-L)}$, that is, the inequality (2.12) is true.

To show that the mapping $\mu : \mathcal{A} \to \mathcal{A}$ is a Jordan triple linear derivation, let us replace $\frac{x}{2^n}$ and $\frac{y}{2^n}$ instead of x and y in (2.5) and multiply by 2^n. Then, by virtue of (2.10), we find that

$$\begin{aligned}
&\lim_{n\to\infty} \Big\| 2^n f\Big(\frac{x+y}{2^n}\Big) - 2^n f\Big(\frac{x}{2^n}\Big) - 2^n f\Big(\frac{y}{2^n}\Big)\Big\| \\
&\quad \leq \lim_{n\to\infty} 2^n \varphi\Big(\frac{x}{2^n}, \frac{y}{2^n}, 0, 0\Big) = 0.
\end{aligned}$$

Thus we see that μ is additive. If we also replace x and y with $2^n x$ and $2^n y$ in (2.6), respectively, and then divide both sides by 2^n, we see that

$$\begin{aligned}
&\lim_{n\to\infty} \Big\| f\Big(\frac{\alpha x + \beta y}{2^n}\Big) - 2^n \alpha f\Big(\frac{x}{2^n}\Big) - 2^n \beta f\Big(\frac{y}{2^n}\Big)\Big\| \\
&\quad \leq \lim_{n\to\infty} 2^n \varphi\Big(\frac{x}{2^n}, \frac{y}{2^n}, 0, 0\Big) = 0.
\end{aligned}$$

So we get $\mu(\alpha x + \beta y) = \alpha\mu(x) + \beta\mu(y)$ for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. Similarly, as we did in the proof of Theorem 2.1, the mapping μ is linear.

Replacing $\frac{z}{2^n}$ and $\frac{w}{2^n}$ instead of z and w in (2.7) and multiplying by 8^n, we obtain by (2.10)

$$\begin{aligned}
&\lim_{n\to\infty} \Big\| 8^n f\Big(\frac{zwz}{8^n}\Big) - 2^n f\Big(\frac{z}{2^n}\Big) wz - 2^n z f\Big(\frac{w}{2^n}\Big) z - 2^n zw f\Big(\frac{z}{2^n}\Big)\Big\| \\
&\quad \leq \lim_{n\to\infty} 8^n \varphi\Big(0, 0, \frac{z}{2^n}, \frac{w}{2^n}\Big) = 0.
\end{aligned}$$

Therefore, μ is a Jordan triple linear derivation.

Following the same fashion as the proof of Theorem 2.1, we can show that μ is unique. □

COROLLARY 2.5. *Let $\mathcal{A}$ be a Banach algebra. Assume that p is given with $p > 3$. Suppose that a mapping $f : \mathcal{A} \to \mathcal{A}$ satisfies the inequality* (2.8). *Then there exists a unique a Jordan triple linear derivation μ :*

$\mathcal{A} \to \mathcal{A}$ such that

$$\|f(x) - \mu(x)\| \leq \frac{3\varepsilon}{|2^p - 2|}\|x\|^p$$

for all $x \in \mathcal{A}$.

THEOREM 2.6. *Let A be a commutative semiprime Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping for which there exists a function $\varphi : \mathcal{A}^4 \to [0, \infty)$ satisfying*

$$\lim_{n\to\infty} 8^n\varphi\Big(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}, \frac{w}{2^n}\Big) = 0, \lim_{n\to\infty} 8^n\varphi\Big(0, 0, \frac{z}{2^n}, w\Big) = 0 \tag{2.13}$$

and (2.2). *If there exists a positive constant $L < 1$ satisfying* (2.11), *then f is a linear derivation which maps $\mathcal{A}$ into its Jacobson radical.*

Proof. It follows from Theorem 2.4 that there exists a unique Jordan triple linear derivation $\mu : \mathcal{A} \to \mathcal{A}$ satisfying (2.12), where $\mu(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{A}$.

If we substitute $z := \frac{z}{2^n}$ in (2.7) and multiply by 4^n, then we obtain by (2.13)

$$\begin{aligned}&\lim_{n\to\infty}\Big\|4^n f\Big(\frac{zwz}{4^n}\Big) - 2^n f\Big(\frac{z}{2^n}\Big)wz - zf(w)z - 2^n zwf\Big(\frac{z}{2^n}\Big)\Big\|\\ &\leq \lim_{n\to\infty} 4^n\varphi\Big(0, 0, \frac{z}{2^n}, w\Big) = 0.\end{aligned}$$

So that $\mu(zwz) = \mu(z)wz + zf(w)z + zw\mu(z)$ for all $z, w \in \mathcal{A}$. Similar to the proof of Theorem 2.3, f is a linear derivation which maps $\mathcal{A}$ into its Jacobson radical. □

References

[1] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan **2** (1950), 64–66.

[2] R. Badora, *On approximate ring homomorphisms*, J. Math. Anal. Appl. **276** (2002), 589–597.

[3] R. Badora, *On approximate derivations*, Math. Inequal. Appl. **9** (2006), no. 1, 167–173.

[4] D. G. Bourgin, *Approximately isometric and multiplicative transformations on continuous function rings*, Duke Math. J. **16** (1949), 385–397.

[5] M. Bresar, *Jordan derivations on semiprime rings*, Proc. Amer. Math. Soc. **104** (1988), 1003–1006.

[6] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Math. Ber. **346** (2004), 43–52.

[7] O. Hatori and J. Wada, *Ring derivations on semi-simple commutative Banach algebras*, Tokyo J. Math. **15** (1992), 223–229.

[8] D. H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. **27** (1941), 222–224.
[9] B. E. Johnson, *Continuity of derivations on commutative Banach algebras*, Amer. J. Math. **91** (1969), 1–10.
[10] B. Margolis and J. B. Diaz, *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc. **126** (1968), 305–309.
[11] T. Miura, G. Hirasawa and S.-E. Takahasi, *A perturbation of ring derivations on Banach algebras*, J. Math. Anal. Appl. **319** (2006), 522–530.
[12] Th. M. Rassias, *On the stability of linear mappings in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.
[13] P. Šemrl, *The functional equation of multiplicative derivation is superstable on standard operator algebras*, Integr. Equat. Oper. Theory **18** (1994), 118–122.
[14] I. M. Singer and J. Wermer, *Derivations on commutative normed algebras*, Math. Ann. **129** (1955), 260–264.
[15] M. P. Thomas, *The image of a derivation is contained in the radical*, Ann. of Math. **128** (1988), 435–460.
[16] S. M. Ulam, *A Collection of Mathematical Problems*, Interscience Publishers, New York (1968), 63.

*

Department of Mathematics
Chungnam National University
Daejeon 305-764, Republic of Korea
E-mail: annans@hanmail.net

**

Department of Mathematics
Chungnam National University
Daejeon 305-764, Republic of Korea
E-mail: sey8405@cnu.ac.kr

***

Department of Mathematics
Mokwon University
Daejeon 305-729, Republic of Korea
E-mail: ischang@mokwon.ac.kr