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SOLVING A CLASS OF GENERALIZED SEMI-INFINITE PROGRAMMING VIA AUGMENTED LAGRANGIANS

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ABSTRACT. Under certain conditions, we use augmented Lagrangians to transform a class of generalized semi-infinite min-max problems into common semi-infinite min-max problems, with the same set of local and global solutions. We give two conditions for the transformation. One is a necessary and sufficient condition, the other is a sufficient condition which can be verified easily in practice. From the transformation, we obtain a new first-order optimality condition for this class of generalized semi-infinite min-max problems.

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1. Introduction

We consider the class of generalized semi-infinite min-max problems of the form

$$(P) \ \min_{x \in R^n} \psi(x), \tag{1}$$

where $\psi : R^n \to R$ is defined by

$$\psi(x) = \sup_{y \in Z(x)} \phi(x, y), \tag{2}$$

where

$$Z(x) = \{y \in R^m | f(x, y) \le 0, g(y) \le 0\},$$

$\phi : R^n \times R^m \to R$, $f : R^n \times R^m \to R^{r_1}$, $g : R^m \to R^{r_2}$, and $v \le 0$ meaning $v^1 \le 0, \cdots, v^q \le 0$, for any $v = (v^1, \cdots, v^q) \in R^q$. We use superscripts to denote components of vectors.

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Let $Y = \{y \in R^m | g(y) \le 0\}$, then (2) can be written as

$$\psi(x) = \sup_{y \in Y} \{\phi(x,y) | f(x,y) \le 0\}. \tag{3}$$

It is well known that generalized semi-infinite min-max problem (P) arises in various engineering design and economic equilibria theory. It has become an active field of research in applied mathematics. Note that it is the dependence of the set-valued map Z(·) on the design variable x that makes (P) a generalized semi-infinite min-max problem. It is difficult to solve (P) because of the existence of Z(·), and there are only a few studies dealing with numerical methods for (P). If the set-valued map Z(·) does not depend on the variable x, i.e., Z(·)=Z, then (P) is a common semi-infinite min-max problem. For common semi-infinite problems, many effective algorithms have been proposed (see [1],[2],[4],[10],[12]). Therefore, many scholars use penalty functions or augmented Lagrangian functions to eliminate the constraints $f(x,y) \le 0$ in(3), and then transform generalized semi-infinite min-max problems into common semi-infinite min-max problems. For example, in [6], a nondifferentiable exact penalty function is used to complete this transformation. Moreover, for this purpose, an augmented Lagrangian function presented in [5] is used in [3]. This approach requires stronger assumptions than the ones in [6], but gives rise to an equivalent common semi-infinite min-max problem which is much easier to solve.

In this paper, we use a class of augmented Lagrangian functions (see [9]) to remove the constraints $f(x,y) \le 0$ and transform (P) into a common semi-infinite min-max problem. The essentially quadratic augmented Lagrangian function in [3] is a special case of this class of augmented Lagrangian functions. We give two conditions for the transformation. One is a necessary and sufficient condition which is different from the one in [3], the other is a sufficient condition which can be verified easily in practice. Compared with [3], our conditions have the following characteristics:(1)They do not need the compactness of Y. (2)The sufficient condition is milder than the one in [3] and it does not require the uniqueness of $\arg \max_{y \in Z(x)} \phi(x,y)$. Based on this, we give a new first-order optimality condition for (P).

This paper is organized as follows. In Section 2, we use a class of augmented Lagrangian functions in [9] to transform (P) into a common semi-infinite min-max problem. To complete this equivalent transformation, we give a necessary and sufficient condition and a sufficient condition. In Section 3, we show that the resulting problem is equivalent to (P). In the process we get a new first-order optimality condition for (P).

2. Augmented Lagrangian function

Now we transform (P) into a common semi-infinite min-max problem by using the class of augmented Lagrangian functions in [9]. We define $\bar{\psi} : R^{n+r_1+1} \mapsto R$ to be given by

$$\overline{\psi}(\overline{x}) = \sup_{y \in Y} \overline{\phi}(\overline{x}, y), \tag{4}$$

where $\overline{x} = (x, \lambda, c) \in R^{n+r_1+1}$, with $x \in R^n$, $\lambda \geq 0$, $c > 0$, and

$$\overline{\phi}(\overline{x}, y) = \phi(x, y) - \frac{1}{c}\sum_{k=1}^{r_1} \min_{\tau \geq cf^k(x,y)} \{\lambda_k \tau + \theta(\tau)\}, \tag{5}$$

where $\theta : R \to R$ satisfies the following conditions:

(i) θ is twice continuously differentiable and convex on R ;

(ii) $\theta(0) = 0, \theta'(0) = 0.\theta''(0) > 0$;

(iii) $\frac{\theta(s)}{|s|} \to +\infty, (|s| \to +\infty)$.

When setting $\theta(s) = \frac{1}{2}s^2$, this special case of (5) becomes the essentially quadratic augmented Lagrangian function in [3].

We define $R_+^{r_1} = \{\lambda \in R^{r_1} |\ \lambda \geq 0\}$ and $R_{++} = \{s \in R|\ s > 0\}$.

Then we obtain a common semi-infinite min-max problem:

$$\min_{\bar{x} \in R^n \times R_+^{r_1} \times R_{++}} \overline{\psi}(\bar{x}) \tag{6}$$

In the following, we study the relation between $\overline{\psi}(\overline{x})$ and $\psi(x)$.

Assumption 1. *We assume that*

(i) $\phi(\cdot,\cdot)$, $f^k(\cdot,\cdot)$, $k \in \mathbf{r}_1 = \{1, ..., r_1\}$, *and* $g^k(\cdot)$, $k \in \mathbf{r}_2 = \{1, ..., r_2\}$ *are continuous, and*

(ii) $Z(x) \neq \emptyset$ *for all* $x \in R^n$.

First, we have the next result.

Theorem 1. *Suppose that Assumption 1 holds. Then for all* $\overline{x} = (x, \lambda, c) \in R^n \times R_+^{r_1} \times R_{++}$, *we get*

$$\overline{\psi}(\overline{x}) \geq \psi(x). \tag{7}$$

Proof. Suppose that $\overline{\psi}(\bar{x}) = +\infty$, then (7) holds obviously. We assume $\overline{\psi}(\bar{x}) < +\infty$, then

$$\begin{aligned}
\overline{\psi}(\overline{x}) &= \sup_{y \in Y}\{\phi(x, y) - \frac{1}{c}\sum_{k=1}^{r_1} \min_{\tau \geq cf^k(x,y)} \{\lambda_k \tau + \theta(\tau)\}\} \\
&\geq \sup_{y \in Y}\{\phi(x, y) - \frac{1}{c}\sum_{k=1}^{r_1} \min_{\tau \geq cf^k(x,y)} \{\lambda_k \tau + \theta(\tau)\} | f^k(x, y) \leq 0, \forall k \in \mathbf{r}_1\} \\
&\geq \sup_{y \in Y}\{\phi(x, y) - \frac{1}{c}\sum_{k=1}^{r_1} (\lambda_k \cdot 0 + \theta(0)) | f^k(x, y) \leq 0, \forall k \in \mathbf{r}_1\} \\
&= \sup_{y \in Y}\{\phi(x, y) | f^k(x, y) \leq 0, \forall k \in \mathbf{r}_1\} \\
&= \psi(x).
\end{aligned}$$

This completes the proof. □

Next, we find a necessary and sufficient condition for the existence of λ and c that ensure equality in (7).

For this purpose, we define

$$v(x,\tau) = \sup_{y\in Y}\{\phi(x,y)|f^k(x,y) \le \tau, \forall k \in \mathbf{r}_1\}, \tag{8}$$

where $v(x,\tau) = -\infty$, if $\{y \in Y|f^k(x,y) \le \tau, \forall k \in \mathbf{r}_1\} = \emptyset$.

$$N(\lambda, c, \delta) = \{\tau \in R|v(x,\tau) - \sum_{k=1}^{r_1}(\lambda_k\tau + \frac{1}{c}\theta(c\tau)) \ge \delta\}.$$

Condition (A). *Let $x \in R^n$ and there exist $\overline{\lambda} \ge 0$, $\overline{c} > 0$ and a neighborhood $\Gamma(0)$ of zero such that*

(*i*) $v(x,\tau) - \sum_{k=1}^{r_1}(\bar{\lambda}_k\tau + \frac{1}{\bar{c}}\theta(\bar{c}\tau)) \le v(x,0)$ *for any* $\tau \in \Gamma(0)$*;*

(*ii*) *there exists* $\delta_0 < \overline{\psi}(x,\overline{\lambda},\overline{c})$ *such that* $\inf\{\theta(\tau)|\tau \in N(\overline{\lambda},\overline{c},\delta)\} = 0$ *for any* $\delta \in [\delta_0,\ \overline{\psi}(x,\overline{\lambda},\overline{c}))$.

Theorem 2. *Suppose that Assumption 1 holds. Then, for any fixed $x \in R^n$, there exist $\overline{\lambda} \in R^{r_1}_+$ and $\overline{c} \in R_{++}$ such that*

$$\psi(x) = \overline{\psi}(x,\overline{\lambda},c) \tag{9}$$

for all $c \ge \overline{c}$ if and only if Condition(A) holds.

Proof. Necessity. Suppose that $\bar{\lambda} \in R^{r_1}_+$ and $\bar{c} \in R_{++}$ satisfy $\psi(x) = \overline{\psi}(x,\overline{\lambda},\overline{c})$, i.e.,

$$v(x,0) = \overline{\psi}(x,\overline{\lambda},\overline{c}). \tag{10}$$

Next, we will show that (i) holds. For any $\bar{\tau} \in \Gamma(0)$, suppose that $\{y \in Y|f^k(x,y) \le \bar{\tau}, \forall k \in \mathbf{r}_1\} = \emptyset$, then $v(x,\bar{\tau}) = -\infty$. Now (i) holds obviously. Otherwise, we have

$$\begin{aligned}
v(x,0) &= \overline{\psi}(x,\overline{\lambda},\overline{c}) \\
&= \sup_{y\in Y}\{\phi(x,y) - \frac{1}{\bar{c}}\sum_{k=1}^{r_1}\min_{\tau\ge\bar{c}f^k(x,y)}\{\bar{\lambda}_k\tau + \theta(\tau)\}\} \\
&= \sup_{y\in Y}\{\phi(x,y) - \sum_{k=1}^{r_1}\min_{\tau\ge f^k(x,y)}\{\bar{\lambda}_k\tau + \frac{1}{\bar{c}}\theta(\bar{c}\tau)\}\} \\
&\ge \sup_{y\in Y}\{\phi(x,y) - \sum_{k=1}^{r_1}\{\bar{\lambda}_k\bar{\tau} + \frac{1}{\bar{c}}\theta(\bar{c}\bar{\tau})\}|\bar{\tau} \ge f^k(x,y), \forall k \in \mathbf{r}_1\} \\
&= \sup_{y\in Y}\{\phi(x,y)|f^k(x,y) \le \bar{\tau}, \forall k \in \mathbf{r}_1\} - \sum_{k=1}^{r_1}\{\bar{\lambda}_k\bar{\tau} + \frac{1}{\bar{c}}\theta(\bar{c}\bar{\tau})\} \\
&= v(x,\bar{\tau}) - \sum_{k=1}^{r_1}\{\bar{\lambda}_k\bar{\tau} + \frac{1}{\bar{c}}\theta(\bar{c}\bar{\tau})\}
\end{aligned}$$

From the arbitrariness of $\overline{\tau}$, we know that (i) holds.

By (10) and the definition of $N(\overline{\lambda}, \overline{c}, \delta)$, for any $\delta < \overline{\psi}(x, \overline{\lambda}, \overline{c})$, we have

$$0 \in N(\overline{\lambda}, \overline{c}, \delta).$$

Furthermore, we can obtain from the properties of $\theta(\tau)$ that $\theta(\tau) > 0$ when $\tau \neq 0$ and $\theta(0) = 0$ when $\tau = 0$. Therefore, for any $\delta < \overline{\psi}(x, \overline{\lambda}, \overline{c})$, we get

$$\inf\{\theta(\tau) | \tau \in N(\overline{\lambda}, \overline{c}, \delta)\} = 0.$$

i.e., (ii) holds.

Sufficiency. Suppose that Condition (A) holds. Let $\{\delta_j\} \nearrow \overline{\psi}(x, \overline{\lambda}, \overline{c})$ and $\{\varepsilon_j\} \searrow 0$ as $j \to +\infty$. From (ii), for any $j \in \{1, 2, 3, ...\}$, we know there exists $\tau_j \in N(\overline{\lambda}, \overline{c}, \delta_j)$ such that

$$\theta(\tau_j) \leq \varepsilon_j \ .$$

From the properties of $\theta(\tau)$, we get $\lim_{j \to +\infty} \tau_j = 0$. Then, $\tau_j \in \Gamma(0)$ as j is large enough. Hence, in view of (i), we have

$$\delta_j \leq v(x, \tau_j) - \sum_{k=1}^{r_1} \{\bar{\lambda}_k \tau_j + \frac{1}{\bar{c}} \theta(\bar{c}\tau_j)\} \leq v(x, 0).$$

Then we get

$$\overline{\psi}(x, \overline{\lambda}, \overline{c}) = \lim_{j \to +\infty} \delta_j \leq v(x, 0) = \psi(x).$$

Therefore, by Theorem 1, we obtain $\psi(x) = \overline{\psi}(x, \overline{\lambda}, \overline{c})$. From [9], we know

$$\frac{1}{c} \sum_{k=1}^{r_1} \min_{\tau \geq c f^k(x,y)} \{\lambda_k \tau + \theta(\tau)\}$$

is a nondecreasing function with respect to c, then $\bar{\psi}(\bar{x})$ is a nonincreasing function with respect to c. Therefore, by Theorem 1, for all $c \geq \bar{c}$, we have

$$\psi(x) = \bar{\psi}(x, \bar{\lambda}, c).$$

This completes the proof. □

Although Condition(A) is a necessary and sufficient condition, it is difficult to verify it in practice. In the following, we give a sufficient condition which can be verified easily in practice.

For $\alpha > 0$, we define

$$G(x, \alpha) = \{y \in Y | \ f^k(x, y) \leq \alpha, \forall k \in \mathbf{r}_1\},$$

$$F(x, \alpha) = \{y \in Y | \ \phi(x, y) \geq v(x, 0) - \alpha\}.$$

We also define

$$\hat{Y}(x) = \arg\max_{y \in Y} \{\phi(x, y) \ |f^k(x, y) \leq 0, \forall k \in \mathbf{r}_1\}.$$

Second-order sufficient conditions. *Suppose that $\phi(\cdot,\cdot)$, $f^k(\cdot,\cdot)$, $k \in \mathbf{r}_1$ are twice continuously differentiable functions. Let $x \in R^n$, $y^* \in R^m$ and $\lambda^* \geq 0$ such that*

$$\nabla_y \phi(x, y^*) - \sum_{k=1}^{r_1} \lambda_k^* \nabla_y f^k(x, y^*) = 0,$$

$$\lambda_k^* f^k(x, y^*) = 0, k \in \mathbf{r}_1.$$

and the Hessian matrix

$$\nabla_{yy}^2 \phi(x, y^*) - \sum_{k=1}^{r_1} \lambda_k^* \nabla_{yy}^2 f^k(x, y^*)$$

is negative definite on the cone

$$M(x, y^*) = \left\{ d \in R^m, d \neq 0 \,\middle|\, \begin{array}{l} \nabla_y f^k(x, y^*)^\top d = 0, k \in J(x, y^*), \\ \nabla_y f^k(x, y^*)^\top d \leq 0, k \in I(x, y^*) \setminus J(x, y^*) \end{array} \right\},$$

where

$$I(x, y^*) = \{k \in \mathbf{r}_1 | f^k(x, y^*) = 0\},$$
$$J(x, y^*) = \{k \in I(x, y^*) | \lambda_k^* > 0\}.$$

Next, we give the sufficient condition.

Condition(B). *Suppose that $x \in R^n$ satisfies the following conditions:*

(i) $\sup_{y \in Y} \phi(x, y) < +\infty$.

(ii) $\hat{Y}(x) \neq \emptyset$ and there exists $\lambda^* \geq 0$ such that for any $y^* \in \hat{Y}(x)$, (x, y^*, λ^*) satisfies second-order sufficient conditions.

(iii) There exists $\alpha_0 > 0$, such that $G(x, \alpha_0) \bigcap F(x, \alpha_0)$ is bounded.

In view of the existence theorems for a global saddle point in [11], we get the next theorem.

Theorem 3. *Suppose that $x \in R^n$ satisfies Condition(B). Then, there exists $c^* > 0$ such that for all $c \geq c^*$, we have*

$$\psi(x) = \bar{\psi}(x, \lambda^*, c).$$

Proof. Suppose that $x \in R^n$ satisfies Condition (B). From the corresponding theorem in [11], we know that for any $y^* \in \hat{Y}(x)$, there exists $c^* > 0$ such that

$$\bar{\phi}((x, \lambda, c), y^*) \geq \bar{\phi}((x, \lambda^*, c), y^*) \geq \bar{\phi}((x, \lambda^*, c), y) \tag{11}$$

for all $c \geq c^*$, any $y \in Y$ and $\lambda \geq 0$.
From the first inequality given above and $f^k(x, y^*) \leq 0 (\forall k \in \mathbf{r}_1)$, for all $c \geq c^*$, we have

$$\frac{1}{c} \sum_{k=1}^{r_1} \min_{\tau \geq c f^k(x, y^*)} \{\lambda_k^* \tau + \theta(\tau)\} = 0. \tag{12}$$

Then by the definition of $\bar{\phi}(\bar{x}, y)$, we get

$$\bar{\phi}((x, \lambda^*, c), y^*) = \phi(x, y^*). \quad (13)$$

From the second inequality of (11), we know

$$\bar{\psi}(x, \lambda^*, c) = \bar{\phi}((x, \lambda^*, c), y^*). \quad (14)$$

By (13)(14) and $y^* \in \hat{Y}(x)$, we know there exists $c^* > 0$ such that for all $c \geq c^*$, we have

$$\psi(x) = \bar{\psi}(x, \lambda^*, c).$$

This completes the proof. □

From Theorem 1 and Theorem 2(or Theorem 3), we obtain the following when Condition (A)(or Condition (B)) holds at x$\in R^n$,

$$\psi(x) = \min_{\lambda \geq 0, c > 0} \bar{\psi}(x, \lambda, c). \quad (15)$$

3. Equivalent problem and first order optimality conditions

In view of (15), we can find that problem (P') is equivalent to (P). Let for any $\hat{x} \in R^n$ and $\rho > 0$, $B(\hat{x}, \rho) = \{x \in R^n | \|x - \hat{x}\| < \rho\}$.

Theorem 4. *Suppose that Assumption 1 holds. Then, the following hold:*

(i) If $\widehat{x} \in R^n$ is a local minimizer for (P) with domain of attraction $B(\widehat{x}, \rho)$ and Condition(A)(or Condition(B)) holds at $\widehat{x}$, then there exist $\widehat{\lambda} \geq 0$ and $\widehat{c} > 0$ such that $\widehat{\bar{x}} = (\widehat{x}, \widehat{\lambda}, \widehat{c})$ is a local minimizer for (P') with domain of attraction $B(\widehat{x}, \rho) \times R_+^{r_1} \times R_{++}$.

(ii) If $\widehat{\bar{x}} = (\widehat{x}, \widehat{\lambda}, \widehat{c})$ is a local minimizer for (P') with domain of attraction $B(\widehat{x}, \rho) \times R_+^{r_1} \times R_{++}$ and Condition(A)(or Condition(B)) holds at any $x \in B(\widehat{x}, \rho)$, then $\widehat{x} \in R^n$ is a local minimizer for (P) with domain of attraction $B(\widehat{x}, \rho)$.

(iii) If Condition(A)(or Condition(B)) holds at any $x \in R^n$, then (P) and (P') have the same optimal value, i.e.,

$$\min_{x \in R^n} \psi(x) = \min_{\bar{x} \in R^n \times R_+^{r_1} \times R_{++}} \bar{\psi}(\bar{x}). \quad (16)$$

Proof. (i) Since $\hat{x} \in R^n$ is a local minimizer for P , we have

$$\psi(x) \geq \psi(\hat{x}) \quad (17)$$

for any $x \in B(\widehat{x}, \rho)$.

Since Condition (A)(or Condition(B)) holds at $\hat{x}$, there exist $\hat{\lambda} \geq 0$ and $\hat{c} > 0$ such that

$$\psi(\hat{x}) = \bar{\psi}(\hat{x}, \hat{\lambda}, \hat{c}). \quad (18)$$

From Theorem 1, for any $x \in R^n, \lambda \geq 0$, $c > 0$, we have

$$\bar{\psi}(x, \lambda, c) \geq \psi(x). \quad (19)$$

From (17)(18) and (19), for any $x \in B(\widehat{x}, \rho), \lambda \geq 0$ and $c > 0$, we obtain

$$\bar{\psi}(x, \lambda, c) \geq \bar{\psi}(\hat{x}, \hat{\lambda}, \hat{c}).$$

i.e., $\widehat{\overline{x}} = (\widehat{x}, \widehat{\lambda}, \widehat{c})$ is a local minimizer for (P') with domain of attraction $B(\widehat{x}, \rho) \times R_+^{r_1} \times R_{++}$.

(ii) Suppose that Condition(A)(or Condition(B)) holds at any $x \in B(\widehat{x}, \rho)$. Let $x^* \in B(\hat{x}, \rho)$ be arbitrary, then there exist $\lambda^* \geq 0$ and $c^* > 0$ such that

$$\psi(x^*) = \bar{\psi}(x^*, \lambda^*, c^*).$$

Furthermore, Since $(\hat{x}, \hat{\lambda}, \hat{c})$ is a local minimizer for (P'), we get

$$\begin{aligned} \bar{\psi}(x^*, \lambda^*, c^*) &\geq \bar{\psi}(\hat{x}, \hat{\lambda}, \hat{c}) \\ &\geq \min_{\lambda \geq 0, c > 0} \bar{\psi}(\hat{x}, \lambda, c) \\ &= \psi(\hat{x}) \end{aligned}$$

Then for any $x^* \in B(\widehat{x}, \rho)$, we have $\psi(x^*) \geq \psi(\hat{x})$. i.e. $\widehat{x} \in R^n$ is a local minimizer for (P) with domain of attraction $B(\widehat{x}, \rho)$.

(iii) Since Condition(A)(or Condition(B)) holds at any $x \in R^n$, we have

$$\psi(x) = \min_{\lambda \geq 0, c > 0} \bar{\psi}(x, \lambda, c).$$

Then (16) holds. This completes the proof. □

The most important part in Theorem 4 is conclusion(i), because a first-order optimality condition for (P) can be deduced from it.

Assumption 2. *We assume that*

(i) $\phi(\cdot,\cdot)$, $f^k(\cdot,\cdot)$, $k \in \mathbf{r}_1$ *and* $g^k(\cdot)$, $k \in \mathbf{r}_2$ *are continuously differentiable, and*

(ii) Y *is compact.*

Suppose that Assumption 2 holds, then (P') is a common semi-infinite min-max problem, and the corresponding optimality condition is available (see [2]).

Theorem 5. *Suppose that Assumption 2 holds. If* $\widehat{\overline{x}} = (\widehat{x}, \widehat{\lambda}, \widehat{c}) \in R^n \times R_+^{r_1} \times R_{++}$ *is a local minimizer of* (P'), *then*

$$0 \in \bar{G}\bar{\psi}(\widehat{\overline{x}}) = conv_{y \in Y} \left\{ \begin{pmatrix} \overline{\psi}(\widehat{\overline{x}}) - \overline{\phi}(\widehat{\overline{x}}, y) \\ \nabla_x \overline{\phi}(\widehat{\overline{x}}, y) \\ \nabla_\lambda \overline{\phi}(\widehat{\overline{x}}, y) \\ \nabla_c \overline{\phi}(\widehat{\overline{x}}, y) \end{pmatrix} \right\}.$$

In view of Theorem 2(or Theorem 3), we deduce the following new optimality condition for (P).

Theorem 6. *Suppose that Assumption 2 holds. If $\widehat{x} \in R^n$ is a local minimizer of (P) and Condition(A)(or Condition(B)) holds at $\hat{x} \in R^n$, then there exist $\widehat{\lambda} \geq 0$ and $\widehat{c} > 0$ such that*

$$0 \in \bar{G}\bar{\psi}(\widehat{\bar{x}}),$$

where $\widehat{\bar{x}} = (\widehat{x}, \widehat{\lambda}, \widehat{c})$.

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