

THE GENERALIZED TRIANGULAR FUZZY SETS

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ABSTRACT. For various fuzzy numbers, many operations have been calculated. We generalize about triangular fuzzy number and calculate four operations based on the Zadeh's extension principle, addition $A(+)B$, subtraction $A(-)B$, multiplication $A(\cdot)B$ and division $A(/)B$ for two generalized triangular fuzzy sets.

1. Introduction

The operations of two fuzzy numbers (A, μ_A) and (B, μ_B) defined in Definition 2.3 are based on the Zadeh's extension principle([4], [5], [6]). We consider four operations, addition $A(+)B$, subtraction $A(-)B$, multiplication $A(\cdot)B$ and division $A(/)B$ for two generalized triangular fuzzy sets A and B .

For two triangular fuzzy numbers, many results for various operations including the above four operations are known([1], [2]). About four operations, we introduce an example in Example 2.5.

In this paper, we generalize the triangular fuzzy number to generalized fuzzy set in Definition 3.1. It is a symmetric fuzzy set and may not have value 1. We study four operations for two generalized fuzzy sets in Theorem 3.2. The addition $A(+)B$ and subtraction $A(-)B$ becomes a generalized trapezoidal fuzzy set defined in Definition 2.5. But the multiplication $A(\cdot)B$ and division $A(/)B$ need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. And we give an example.

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2. Preliminaries

DEFINITION 2.1. The set $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ is said to be the α -cut of a fuzzy set A .

The membership function of a fuzzy set A can be expressed in terms of the characteristic functions of its α -cuts according to the formula

$$\mu_A(x) = \sup_{\alpha \in (0,1]} \min(\alpha, \mu_{A_\alpha}(x)),$$

where

$$\mu_{A_\alpha}(x) = \begin{cases} 1, & x \in A_\alpha, \\ 0, & \text{otherwise.} \end{cases}$$

It is easily checked that the following properties hold

$$(A \cup B)_\alpha = A_\alpha \cup B_\alpha, \quad (A \cap B)_\alpha = A_\alpha \cap B_\alpha.$$

DEFINITION 2.2. A triangular fuzzy number is a fuzzy number A having membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \quad a_3 \leq x, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x < a_3. \end{cases}$$

The above triangular fuzzy number is denoted by $A = (a_1, a_2, a_3)$.

DEFINITION 2.3. The addition, subtraction, multiplication, and division of two fuzzy numbers are defined as

1. Addition $A(+)B$:

$$\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}, \quad x \in A, y \in B.$$

2. Subtraction $A(-)B$:

$$\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}, \quad x \in A, y \in B.$$

3. Multiplication $A(\cdot)B$:

$$\mu_{A(\cdot)B}(z) = \sup_{z=x \cdot y} \min\{\mu_A(x), \mu_B(y)\}, \quad x \in A, y \in B.$$

4. Division $A(/)B$:

$$\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\}, \quad x \in A, y \in B.$$

EXAMPLE 2.4. For two triangular fuzzy numbers $A = (1, 2, 4)$ and $B = (2, 4, 5)$, we have

1. Addition : $A(+)B = (3, 6, 9)$.
2. Subtraction : $A(-)B = (-4, -2, 2)$.
3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \quad 20 \leq x, \\ \frac{-2+\sqrt{2x}}{2}, & 2 \leq x < 8, \\ \frac{7-\sqrt{9+2x}}{2}, & 8 \leq x < 20. \end{cases}$$

Note that $A(\cdot)B$ is not a triangular fuzzy number.

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{1}{5}, \quad 2 \leq x, \\ \frac{5x-1}{x+1}, & \frac{1}{5} \leq x < \frac{1}{2}, \\ \frac{-x+2}{x+1}, & \frac{1}{2} \leq x < 2. \end{cases}$$

Note that $A(/)B$ is not a triangular fuzzy number.

DEFINITION 2.5. A fuzzy set A having membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \quad a_4 \leq x, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ c, & a_2 \leq x < a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x < a_4, \end{cases}$$

where $a_i \in \mathbb{R}, i = 1, 2, 3, 4$ and $0 < c \leq 1$, is called a *generalized trapezoidal fuzzy set*.

The above generalized trapezoidal fuzzy set is denoted by $A = (a_1, a_2, c, a_3, a_4)$.

3. Generalized triangular fuzzy set

We generalize the triangular fuzzy number. A generalized triangular fuzzy set is symmetric and may not have value 1.

DEFINITION 3.1. *A generalized triangular fuzzy set is a symmetric fuzzy set A having membership function*

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \quad a_2 \leq x, \\ \frac{2c(x-a_1)}{a_2-a_1}, & a_1 \leq x < \frac{a_1+a_2}{2}, \\ \frac{-2c(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2} \leq x < a_2, \end{cases}$$

where $a_1, a_2 \in \mathbb{R}$ and $0 < c \leq 1$.

The above generalized triangular fuzzy set is denoted by $A = ((a_1, c, a_2))$.

THEOREM 3.2. *For two generalized triangular fuzzy sets $A = ((a_1, c_1, a_2))$ and $B = ((b_1, c_2, b_2))$, if $c_1 \leq c_2$ and $\mu_B(x) \geq c_1$ in $[k_1, k_2]$, we have the followings.*

1. $A(+)B = (a_1 + b_1, \frac{1}{2}(a_1 + a_2) + k_1, c_1, \frac{1}{2}(a_1 + a_2) + k_2, a_2 + b_2)$, i.e., $A(+)B$ is a generalized trapezoidal fuzzy set.

2. $A(-)B = (a_1 - b_2, \frac{1}{2}(a_1 + a_2) - k_2, c_1, \frac{1}{2}(a_1 + a_2) - k_1, a_2 - b_1)$, i.e., $A(-)B$ is a generalized trapezoidal fuzzy set.

3. $A(\cdot)B$ is a fuzzy set on (a_1b_1, a_2b_2) , but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. The membership function of $A(\cdot)B$ is

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < a_1b_1, \quad a_2b_2 \leq x, \\ \frac{1}{2pq}(-pb_1 - qa_1 + \sqrt{(pb_1 + qa_1)^2 - 4pq(a_1b_1 - x)}), & a_1b_1 \leq x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1, \\ \frac{1}{2}, & a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1 \leq x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2, \\ \frac{1}{2pq}(pb_2 + qa_2 - \sqrt{(pb_2 + qa_2)^2 - 4pq(a_2b_2 - x)}), & a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2 \leq x < a_2b_2, \end{cases}$$

where $p = \frac{a_2 - a_1}{2c_1}$ and $q = \frac{b_2 - b_1}{2c_2}$.

4. $A(/)B$ is a fuzzy set on $(\frac{a_1}{b_2}, \frac{a_2}{b_1})$, but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. The membership function of $A(/)B$ is

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{a_1}{b_2}, \frac{a_2}{b_1} \leq x, \\ \frac{2c_1c_2(b_2x-a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1}{b_2} \leq x < \frac{a_1+a_2}{2k_2}, \\ \frac{1}{2}, & \frac{a_1+a_2}{2k_2} \leq x < \frac{a_1+a_2}{2k_1}, \\ \frac{-2c_1c_2(b_1x-a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1+a_2}{2k_1} \leq x < \frac{a_2}{b_1}. \end{cases}$$

Proof. Note that

$$\mu_A(x) = \begin{cases} 0, & x < a_1, a_2 \leq x, \\ \frac{2c_1(x-a_1)}{a_2-a_1}, & a_1 \leq x < \frac{a_1+a_2}{2}, \\ \frac{-2c_1(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2} \leq x < a_2, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < b_1, b_2 \leq x, \\ \frac{2c_2(x-b_1)}{b_2-b_1}, & b_1 \leq x < \frac{b_1+b_2}{2}, \\ \frac{-2c_2(x-b_2)}{b_2-b_1}, & \frac{b_1+b_2}{2} \leq x < b_2. \end{cases}$$

We calculate exactly four operations using α -cuts. Let $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$ are the α -cuts of A and B, respectively. Since $\alpha = \frac{2c_1(a_1^{(\alpha)}-a_1)}{a_2-a_1}$ and $\alpha = \frac{-2c_1(a_2^{(\alpha)}-a_2)}{a_2-a_1}$, we have

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = \left[\frac{(a_2 - a_1)\alpha}{2c_1} + a_1, \frac{(a_2 - a_1)\alpha}{-2c_1} + a_2 \right].$$

Similarly, we have

$$B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}] = \left[\frac{(b_2 - b_1)\alpha}{2c_2} + b_1, \frac{(b_2 - b_1)\alpha}{-2c_2} + b_2 \right].$$

1. Addition : Since $c_1 \leq c_2$ and $\mu_B(x) \geq c_1$ in $[k_1, k_2]$, $\mu_{A(+)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2} + k_1, \frac{a_1+a_2}{2} + k_2]$. By the above facts,

$$\begin{aligned}
A_{\alpha}(+)B_{\alpha} &= [a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}] \\
&= \left[\frac{(a_2 - a_1)\alpha}{2c_1} + a_1 + \frac{(b_2 - b_1)\alpha}{2c_2} + b_1, \right. \\
&\quad \left. \frac{(a_2 - a_1)\alpha}{-2c_1} + a_2 + \frac{(b_2 - b_1)\alpha}{-2c_2} + b_2 \right].
\end{aligned}$$

If $x \in [a_1 + b_1, \frac{a_1+a_2}{2} + k_1]$, then $\frac{(a_2-a_1)\alpha}{2c_1} + a_1 + \frac{(b_2-b_1)\alpha}{2c_2} + b_1 = x$. Thus calculating for α , $\alpha = \frac{2c_1c_2(x-a_1-b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Similarly, for $x \in [\frac{a_1+a_2}{2} + k_2, a_2 + b_2]$, we have $\alpha = \frac{-2c_1c_2(x-a_2-b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Therefore

$$\mu_{A(+)B}(x) = \begin{cases} 0, & x < a_1 + b_1, \quad a_2 + b_2 \leq x, \\ \frac{2c_1c_2(x-a_1-b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & a_1 + b_1 \leq x < \frac{a_1+a_2}{2} + k_1, \\ \frac{1}{2}, & \frac{a_1+a_2}{2} + k_1 \leq x < \frac{a_1+a_2}{2} + k_2, \\ \frac{-2c_1c_2(x-a_2-b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & \frac{a_1+a_2}{2} + k_2 \leq x < a_2 + b_2. \end{cases}$$

Hence $A(+)B = (a_1 + b_1, \frac{1}{2}(a_1 + a_2) + k_1, c_1, \frac{1}{2}(a_1 + a_2) + k_2, a_2 + b_2)$, i.e., $A(+)B$ is a generalized trapezoidal fuzzy set.

2. Subtraction : Since $c_1 \leq c_2$ and $\mu_B(x) \geq c_1$ in $[k_1, k_2]$,

$\mu_{A(-)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2} - k_2, \frac{a_1+a_2}{2} - k_1]$. By the above facts,

$$\begin{aligned}
A_{\alpha}(-)B_{\alpha} &= [a_1^{(\alpha)} - b_2^{(\alpha)}, a_2^{(\alpha)} - b_1^{(\alpha)}] \\
&= \left[\frac{(a_2 - a_1)\alpha}{2c_1} + a_1 - \frac{(b_2 - b_1)\alpha}{-2c_2} - b_2, \right. \\
&\quad \left. \frac{(a_2 - a_1)\alpha}{-2c_1} + a_2 - \frac{(b_2 - b_1)\alpha}{2c_2} - b_1 \right].
\end{aligned}$$

If $x \in [a_1 - b_2, \frac{a_1+a_2}{2} - k_2]$, then $\frac{(a_2-a_1)\alpha}{2c_1} + a_1 - \frac{(b_2-b_1)\alpha}{-2c_2} - b_2 = x$. Thus calculating for α , $\alpha = \frac{2c_1c_2(x-a_1+b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Similarly, for $x \in [\frac{a_1+a_2}{2} - k_1, a_2 - b_1]$, we have $\alpha = \frac{-2c_1c_2(x-a_2+b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Therefore

$$\mu_{A(-)B}(x) = \begin{cases} 0, & x < a_1 - b_2, \quad a_2 - b_1 \leq x, \\ \frac{2c_1c_2(x-a_1+b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & a_1 - b_2 \leq x < \frac{a_1+a_2}{2} - k_2, \\ \frac{1}{2}, & \frac{a_1+a_2}{2} - k_2 \leq x < \frac{a_1+a_2}{2} - k_1, \\ \frac{-2c_1c_2(x-a_2+b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & \frac{a_1+a_2}{2} - k_1 \leq x < a_2 - b_1. \end{cases}$$

Hence $A(-)B = (a_1 - b_2, \frac{1}{2}(a_1 + a_2) - k_2, c_1, \frac{1}{2}(a_1 + a_2) - k_1, a_2 - b_1)$, i.e., $A(-)B$ is a generalized trapezoidal fuzzy set.

3. Multiplication : Since $c_1 \leq c_2$ and $\mu_B(x) \geq c_1$ in $[k_1, k_2]$, $\mu_{A(\cdot)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2}k_1, \frac{a_1+a_2}{2}k_2]$. By the above facts,

$$\begin{aligned} A_\alpha(\cdot)B_\alpha &= [a_1^{(\alpha)}b_1^{(\alpha)}, a_2^{(\alpha)}b_2^{(\alpha)}] \\ &= \left[\left(\frac{(a_2 - a_1)\alpha}{2c_1} + a_1 \right) \left(\frac{(b_2 - b_1)\alpha}{2c_2} + b_1 \right), \right. \\ &\quad \left. \left(\frac{(a_2 - a_1)\alpha}{-2c_1} + a_2 \right) \left(\frac{(b_2 - b_1)\alpha}{-2c_2} + b_2 \right) \right]. \end{aligned}$$

If $x \in [a_1b_1, \frac{a_1+a_2}{2}k_1]$, then $(\frac{(a_2-a_1)\alpha}{2c_1} + a_1)(\frac{(b_2-b_1)\alpha}{2c_2} + b_1) = x$. Thus calculating for α ,

$$\alpha = \frac{1}{2pq}(-pb_1 - qa_1 + \sqrt{(pb_1 + qa_1)^2 - 4pq(a_1b_1 - x)}), \text{ where } p = \frac{a_2-a_1}{2c_1}$$

and $q = \frac{b_2-b_1}{2c_2}$. Similarly,

$$\alpha = \frac{1}{2pq}(pb_2 + qa_2 - \sqrt{(pb_2 + qa_2)^2 - 4pq(a_2b_2 - x)})$$

for $x \in [\frac{a_1+a_2}{2}k_2, a_2b_2]$. Therefore

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < a_1b_1, \quad a_2b_2 \leq x, \\ \frac{1}{2pq}(-pb_1 - qa_1 + \sqrt{(pb_1 + qa_1)^2 - 4pq(a_1b_1 - x)}), & a_1b_1 \leq x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1, \\ \frac{1}{2}, & a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1 \leq x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2, \\ \frac{1}{2pq}(pb_2 + qa_2 - \sqrt{(pb_2 + qa_2)^2 - 4pq(a_2b_2 - x)}), & a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2 \leq x < a_2b_2, \end{cases}$$

where $p = \frac{a_2-a_1}{2c_1}$ and $q = \frac{b_2-b_1}{2c_2}$.

Hence $A(\cdot)B$ is a fuzzy set on (a_1b_1, a_2b_2) , but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

4. Division : Since $c_1 \leq c_2$ and $\mu_B(x) \geq c_1$ in $[k_1, k_2]$, $\mu_{A(/)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2k_2}, \frac{a_1+a_2}{2k_1}]$. By the above facts,

$$\begin{aligned} A_\alpha(/)B_\alpha &= [a_1^{(\alpha)}/b_2^{(\alpha)}, a_2^{(\alpha)}/b_1^{(\alpha)}] \\ &= \left[\frac{\frac{(a_2-a_1)\alpha}{2c_1} + a_1}{\frac{(b_2-b_1)\alpha}{-2c_2} + b_2}, \frac{\frac{(a_2-a_1)\alpha}{-2c_1} + a_2}{\frac{(b_2-b_1)\alpha}{2c_2} + b_1} \right]. \end{aligned}$$

If $x \in [\frac{a_1}{b_2}, \frac{a_1+a_2}{2k_2}]$, then $(\frac{(a_2-a_1)\alpha}{2c_1} + a_1) / (\frac{(b_2-b_1)\alpha}{-2c_2} + b_2) = x$. Thus calculating for α , $\alpha = \frac{2c_1c_2(b_2x-a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}$. Similarly, for $x \in [\frac{a_1+a_2}{2k_1}, \frac{a_2}{b_1}]$, we have $\alpha = \frac{-2c_1c_2(b_1x-a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}$. Therefore

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{a_1}{b_2}, \frac{a_2}{b_1} \leq x, \\ \frac{2c_1c_2(b_2x-a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1}{b_2} \leq x < \frac{a_1+a_2}{2k_2}, \\ \frac{1}{2}, & \frac{a_1+a_2}{2k_2} \leq x < \frac{a_1+a_2}{2k_1}, \\ \frac{-2c_1c_2(b_1x-a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1+a_2}{2k_1} \leq x < \frac{a_2}{b_1}. \end{cases}$$

Hence $A(/)B$ is a fuzzy set on $(\frac{a_1}{b_2}, \frac{a_2}{b_1})$, but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. \square

EXAMPLE 3.3. Let $A = ((2, \frac{1}{2}, 8))$ and $B = ((1, \frac{4}{5}, 5))$ be generalized triangular fuzzy sets, i.e.,

$$\mu_A(x) = \begin{cases} 0, & x < 2, 8 \leq x, \\ \frac{1}{6}(x-2), & 2 \leq x < 5, \\ -\frac{1}{6}(x-8), & 5 \leq x < 8, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < 1, 5 \leq x, \\ \frac{2}{5}(x-1), & 1 \leq x < 3, \\ -\frac{2}{5}(x-5), & 3 \leq x < 5. \end{cases}$$

Let A_α and B_α be the α -cuts of A and B , respectively. Let $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$. Since $\alpha = \frac{1}{6}(a_1^{(\alpha)} - 2)$ and $\alpha = -\frac{1}{6}(a_2^{(\alpha)} - 8)$, $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = [6\alpha + 2, -6\alpha + 8]$. Since $\alpha = \frac{2}{5}(b_1^{(\alpha)} - 1)$ and $\alpha = -\frac{2}{5}(b_2^{(\alpha)} - 5)$, we have $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}] = [\frac{5}{2}\alpha + 1, -\frac{5}{2}\alpha + 5]$. Then we have the followings.

1. Addition :

$$\mu_{A(+)B}(x) = \begin{cases} 0, & x < 3, 13 \leq x, \\ \frac{2}{17}(x-3), & 3 \leq x < \frac{29}{4}, \\ \frac{1}{2}, & \frac{29}{4} \leq x < \frac{35}{4}, \\ \frac{-2}{17}(x-13), & \frac{35}{4} \leq x < 13, \end{cases}$$

i.e., $A(+)B = (3, \frac{29}{4}, \frac{1}{2}, \frac{35}{4}, 13)$.

2. Subtraction :

$$\mu_{A(-)B}(x) = \begin{cases} 0, & x < -3, \quad 7 \leq x, \\ \frac{2}{17}(x+3), & -3 \leq x < \frac{5}{4}, \\ \frac{1}{2}, & \frac{5}{4} \leq x < \frac{11}{4}, \\ \frac{-2}{17}(x-7), & \frac{11}{4} \leq x < 7, \end{cases}$$

i.e., $A(-)B = (-3, \frac{5}{4}, \frac{1}{2}, \frac{11}{4}, 7)$.

3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \quad 40 \leq x, \\ \frac{1}{30}(-11 + \sqrt{121 - 60(2-x)}), & 2 \leq x < \frac{45}{4}, \\ \frac{1}{2}, & \frac{45}{4} \leq x < \frac{75}{4}, \\ \frac{1}{30}(50 - \sqrt{2500 - 60(40-x)}), & \frac{75}{4} \leq x < 40, \end{cases}$$

Thus $A(\cdot)B$ is a fuzzy set on $(2, 40)$, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{2}{5}, \quad 8 \leq x, \\ \frac{10x-4}{5x+12}, & \frac{2}{5} \leq x < \frac{4}{3}, \\ \frac{1}{2}, & \frac{4}{3} \leq x < \frac{20}{9}, \\ \frac{-2(x-8)}{5x+12}, & \frac{20}{9} \leq x < 8, \end{cases}$$

Thus $A(/)B$ is a fuzzy set on $(\frac{2}{5}, 8)$, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

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