Korean J. Math. **17** (2009), No. 1, pp. 103–112

THE SOLUTIONS OF A PERTURBED ELLIPTIC EQUATION WITH EXPONENTIAL GROWTH

Tacksun Jung and Q-Heung Choi*

Abstract. We show the existence of the solutions for the following nonlinear elliptic problem under the Dirichlet boundary condition. To show the existence of the solutions we use the variational formulation.

1. Introduction and statement of main result

In this paper we study the following nonlinear elliptic problem under the Dirichlet boundary condition

$$\begin{cases} -\Delta u &= a(x)g(u) + f(x) \qquad \text{in } \Omega, \qquad u \in C^2(\overline{\Omega}) \\ u &= 0 \qquad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^N$ is a bounded domain with a smooth boundary $\partial\Omega$, f is a given function in $L^2(\Omega)$, and $a : \overline{\Omega} \to R$ is a continuous function which changes sign on Ω, so that the open sets

$$\Omega^+ = \{x \in \Omega \mid a(x) > 0\} \qquad \textit{and} \qquad \Omega^- = \{x \in \Omega | a(x) < 0\}$$

are both nonempty. We shall write $a = a^+ - a^-$, where $a^+ = a \cdot \chi_{\Omega^+}$ and $a^- = -a \cdot \chi_{\Omega^-}$. We mainly deal with the case $N = 2$. We assume that g satisfies the following conditions:
$(g1)$ $g \in C(\mathrm{R},\mathrm{R})$,
$(g2)$ there is a constant $A_0 > 0$ such that

$$|g(\xi)| \leq A_0 \exp^{\phi(\xi)} \qquad \text{for } \xi \in R,$$

Received March 6, 2009. Revised March 16, 2009.
2000 Mathematics Subject Classification: 35J15, 35J20.
Key words and phrases: exponential growth, existence of solution,eigenvalue, critical point, boundary value problem.
*Corresponding author.

where $\phi : R \to R$ is a function satisfying $\phi(\xi)\xi^{-2} \to 0$ as $|\xi| \to \infty$,
$(g3)$ there are constants $\mu > 2$ and $r_0 \geq 0$ such that

$$0 < \mu G(\xi) = \mu \int_0^\xi g(t)dt \leq \xi g(\xi) \qquad \text{for } |\xi| \geq r_0.$$

$(g4)$ there exist $0 < \alpha_1 \leq \alpha_2 < 2$, $A_1, A_2 > 0$, and $B_1, B_2 \geq 0$ such that

$$A_1 \exp^{|\xi|^{\alpha_1}} - B_1 \leq G(\xi) = \int_0^\xi g(t)dt \leq A_2 \exp^{|\xi|^{\alpha_2}} + B_2 \qquad \text{for } \xi \in R,$$

where α_1, α_2 are further restricted by

$$\frac{2}{\alpha_2} - 2 > \frac{1}{\alpha_1}.$$

Remark that the conditions $0 < \alpha_1 \leq \alpha_2 < 2$ and $\frac{2}{\alpha_2} - 2 > \frac{1}{\alpha_1}$ imply $\alpha_2 < \frac{1}{2}$. Note that $(g3)$ implies the existence of positive constants a_1, a_2, a_3 such that

$$\frac{1}{\mu}\left(\xi g(\xi) + a_1\right) \geq G(\xi) + a_2 \geq a_3|\xi|^\mu \qquad \text{for } \xi \in R. \tag{1.2}$$

Khanfir and Lassoued [10] prove that if
$f = 0$,
g is locally Hölder continuous on R_+,
$g(u) = o(u)$ as $u \to 0^+$,
$g(u)u \geq \beta \int_0^u g(t)dt > 0 \qquad$ for $u > 0$, $\beta \in]2, \frac{2N}{N-2}[$,
$g(u) \leq C_1\left(u^{\beta-1} + 1\right)$,
then there exists a sequence $(\beta_n)_{n\geq 1}$ of nonnegative reals strictly increasing to the real $2^* = \frac{2N}{N-2}$ such that $\beta < \beta_n$ and $M_R = o\left(R^{\frac{2}{(\beta-1)^n}}\right)$, the problem (1.1) has at least one solution. They also prove that there exists an $\epsilon > 0$ depending on g and a^+ such that if $\int_{\Omega^-} a^- dx < \epsilon$, problem (1.1) has at least one solution.

In this paper, we shall investigate the existence of solutions of the problem (1.1) by the variational method. It is well known that the solution of problem (1.1) corresponds to the nonzero critical point of the functional

$$I(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 - \int_\Omega a(x)G(u)dx - \int_\Omega fudx.$$

We will prove the following: If the function $g(u)u - \mu G(u)$ is bounded, then I satisfies the Palais-Smale condition and the mountain pass theorem gives a solution of (1.1), which is bounded (cf. Theorem 1.1).

Moreover, if $g(u)u - \mu G(u)$ is not bounded and there exists an $\epsilon > 0$ such that $\int_{\Omega^-} a^- dx < \epsilon$, then I also satisfies the Palais-Smale condition and the mountain pass theorem and the variational method [13] give at least two solution of (1.1), one of which is bounded and the other solution is large norm(cf. Theorem 1.2(1) and (2)).
Our main results are as follows:

THEOREM 1.1. *Assume that g satisfies $(g1) - (g4), g(u)u - \mu G(u)$ is bounded, and $f \in L^2(\Omega)$. Then problem (1.1) has at least one solution, which is bounded.*

THEOREM 1.2. *Assume that g satisfies $(g1) - (g4), g(u)u - \mu G(u)$ is not bounded, and $f \in L^2(\Omega)$. We also assume that there exists a small $\epsilon > 0$ such that $\int_{\Omega^-} a^-(x) < \epsilon$. Then problem (1.1) has at least two solutions, (1) one of which is bounded and (2) the other solution is large norm such that for any M,*

$$\max_{x \in \Omega} |u(x)| > M.$$

Theorem 1.1 and 1.2 will be proved in Section 2 and Section 3 via variational methods. An outline of this paper is as follows: in Section 2 we introduce a functional I whose critical points and weak solutions of (1.1) possess one-to-one correspondence. Next we prove that $I \in C^1(E, R)$ and satisfies the Palais-Smale condition. We introduce subspaces E_n such that there exists an $R_n > 0$ such that $I(u_n) \leq 0$ for $u_n \in E_n \backslash B_{R_n}$. Next we introduce minimax values b_n and use the fact that there exist $\theta_1 > 0$ and $N_1 \in N$ such that

$$b_n \geq \theta_1 n \left(\log n\right)^{\frac{2}{\alpha_2} - 2} \qquad \text{for all } n \geq N_1,$$

to show that for n large enough, $b_n > 0$ is a critical value of I in each $D_n = B_{R_n} \cap E$. Furthermore we prove that if $g(u)u - \mu G(u)$ is bounded and u is a critical point of I, then $I(u)$ is bounded, from which u is bounded. In Section 3 we prove Theorem 1.2(2). From the fact that for n large enough, $b_n > 0$ is a critical value of I and $\lim_{n \to \infty} b_n = +\infty$, we prove that if $g(u)u - \mu G(u)$ is not bounded, u is a critical point of I, and for some real K, $\max_{\Omega} |u(x)| \leq K$, then $I(u)$ is bounded, which contradicts to the fact that $\lim_{n \to \infty} b_n = +\infty$. So we complete the proof of Theorem 1.2(2).

2. Variational formulation and proof of Theorem 1.1

$E = H_0^1(\Omega)$ be the completion of $C_0^\infty(\Omega)$ with respect to the norm

$$\|u\| = \left(\int_\Omega |\nabla u|^2 dx \right)^{\frac{1}{2}},$$

and let us use the notation:

$$\|u\|_{L^p(\Omega)} = \left(\int_\Omega |u|^p dx \right)^{\frac{1}{p}} \qquad \text{for } p \in [1, \infty).$$

We consider the following functional associated with (1.1)

$$\begin{aligned} I(u) = \int_\Omega & \frac{1}{2}|\nabla u|^2 dx - \int_\Omega a(x)G(u)dx \\ & - \int_\Omega f(x)udx \qquad \text{for } u \in E, \qquad (2.1) \end{aligned}$$

where

$$G(u) = \int_0^u g(t)dt.$$

From $(g1)$ and $(g2)$, I is well defined. The solutions of (1.1) coincide with the nonzero critical points of $I(u)$. From the assumptions $(g1)-(g4)$ we can obtain the following propositions (For the proof of Proposition 2.1, we refer to Appendix B in [13]).

PROPOSITION 2.1. *Assume that g satisfies $(g1)-(g4)$ and $f \in L^2(\Omega)$. Then $I(u)$ is continuous and Fréchet differentiable in E with Fréchet derivative*

$$I'(u)h = \int_\Omega [\nabla u \cdot \nabla h - a(x)g(u)h - f(x)h]dx \qquad \text{for } h \in E. \qquad (2.2)$$

If we set

$$H(u) = \int_\Omega a(x)G(u)dx,$$

then $H'(u)$ is continuous with respect to weak convergence, $H'(u)$ is compact, and

$$H'(u)h = \int_\Omega a(x)g(u)hdx \qquad \text{for all } h \in E.$$

This implies that $I \in C^1(E, R)$ and $H(u)$ is weakly continuous.

PROPOSITION 2.2. *Assume that g satisfies $(g1)$-$(g4)$ and $f \in L^2(\Omega)$. We also assume that $g(u)u - \mu G(u)$ is bounded, or there exists an $\epsilon > 0$ such that $\int_{\Omega^-} a^-(x)dx < \epsilon$ even if $g(u)u - \mu G(u)$ is unbounded. Then $I(u)$ satisfies the Palais-Smale condition: If for a sequence (u_m), $I(u_m)$ is bounded from above and $I'(u_m) \to 0$ as $m \to \infty$, then (u_m) is bounded.*

Proof. Suppose that (u_m) is a sequence with $I(u_m) \leq M$ and $I'(u_m) \to 0$ as $m \to \infty$. Then by $(g2)$, $(g3)$, $(g4)$, and Hölder inequality and Sobolev Embedding Theorem, for large m and $\mu > 2$ with $u = u_m$, we have

$$
\begin{aligned}
& M\mu + \|u\| \geq \mu I(u) - I'(u)u \\
= \; & \left(\frac{\mu}{2} - 1\right) \|u\|^2 + \int_\Omega a(x)[g(u)u - \mu G(u)]dx - (\mu - 1) \int_\Omega f(x)udx \\
= \; & \left(\frac{\mu}{2} - 1\right) \|u\|^2 + \int_\Omega a^+(x)[g(u)u - \mu G(u)]dx \\
& - \int_\Omega a^-(x)[g(u)u - \mu G(u)]dx \, (\mu - 1) \, \|f\|_{L^2(\Omega)} \|u\|_{L^2(\Omega)} \\
\geq \; & \left(\frac{\mu}{2} - 1\right) \|u\|^2 - \max_\Omega |g(u)u - \mu G(u)| \int_{\Omega^-} a^-(x)dx - C\|u\|.
\end{aligned}
$$

Thus if $g(u)u - \mu G(u)$ is bounded, or if there exists an $\epsilon > 0$ such that $\int_{\Omega^-} a^-(x) < \epsilon$, then we have

$$
M_1 (1 + \|u\|) \geq \left(\frac{\mu}{2} - 1\right) \|u\|^2 \qquad \text{for } M_1 > 0,
$$

from which the boundedness of (u_m) follows. On the other hand, let $D : E \to E^*$ denote the duality map between E and E^*. Then for $u, \varphi \in E$,

$$
(Du_m) \varphi_m = \int_\Omega \nabla u_m \cdot \nabla \varphi_m dx.
$$

Then $D^{-1} I'(u_m) = u_m - D^{-1}\mathcal{P}(u_m)$, where $\mathcal{P} : E \to E^*$ is compact. Therefore, boundedness of (u_m) implies $D^{-1}\mathcal{P}(u_m)$ converges along a subsequence. Hence by the assumption $I'(u_m) \to 0$, we can conclude that $\{u_m\}$ is relatively compact in E. $\square$

Now, we consider the following eigenvalue problem

$$
\begin{aligned}
-\Delta u &= \lambda u \qquad \text{in } \Omega, \\
u &= 0 \qquad \text{on } \partial\Omega.
\end{aligned}
\tag{2.3}
$$

The eigenvalue problem (2.3) possesses a sequence of eigenvalues such that $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \ldots$ (with repetitions according to the multiplicity of each eigenvalue). We denote by e_j the eigenfunction which corresponds to λ_j, where we may assume $\int_\Omega \nabla e_i \cdot \nabla e_j dx = \delta_{ij}$ for $i, j \in N$. Let $E_n \equiv \text{span}\{e_1, \ldots, e_n\}$. We note that

$$\|u\| \leq \lambda_n^{\frac{1}{2}} \|u\|_{L^2(\Omega)} \qquad \text{for } u \in E_n. \tag{2.4}$$

From now on we will assume that $g(u)u - \mu G(u)$ is bounded. The next proposition shows that I satisfies the one of the geometrical assumptions of the mountain pass theorem (cf. [2])

Proposition 2.3. *Assume that g satisfies $(g1)-(g4)$, and $f \in L^2(\Omega)$. We also assume that there exists an $\epsilon > 0$ such that $a^-(x) < \epsilon$. Then there exists an $R_n > 0$ such that*

$$I(u) \leq 0 \qquad \textit{for } u \in E_n \backslash B_{R_n}, \tag{2.5}$$

where $B_{R_n} \equiv \{u \in E \mid \|u\| \leq R_n\}$.

Proof. If we choose $\psi \in E$ such that $\|\psi\| = 1, \psi \geq 0$ in Ω and $\text{supp}(\psi) \subset \Omega^+$, then, by (1.2), (2.4), the Hölder inequality, and the Sobolev Embedding Theorem, we have

$$\begin{aligned}
I(t\psi) &\leq \frac{1}{2}t^2 - \int_{\Omega^+} a(x)\left(a_3 t^\mu \psi^\mu - a_4\right) + \|f\|_{L^2(\Omega)}\|u\|_{L^2(\Omega)} \\
&\leq \frac{1}{2}t^2 - a_3 t^\mu \int_{\Omega^+} a(x)\psi^\mu + C_1 + C_2 t \\
&\leq \frac{1}{2}t^2 - a_3 t^\mu \|a(x)^{\frac{1}{\mu}\cdot\psi}\|^\mu_{L^\mu(\Omega)} + C_1 + C_2 t \\
&\leq \frac{1}{2}t^2 - a_3 t^\mu \|a(x)^{\frac{1}{\mu}} \cdot \psi\|^\mu_{L^2(\Omega)} + C_1 + C_2 t \\
&\leq \frac{1}{2}t^2 - a_3 \lambda_n^{-\frac{\mu}{2}} t^\mu \|a(x)^{\frac{1}{\mu}} \cdot \psi\|^\mu + C_1 + C_2.
\end{aligned}$$

Since $\mu > 2$, there exist a t_n great enough for each n and an $R_n > 0$ such that $u_n = t_n\psi$ and $I(u) \leq 0$ if $u_n \in E_n \backslash B_{R_n}$ and $\|u_n\| > R_n$. $\square$

Here, we may assume that $R_n < R_{n+1}$ for all $n \in N$.
Now we set $D_n = B_{R_n} \cap E_n$, $\partial D_n = \partial B_{R_n} \cap E_n$, and

$$\Gamma_n = \{\gamma \in C([0,1], E) \mid \gamma(0) = 0 \text{ and } \gamma(1) = u_n\}.$$

Define

$$b_n = \inf_{\gamma\in\Gamma_n} \max_{[0,1]} I(\gamma(u)), \qquad n \in N.$$

As in Proposition 4.1 in [16], we have the following proposition.

Proposition 2.4. *There exist $\theta_1 > 0$ and $N_1 \in N$ such that*

$$b_n \geq \theta_1 \cdot n \cdot (\log n)^{\frac{2}{\alpha_2}-2} \qquad \textit{for all } n \geq N_1.$$

By Proposition 2.4, there exist θ_1 and $\tilde{N} \in N$ such that

$$b_n > 0 \qquad \text{for all } n \in \tilde{N}. \tag{2.6}$$

Proof of Theorem 1.1 and 1.2(1). (1) Assume that $g(u)u-\mu G(u)$ is bounded. From Proposition 2.1 and 2.2, $I \in C^1(E, R)$ and satisfies the Palais-Smale condition. From Proposition 2.3 and 2.4, there exists an $\tilde{\tilde{N}}$ such that for all $n \geq \tilde{\tilde{N}} \geq \tilde{N}$,
(1) $b_n > 0$,
(2) there exists an $u_n \in E_n \backslash B_{R_n}$ such that $I(u_n) \leq 0$.
We note that $I(0) = 0$. The above facts shows that $I(u)$ satisfies the geometrical assumptions of the mountain pass theorem. Therefore by the mountain pass theorem, $I(u)$ has a critical value $b_n > 0$ for $n \geq \tilde{\tilde{N}}$. We denote by $\tilde{u}_n$ a critical point of I such that $I(\tilde{u}_n) = b_n$. Now, we claim that there exists a constant $C_n > 0$ such that

$$\|\tilde{u}_n\|^2 \leq C_n \left(1 + L_n \int_\Omega a^- dx\right), \tag{2.7}$$

where $L_n = \max\limits_{\Omega} |g(\tilde{u}_n)\tilde{u}_n - \mu G(\tilde{u}_n)|$. In fact, we have

$$b_n \leq \max I(tu_n),$$

and

$$\begin{aligned} I(tu_n) &= \frac{1}{2}t^2\|u_n\|^2 - \int_\Omega a^+(x)G(tu_n)dx \\ &\quad + \int_\Omega a^-(x)G(tu_n)dx + t\|f\|_{L^2(\Omega)}\|u\|_{L^2(\Omega)} \\ &\leq t^2\|u_n\|^2 - a_3t^\mu \int_\Omega a^+(x)|u_n|^\mu + a_4 \int_\Omega a^+(x)dx \\ &\quad + A_2e^{t^{\alpha_2}} \int_\Omega a^-(x)e^{|u_n|^{\alpha_2}}dx + B_2 \int_\Omega a^-(x)dx + C_1t \\ &\leq Ct^2 - Ct^\mu + Ce^{t^{\alpha_2}} + C + Ct. \end{aligned}$$

Since $0 \le t \le 1$, b_n is bounded: $b_n \le C_n$ for all $n \ge \tilde{\tilde{N}}$.
We can write

$$\begin{aligned}
\mu b_n &= \mu I(\tilde{u}_n) - I'(\tilde{u}_n)\tilde{u}_n \\
&= \left(\frac{\mu}{2} - 1\right)\|\tilde{u}_n\|^2 + \int_\Omega a(x)[g(\tilde{u}_n)\tilde{u}_n - \mu G(\tilde{u}_n)] - (\mu - 1)\int_\Omega f\tilde{u}_n \\
&\ge \left(\frac{\mu}{2} - 1\right)\|\tilde{u}_n\|^2 - \int_\Omega a^-(x)[g(\tilde{u}_n)\tilde{u}_n - \mu G(\tilde{u}_n)] \\
&\quad - b_1(\mu - 1)\|f\|\|\tilde{u}_n\|
\end{aligned}$$

So

$$\left(\frac{\mu}{2} - 1\right)\|\tilde{u}_n\|^2 \le \mu b_n + L_n \int_{\Omega^-} a^-(x)dx + b_2.$$

Thus we have

$$\|\tilde{u}_n\|^2 \le C_n \left(1 + L_n \int_{\Omega^-} a^-(x)dx\right)$$

and the proof of Theorem 1.1 is complete.
On the other hand, by Proposition 2.2, if $g(u)u - \mu G(u)$ is not bounded and there exists an $\epsilon > 0$ such that $\int_{\Omega^-} a^-(x)dx < \epsilon$, then $I(u)$ satisfies the Palais-Smale condition. Proposition 2.3, 2.4, and the mountain pass theorem show that there exists a $\tilde{\tilde{N}}$ such that $I(u)$ has a critical value b_n with critical point $\tilde{u}_n$ such that $I(\tilde{u}_n) = b_n$ for all $n \ge \tilde{\tilde{N}}$. If $\int_{\Omega^-} a^-(x)dx$ is sufficiently small, by (2.7), we have

$$\|\tilde{u}_n\|^2 \le C_n \qquad \text{for } C_n > 0,$$

from which we can conclude that $\tilde{u}_n$ is bounded and the proof of Theorem 1.2(1) is complete.

3. Proof of Theorem 1.2(2)

We assume that $g(u)u - \mu G(u)$ is not bounded, and there exists an $\epsilon > 0$ such that $\int_{\Omega^-} a^-(x)dx < \epsilon$. From Proposition 2.1 and 2.2, $I \in C^1(E, R)$ and satisfies the Palais-Smale condition. From Proposition 2.3, there exists an $R_n > 0$ such that $I(u_n) \le 0$ for $u_n \in E_n \backslash B_{R_n}$. We note that $I(0) = 0$. By Proposition 2.4 and Mountain pass theorem, we see that for n large enough $b_n > 0$ is a critical value of I and

$$\lim_{n \to \infty} b_n = +\infty. \tag{3.1}$$

We denote by $\tilde{u}_n$ a critical point of I such that $I(\tilde{u}_n) = b_n$. Then we claim that for each real number M, $\max_{\Omega} |\tilde{u}_n(x)| \geq M$. In fact, $-\Delta u = a(x)g(u) + f(x)$ and $\max_{\Omega} |\tilde{u}_n(x)| \leq K$ imply

$$I(\tilde{u}_n) \leq \max_{|\tilde{u}_n| \leq K} \left(\frac{1}{2} g(\tilde{u}_n)\tilde{u}_n - G(\tilde{u}_n) \right) \int_{\Omega} |a(x)| - \frac{1}{2} \int_{\Omega} f \tilde{u}_n,$$

which means that b_n is bounded, which contradicts to the fact (3.1). Thus the proof is complete.

References

[1] Amann, H., Zehnder, E., *Multiple periodic solutions for a class of nonlinear-autonomous wave equations*, Houston J. Math. **7**, 147-174(1981).

[2] Ambrosetti, A., Rabinowitz, P. H., *Dual variational methods in critical point theory and application*, J. funct. Analysis **14**, 349-381(1973).

[3] Benci, V., Rabinowitz, P. H., *Critical point theorems for indefinite functionals*, Inventions Math. **52**, 241-273(1979).

[4] Berger, M. S., *Nonlinearity and Functional Analysis*, Academic Press, New York(1977).

[5] Q. H. Choi, T. S. Jung, *An application of a variational reduction method to a nonlinear wave equation*, J. Differential Equations **7**, 390-410(1995).

[6] Coffman, C. V., *On a class of nonlinear elliptic boundary value problems*, J.Math. Mech. **19**, 351-356(1970).

[7] Crandall,M. G., Rabinowitz, P. H., *Continuation and variational methods for the existence of positive solutions of nonlinear elliptic eigenvalue problems*, Arch. Rat. Mech. Anal. **58**, 201-218(1975).

[8] Ding W. Y, Ni W. M, *On the existence of positive entire solution of a semilinear elliptic equation*, Arch. Rat. Mech. Anal. **91**, 288-308(1986).

[9] T. S. Jung, Q. H. Choi, *Multiplicity results on a nonlinear biharmonic equation*, Nonlinear Analysis, Theory, Methods and Applications, **30**, No.8, 5083-5092(1997).

[10] Khanfir, S., Lassoued, L., *On the existence of positive solutions of a semilinear elliptic equation with change of sign*, Nonlinear Analysis, Theory, Methods and Applications, **22**, No.11, 1309-1314(1994).

[11] Long. Y, *Periodic solutions of perturbed superquadratic hamiltonian systems*, Ann. Scu. Norm. Sup. Pisa Cl. Sci. **17**, No. 4, 35-77(1990).

[12] Rabinowitz, P. H., *Periodic solutions of Hamiltonian systems*, Comm. Pure Appl. Math. **31**, 157-184(1978).

[13] Rabinowitz, P. H. *Minimax methods in critical point theory with applications to differential equations*, CBMS. Regional conf. Ser. Math. **65**, Amer. Math. Soc., Providence, Rhode Island(1986).

[14] Rabinowitz, P. H. *Periodic solutions of large norm of hamiltonian systems*, J. of Differential Equations **50**, 33-48(1983).

[15] Schwartz, J. T. *Nonlinear functional analysis*, Gordon and Breach, New York, 1969.

[16] Sugimura Kunihiko, *Existence of infinitely many solutions for a perturbed elliptic equation with exponential growth*, Nonlinear Analysis, Theory, Methods and Applications, **22**, 277-293(1994).

Department of Mathematics
Kunsan National University
Kunsan 573-701, Korea
E-mail: tsjung@kunsan.ac.kr

Department of Mathematics Education
Inha University
Incheon 402-751, Korea
E-mail: qheung@inha.ac.kr