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DOUBLY SIMULATIVE WFI-ALGEBRAS

YOUNG BAE JUN†, KYOUNG JA LEE AND CHUL HWAN PARK*

ABSTRACT. Characterizations of simulative WFI-algebras are provided. The notion of commutators, doubly simulative parts, doubly simulative WFI-algebras, and WFI-morphisms are introduced. Using the notion of commutators, the conditions for a WFI-algebra to be simulative are given. Characterizations of doubly simulative WFI-algebras are discussed. Using the notion of doubly simulative WFI-algebras, a commutative pomonoid is established.

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1. Introduction

In 1990, W. M. Wu [7] introduced the notion of fuzzy implication algebras (FI-algebra, for short), and investigated several properties. In [6], Z. Li and C. Zheng introduced the notion of distributive (resp. regular, commutative) FI-algebras, and investigated the relations between such FI-algebras and MV-algebras. In [1], the first author discussed several aspects of WFI-algebras. He introduced the notion of associative (resp. normal, medial) WFI-algebras, and investigated several properties. He gave conditions for a WFI-algebra to be associative/medial, and provided characterizations of associative/medial WFI-algebras, and showed that every associative WFI-algebra is a group in which every element is an involution. He also verified that the class of all medial WFI-algebras is a variety. Y. B. Jun and S. Z. Song [5] introduced the notions of simulative and/or mutant WFI-algebras and investigated some properties. They established characterizations of a simulative WFI-algebra, and gave a relation between an associative WFI-algebra and a simulative WFI-algebra. They

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also found some types for a simulative WFI-algebra to be mutant. Jun et al. [4] introduced the concept of ideals of WFI-algebras, and gave relations between a filter and an ideal. Moreover, they provided characterizations of an ideal, and established an extension property for an ideal. In [2] and [3], the first author discussed perfect, weak and concrete filters. In this paper, we give characterizations of simulative WFI-algebras. We introduce the notion of commutators, doubly simulative parts, doubly simulative WFI-algebras, and WFI-morphisms. Using the notion of commutators, we give conditions for a WFI-algebra to be simulative. We discuss characterizations of doubly simulative WFI-algebras, and we establish a commutative pomonoid by using the notion of doubly simulative WFI-algebras.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2, 0)$. By a *WFI-algebra* we mean a system $\mathfrak{X} := (X, \ominus, 1) \in K(\tau)$ in which the following axioms hold:

(a1) $(x \in X)\ (x \ominus x = 1)$,
(a2) $(x, y \in X)\ (x \ominus y = y \ominus x = 1 \Rightarrow x = y)$,
(a3) $(x, y, z \in X)\ (x \ominus (y \ominus z) = y \ominus (x \ominus z))$,
(a4) $(x, y, z \in X)\ ((x \ominus y) \ominus \big((y \ominus z) \ominus (x \ominus z)\big) = 1)$.

We call the special element 1 the *unit*. For the convenience of notation, we shall write $[x, y_1, y_2, \cdots, y_n]$ for $(\cdots((x \ominus y_1) \ominus y_2) \ominus \cdots) \ominus y_n$. We define $[x, y]^n = [x, y, y, \cdots, y]$, for $n \geq 2$, where y occurs n-times. We use the notation $x^n \ominus y$ instead of $x \ominus (\cdots (x \ominus (x \ominus y)) \cdots)$ in which x occurs n-times.

Proposition 2.1. [1] *In a WFI-algebra $\mathfrak{X}$, the following are true:*

(b1) $x \ominus [x, y]^2 = 1$,
(b2) $1 \ominus x = 1 \Rightarrow x = 1$,
(b3) $1 \ominus x = x$,
(b4) $x \ominus y = 1 \Rightarrow [y, z, x \ominus z] = 1 \,\&\, [z, x, z \ominus y] = 1$,
(b5) $[x, y, 1] = [x, 1, y \ominus 1]$,
(b6) $[x, y]^3 = x \ominus y$.

A WFI-algebra $\mathfrak{X}$ is said to be *normal* (see [1]) if it satisfies:

$$(\forall a \in X)\,(X \ominus a = X = a \ominus X).$$

A nonempty subset S of a WFI-algebra $\mathfrak{X}$ is called a *subalgebra* of $\mathfrak{X}$ if $x \ominus y \in S$ whenever $x, y \in S$. A nonempty subset F of a WFI-algebra $\mathfrak{X}$ is called a *filter* of $\mathfrak{X}$ if it satisfies:

(c1) $1 \in F$,
(c2) $(\forall x \in F)\ (\forall y \in X)\ (x \ominus y \in F \Rightarrow y \in F)$.

A filter F of a WFI-algebra $\mathfrak{X}$ is said to be *closed* [1] if F is also a subalgebra of $\mathfrak{X}$.

Proposition 2.2. [1] *Let F be a filter of a WFI-algebra $\mathfrak{X}$. Then F is closed if and only if $x \ominus 1 \in F$ for all $x \in F$.*

Proposition 2.3. [1] *In a finite WFI-algebra, every filter is closed.*

We now define a relation "$\preceq$" on $\mathfrak{X}$ by $x \preceq y$ if and only if $x \ominus y = 1$. It is easy to verify that a WFI-algebra is a partially ordered set with respect to $\preceq$.

3. Simulative WFI-algebras

In what follows, let $\mathfrak{X}$ denote a WFI-algebra unless otherwise specified. For a WFI-algebra $\mathfrak{X}$, the set

$$\mathcal{S}(\mathfrak{X}) := \{x \in X \mid x \preceq 1\}$$

is called the *simulative part* of $\mathfrak{X}$. A WFI-algebra $\mathfrak{X}$ is said to be *simulative* [5] if it satisfies

(S) $x \preceq 1 \Rightarrow x = 1$.

Note that the condition (S) is equivalent to $\mathcal{S}(\mathfrak{X}) = \{1\}$.

Lemma 3.1. [5] *The following assertions are equivalent:*

(i) *$\mathfrak{X}$ is simulative.*
(ii) $(\forall x \in X)\ ([x, 1, 1] = x)$.
(iii) $(\forall x, y \in X)\ ([x, 1, y] = [y, 1, x])$.
(iv) $(\forall x, y \in X)\ ([x, y, 1] = y \ominus x)$.
(v) $(\forall x, y \in X)\ ([x, y, y] = x)$.

Proposition 3.2. *Every simulative WFI-algebra $\mathfrak{X}$ satisfies the following assertion:*

$$(\forall x, y, z \in X)\,(x \ominus (y \ominus z) = [x, 1, y, z]).$$

Proof. Using (a3) and Lemma 3.1, we have

$$\begin{aligned} x \ominus (y \ominus z) &= [x, 1, 1] \ominus (y \ominus z) = y \ominus ([x, 1, 1] \ominus z) \\ &= y \ominus [z, 1, x \ominus 1] = [z, 1, y \ominus (x \ominus 1)] \\ &= [y \ominus (x \ominus 1), 1, z] = [y, x \ominus 1, 1] \ominus z \\ &= ((x \ominus 1) \ominus y) \ominus z = [x, 1, y, z]. \end{aligned}$$

This completes the proof. □

We provide characterizations of a simulative WFI-algebra.

Theorem 3.3. *A WFI-algebra $\mathfrak{X}$ is simulative if and only if it satisfies the following assertion:*

$$(\forall x, y, z \in X)\,([x, y, z] = [z, y, x]). \tag{3.1}$$

Proof. Assume that $\mathfrak{X}$ is simulative and let $x, y, z \in X$. Then

$$\begin{aligned}[x,y,z] &= [y,x,1,z] = [z,1,y\ominus x] \\ &= y\ominus[z,1,x] = y\ominus[x,1,z] \\ &= [x,1,y\ominus z] = [y,z,1,x] \\ &= [z,y,x],\end{aligned}$$

which proves (3.1). Conversely, suppose (3.1) is valid. If we put $y = z = 1$ in (3.1), then $[x, 1, 1] = x$ for all $x \in X$. Hence $\mathfrak{X}$ is simulative by Lemma 3.1. □

Theorem 3.4. *A WFI-algebra $\mathfrak{X}$ is simulative if and only if it satisfies the following assertion:*

$$(\forall x, y, z \in X)\,(x \ominus (y \ominus z) = [z, [x, 1, y], 1]). \tag{3.2}$$

Proof. Assume that $\mathfrak{X}$ is simulative. Using Lemma 3.1 and Proposition 3.2, we have $x \ominus (y \ominus z) = [x, 1, y, z] = [x, 1, y] \ominus z = [z, [x, 1, y], 1]$. If we put $x = y = 1$ in (3.2), then $[z, 1, 1] = z$ for all $z \in X$. It follows from Lemma 3.1 that $\mathfrak{X}$ is simulative. □

For any $a, b \in X$, consider the set $L(a, b) := \{x \in X \mid a \ominus (b \ominus x) = 1\}$.

Theorem 3.5. *If $\mathfrak{X}$ is simulative, then there exists the least element in $L(a, b)$, and it is $[a, 1, b]$. Moreover, $L(a, b) = \{[a, 1, b]\}$.*

Proof. Using Proposition 3.2, we have $a \ominus (b \ominus [a, 1, b]) = [a, 1, b, [a, 1, b]] = 1$, and so $[a, 1, b] \in L(a, b)$. Now let $y \in L(a, b)$. Then $1 = a \ominus (b \ominus y) = [a, 1, b, y]$ by Proposition 3.2. Thus $[a, 1, b] \preceq y$, which proves that $[a, 1, b]$ is the least element of $L(a, b)$. On the other hand, if $y \in L(a, b)$, then $1 = a \ominus (b \ominus y) = [y, [a, 1, b], 1]$ by Theorem 3.4, i.e., $y \ominus [a, 1, b] \preceq 1$. It follows that $y \ominus [a, 1, b] \in \mathcal{S}(\mathfrak{X}) = \{1\}$ since $\mathfrak{X}$ is simulative. Hence $y \preceq [a, 1, b]$, and therefore $y = [a, 1, b]$. Thus $L(a, b) = \{[a, 1, b]\}$. □

Defitinition 3.6. The element of the form $[x, y, y \ominus x]$ is called a *commutator* of $\mathfrak{X}$ and is denoted by $C(x, y)$.

Denote by $C(\mathfrak{X})$ the set of all commutators of $\mathfrak{X}$, i.e.,

$$C(\mathfrak{X}) = \{C(x, y) \mid x, y \in X\}.$$

Example 3.7. Let $X = \{1, a, b, c\}$ be a set with the following Cayley table.

$\ominus$	1	a	b	c
1	1	a	b	c
a	c	1	a	b
b	b	c	1	a
c	a	b	c	1

Then $\mathfrak{X} := (X, \ominus, 1)$ is a WFI-algebra, and $C(\mathfrak{X}) = \{1, b\}$.

Theorem 3.8. *A WFI-algebra $\mathfrak{X}$ is simulative if and only if for every $a, b \in X$, the commutator $C(a,b)$ of $\mathfrak{X}$ is the unique solution of the following equation:*

$$x \ominus (b \ominus a) = a \ominus b. \tag{3.3}$$

Proof. Suppose that $\mathfrak{X}$ is simulative. Then $(X, \cdot, 1)$ is a commutative group (see [5]). Hence

$$x \ominus (b \ominus a) = a \ominus b \Longleftrightarrow x^{-1} \cdot (b^{-1} \cdot a) = a^{-1} \cdot b$$
$$\Longleftrightarrow x = (a^{-1} \cdot b^{-1})^{-1} \cdot (b^{-1} \cdot a) = [a, b, b \ominus a] = C(a, b).$$

This proves that $C(a,b)$ is the unique solution of the equation (3.3). Conversely, assume that

$$C(a, b) \ominus (b \ominus a) = a \ominus b \tag{3.4}$$

for all $a, b \in X$. Let $a = 1$ and $b \in \mathcal{S}(\mathfrak{X})$. Then $b = 1 \ominus b = C(1, b) \ominus (b \ominus 1) = 1$, and so $\mathcal{S}(\mathfrak{X}) = \{1\}$. Hence $\mathfrak{X}$ is simulative. □

Theorem 3.9. *If $\mathfrak{X}$ satisfies the following identity*

$$(\forall x, y \in X)\,(C(x, y) = x \ominus y), \tag{3.5}$$

then $\mathfrak{X}$ is simulative.

Proof. Let $b \in \mathcal{S}(\mathfrak{X})$. Then $b \ominus 1 = 1$, and so

$$b = 1 \ominus b = [b, 1, 1 \ominus b] = C(b, 1) = b \ominus 1 = 1.$$

Hence $\mathcal{S}(\mathfrak{X}) = \{1\}$, which proves that $\mathfrak{X}$ is simulative. □

Example 3.10. Let $X = \{1, a, b\}$ be a set with the following Cayley table.

$\ominus$	1	a	b
1	1	a	b
a	b	1	a
b	a	b	1

Then $\mathfrak{X} := (X, \ominus, 1)$ is a WFI-algebra in which (3.5) is valid. Hence $\mathfrak{X}$ is a simulative WFI-algebra.

Proposition 3.11. In a simulative WFI-algebra $\mathfrak{X}$, the following holds:

$$(\forall x, y \in X)\,(x \ominus y = x \Longleftrightarrow [x, 1, x] = y).$$

Proof. Let $x, y \in X$ be such that $x \ominus y = x$. Then $y \ominus x = y \ominus (x \ominus y) = x \ominus (y \ominus y) = x \ominus 1$, which implies that $[x, 1, x] = [y, x, x] = y$. Conversely, suppose that $[x, 1, x] = y$ for all $x, y \in X$. Then

$$x \ominus y = x \ominus [x, 1, x] = [x, 1, x \ominus x] = [x, 1, 1] = x.$$

This completes the proof. □

4. Doubly simulative WFI-algebras

Defitinition 4.1. For $a \in X$, the set

$$\mathcal{DS}_a(\mathfrak{X}) := \{x \in X \mid [x, a]^2 = x\}$$

is called the *doubly simulative part* of $\mathfrak{X}$ with respect to a (briefly, *a-doubly simulative part* of $\mathfrak{X}$).

Proposition 4.2. *For any $a, x \in X$, we have $1, a, x \ominus a \in \mathcal{DS}_a(\mathfrak{X})$ and $\mathcal{DS}_a(\mathfrak{X}) = X \ominus a$.*

Proof. Straightforward. □

Theorem 4.3. *For $a \in X$, the a-doubly simulative part of $\mathfrak{X}$ is a subalgebra of $\mathfrak{X}$.*

Proof. Let $x, y \in \mathcal{DS}_a$. Then $[x, a]^2 = x$ and $[y, a]^2 = y$. Note that

$$[x, y, [x \ominus y, a]^2] = 1$$

by (b1). Using (a3) and (b6), we have

$$\begin{aligned} [x \ominus y, a]^2 \ominus (x \ominus y) &= x \ominus ([x \ominus y, a]^2 \ominus y) \;=\; x \ominus ([x \ominus y, a]^2 \ominus [y, a]^2) \\ &= x \ominus [y, a, [x \ominus y, a]^3] \;=\; x \ominus [y, a, [x, y, a]] \\ &= x \ominus [x, y, [y, a]^2] \;=\; x \ominus [x, y]^2 \;=\; 1. \end{aligned}$$

Hence $[x \ominus y, a]^2 = x \ominus y$, i.e., $x \ominus y \in \mathcal{DS}_a(\mathfrak{X})$. Therefore $\mathcal{DS}_a(\mathfrak{X})$ is a subalgebra of $\mathfrak{X}$. □

Defitinition 4.4. For any $a \in X$, if $\mathcal{DS}_a(\mathfrak{X}) = X$, then $\mathfrak{X}$ is called an *a-doubly simulative WFI-algebra.*

Example 4.5. Let $X = \{1, a, b, c\}$ be a set with the following Cayley table.

$\ominus$	1	a	b	c
1	1	a	b	c
a	1	1	c	c
b	c	c	1	1
c	c	b	a	1

Then $\mathfrak{X} := (X, \ominus, 1)$ is an a-doubly simulative WFI-algebra. Note that $\mathcal{DS}_c(\mathfrak{X}) = \{1, c\} \neq X$. Hence $\mathfrak{X}$ is not a c-doubly simulative WFI-algebra.

Proposition 4.6. The 1-doubly simulative part of $\mathfrak{X}$ is contained in the a-doubly simulative part of $\mathfrak{X}$ for all $a \in X$.

Proof. Let $a \in X$ and $x \in \mathcal{D}S_1(\mathfrak{X})$. Then

$$x = [x, 1]^2 = [x, a \ominus a, 1] = [a, x \ominus a, 1] \in \mathcal{D}S_a(\mathfrak{X}).$$

Hence $\mathcal{D}S_1(\mathfrak{X}) \subseteq \mathcal{D}S_a(\mathfrak{X})$ for all $a \in X$. □

The following is a characterization of doubly simulative WFI-algebras.

Theorem 4.7. *For any $a \in X$, the following assertions are equivalent.*

(i) *$\mathfrak{X}$ is an a-doubly simulative.*
(ii) $(\forall x, y \in X)\ ([x, a, y] = [y, a, x])$.
(iii) $(\forall x, y \in X)\ [x, a, y \ominus a] = y \ominus x)$.

Proof. (i) ⇒ (ii). Let $x, y \in X$. Since $\mathcal{D}S_a(\mathfrak{X}) = X$, we have $[x, a]^2 = x$ and $[y, a]^2 = y$. Hence $[y, a, x] = [y, a, [x, a]^2] = [x, a, [y, a]^2] = [x, a, y]$, which proves (ii)

(ii) ⇒ (iii). Assume that (ii) is valid and let $x, y \in X$. Then

$$x = 1 \ominus x = [a, a, x] = [x, a, a]$$

and so $y \ominus x = y \ominus [x, a, a] = [x, a, y \ominus a]$. This proves (iii).

(iii) ⇒ (i). Suppose (iii) is valid. For any $x \in X$, we get

$$x = 1 \ominus x = [x, a, 1 \ominus a] = [x, a]^2.$$

Thus $x \in \mathcal{D}S_a(\mathfrak{X})$. This completes the proof. □

Theorem 4.8. *For any $a \in X$, we have*

$$\mathcal{D}S_1(\mathfrak{X}) = \mathcal{D}S_a(\mathfrak{X}) \iff a \in \mathcal{D}S_1(\mathfrak{X}). \tag{4.1}$$

Proof. Since $a \in \mathcal{D}S_a(\mathfrak{X})$, the necessity is clear. Assume that $a \in \mathcal{D}S_1(\mathfrak{X})$. For every $x \in X$, we have

$$x \ominus a = x \ominus [a, 1]^2 = [a, 1, x \ominus 1] = [a, x, 1] \in \mathcal{D}S_1(\mathfrak{X})$$

by (a3), (b5) and (b6). This shows that $\mathcal{D}S_a(\mathfrak{X}) = X \ominus a \subseteq \mathcal{D}S_1(\mathfrak{X})$. This completes the proof. □

Theorem 4.9. *The following assertions are equivalent.*

(i) *$\mathfrak{X}$ is a normal WFI-algebra.*
(ii) $\mathcal{D}S_1(\mathfrak{X}) = X$.

Proof. (i) ⇒ (ii). Straightforward.

(ii) ⇒ (i). Suppose $\mathcal{D}S_1(\mathfrak{X}) = X$. Then $X \ominus a = \mathcal{D}S_a(\mathfrak{X}) = \mathcal{D}S_1(\mathfrak{X}) = X$ for all $a \in X$. On the other hand,

$$x = a^{-1} \cdot (a \cdot x) = a \ominus (a \cdot x) = a \ominus [x, 1, a] \in a \ominus X$$

for all $a \in \mathcal{D}S_1(\mathfrak{X}) = X$. Hence $\mathfrak{X}$ is a normal WFI-algebra. □

Corollary 4.10. *In a normal WFI-algebra* $\mathfrak{X}$, *we have*

$$(\forall a \in X)\,(\mathcal{D}S_a(\mathfrak{X}) = \mathcal{D}S_1(\mathfrak{X})).$$

Proposition 4.11. *Let* $\mathfrak{X}$ *be an a-doubly simulative WFI-algebra for an element a in the simulative part of* $\mathfrak{X}$. *Then the following assertions are valid.*

(i) $(\forall x, y \in X)\ ([x, y, a] \preceq y \ominus x)$.
(ii) $(\forall x, y, z \in X)\ ([x, a, y, z] \preceq x \ominus (y \ominus z))$.
(iii) $(\forall x, y, z \in X)\ ([z, [x, a, y], a] \preceq x \ominus (y \ominus z))$.
(iv) $(\forall x, y \in X)\ ([a, x, y] \preceq x \ominus y)$.
(v) $(\forall x, y \in X)\ ([a, x, a \ominus y] \preceq a \ominus (x \ominus y))$.

Proof. Since $a \in \mathcal{S}(\mathfrak{X})$, we get

$$\begin{aligned} 1 &= a \ominus 1 = a \ominus (y \ominus y) = y \ominus (a \ominus y) \\ &\preceq y \ominus [x, a, x \ominus y] = y \ominus [x \ominus y, a, x] \\ &= [x, y, a] \ominus (y \ominus x), \end{aligned}$$

and so $[x, y, a] \ominus (y \ominus x) = 1$, i.e., $[x, y, a] \preceq y \ominus x$. Thus (i) is valid. Using (i), we have $[y, x \ominus a, a] \preceq [x, a, y]$. It follows from (a3), (b4) and Theorem 4.7 that

$$\begin{aligned} [x, a, y, z] &\preceq [y \ominus (x \ominus a), a, z] = [z, a, y \ominus (x \ominus a)] \\ &= y \ominus [z, a, x \ominus a] = y \ominus (x \ominus z) \\ &= x \ominus (y \ominus z), \end{aligned}$$

which proves (ii).

(iii) is an immediate consequence of (i) and (ii). Using (b3) and (ii), we have

$$[a, x, y] = [1, a, x, y] \preceq 1 \ominus (x \ominus y) = x \ominus y$$

for all $x, y \in X$. Hence (iv) is valid. Finally, if we use (b4) and (iv), then

$$[a, x, a \ominus y] = a \ominus [a, x, y] \preceq a \ominus (x \ominus y)$$

for all $x, y \in X$, which proves (v). □

Theorem 4.12. *Let* $\mathfrak{X}$ *be an a-doubly simulative WFI-algebra for* $a \in X$. *If we define a binary operation* $\dotplus$ *on* X *by*

$$(\forall x, y \in X)\,(x \dotplus y = [y, a, x]),$$

then $\mathfrak{X}$ *is a commutative pomonoid.*

Proof. The commutativity of $\dotplus$ follows from Theorem 4.7(ii). For any $x, y, z \in X$, we have

$$\begin{aligned} x \dotplus (y \dotplus z) &= (y \dotplus z) \dotplus x = [x, a, y \dotplus z] \\ &= [x, a, [z, a, y]] = [z, a, [x, a, y]] \\ &= [z, a, x \dotplus y] = (x \dotplus y) \dotplus z, \end{aligned}$$

i.e., the associative law is valid. Note that $x \dotplus a = [a, a, x] = x$ for all $x \in X$. Hence a is the identity element of X. Now let $x, y, z \in X$ be such that $x \preceq y$. Then $x \dotplus z = [z, a, x] \preceq [z, a, y] = y \dotplus z$. By the commutativity of $\dotplus$, $z \dotplus x \preceq z \dotplus y$. Therefore $\mathfrak{X} := (X, \dotplus, \preceq)$ is a commutative pomonoid. □

Defitinition 4.13. Let $\mathfrak{X}$ and $\mathfrak{Y}$ be WFI-algebras. A mapping $f : \mathfrak{X} \to \mathfrak{Y}$ is called a *WFI-morphism* if it satisfies:

$$(\forall x, y \in X)\,(f(x \ominus y) = f(x) \ominus f(y)).$$

Theorem 4.14. Let $f : \mathfrak{X} \to \mathfrak{Y}$ be an onto WFI-morphism of WFI-algebras. For any $a \in X$, if $\mathfrak{X}$ is an a-doubly simulative WFI-algebra, then $\mathfrak{Y}$ is an $f(a)$-doubly simulative WFI-algebra.

Proof. Let $y \in Y$. Then there exists $x \in X$ such that $f(x) = y$. Since $\mathcal{DS}_a(\mathfrak{X}) = X$, it follows that

$$y = f(x) = f([x, a]^2) = [f(x), f(a)]^2 = [y, f(a)]^2.$$

Hence $y \in \mathcal{DS}_{f(a)}(\mathfrak{Y})$, and so $\mathcal{DS}_{f(a)}(\mathfrak{Y}) = Y$. This completes the proof. □

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Young Bae Jun has been an educator and research mathematician since 1982, mostly at the Gyeongsang National University; and a member of the editorial board of Far East Journal of Mathematical Science (India) since 1998, and Quasigroups and Related Systems (Moldova) since 2000. He did postdoctoral work (one year, 1989-90, supported by KOSEF) at the University of Albert in Albert, Canada; and worked for one year (1996-97) as a visiting professor at the Northwest University in Xian, China (supported by LG Yonam Foundation). His research interests focus on the structure theory of BCK/BCI-algebras, Hilbert algebras, (lattice) implication algebras and negatively partially ordered semigroups, and fuzzy and hyper theory of such algebraic structures. Jun is a co-author of the text BCK-algebras with J. Meng which is an approachable introduction to BCK/BCI-algebras.

Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea.

e-mail :skywine@gmail.com

Kyoung Ja Lee received her Ph.D degree from Yonsei University, Korea, in 2000. From 2001 to 2002, she was a Post-Doc researcher at Korea Universty, Korea. She is currently a faculty member of the Hannam University in Daejeon, Korea. Her research interests are in the areas of Fuzzy algebraic structure, BCK/BCI/d-algebraiac structure, Homological algebraic structure, and Representation theory.

Department of Mathematics Education, Hannam University, Daejeon 306-791, Korea
e-mail: lsj1109@hotmail.com

Chul Hwan Park received his B.S., M.S. and Ph.D. degree from the Department of Mathematics of University of Ulsan,Korea, in 1986,1988 and 1997 respectively. From 1997 to 1998, he was a researcher at the Institute of Basic Science,The Kyungpook National Universty,Korea(supported by KOSEF). He is currently a full time lecture at the Department of Mathematics in University of Ulsan,Korea since 2005. His research interests are in the areas of Fuzzy Algebraic Structure,BCK-algebra,Quantum Structure ,semigroup and Commutative ring.

Department of Mathematics, University of Ulsan, Ulsan 680-749, Korea
e-mail: skyrosemary@gmail.com