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OSCILLATION OF NONLINEAR EQUATIONS ON TIME SCALES

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ABSTRACT. By means of Riccati transformation techniques, we obtain some criteria which ensure that every solution of a nonlinear equation on time scales oscillates.

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1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger [5] in his Ph.D. Thesis in 1988 in order to unify continuous and discrete analysis. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the reals. Many authors have expanded on various aspects of this new theory; see the survey paper by Agarwal et al. [1] and the book by Bohner and Peterson [2] which summarizes and organizes much of the time scale calculus. For the notion used below we refer to the next section that provides some basic facts on time scale extracted from [2].

In recent years, there has been an increasing interest in studying the oscillation and nonoscillation of solutions of various equations on time scales, and we refer the reader to the papers [3,4,6,7,8,9] and the references cited therein. For oscillation of nonlinear delay dynamic equations, Zhang and Shanliang [10] considered the equation

$$x^{\Delta\Delta}(t) + p(t)f(x(t-\tau)) = 0, \quad t \in \mathbb{T},$$

where $\tau \in \mathbb{R}$ and $t - \tau \in \mathbb{T}$, $f : \mathbb{R} \to \mathbb{R}$ is continuous and nondecreasing ($f'(u) > k > 0$), and $uf(u) > 0$ for $u \neq 0$. They established some sufficient conditions for oscillation.

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In this paper, we consider the following equation:

$$x^{\Delta\Delta} + r(t)x^{\Delta^{\sigma}} + q(t)f(x \circ g) = 0, \quad t \in [t_0, \infty)_{\mathbb{T}} \tag{1.1}$$

where $r \in \mathcal{R}^+$, $q(t) \geq 0$ and q is real-valued, right-dense continuous function on a time scale $\mathbb{T} \subset \mathbb{R}$, with $\sup \mathbb{T} = \infty$. We also assume that $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and satisfies $\frac{f(x)}{x} \geq M > 0$, $g : \mathbb{T} \to \mathbb{T}$ is right-dense continuous function.

For completeness, we recall the following concepts related to the notion of time scales. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. We define the forward and backward jump operators by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$, $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$, where $\inf \emptyset = \sup \mathbb{T}$, $\sup \emptyset = \inf \mathbb{T}$, and $\emptyset$ denotes the empty set. A nonmaximal element $t \in \mathbb{T}$ is called right-dense if $\sigma(t) = t$ and right-scattered if $\sigma(t) > t$. A nonminimal element $t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$ and left-scattered if $\rho(t) < t$. The graininess μ of the time scale $\mathbb{T}$ is defined by $\mu(t) := \sigma(t) - t$. We also need below the set $\mathbb{T}^k$ as follows: if $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} - m$, otherwise $\mathbb{T}^k = \mathbb{T}$.

A mapping $f : \mathbb{T} \to \mathbb{R}$ is said to be differentiable at $t \in \mathbb{T}^k$, if there exists $b \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists a neighborhood $\mathbf{U}$ of t satisfying $|[f(\sigma(t) - f(s)] - b[\sigma(t) - s]| \leq \varepsilon|\sigma(t) - s|$, for all $s \in \mathbf{U}$. We say that f is delta differentiable (or in short: differentiable) on $\mathbb{T}^k$ provided $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}^k$.

A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.

The derivative and forward jump operator σ are related by the formula $f(\sigma(t)) = f(t) + \mu(t)f^{\Delta}(t)$.

We use the following product and quotient rules for derivative of two differentiable functions f and g

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)),$$

$$(\frac{f}{g})^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g(\sigma(t))},$$

where $g(t)g(\sigma(t)) \neq 0$. Let f be a differentiable function on $[a, b]$. If $f^{\Delta} > 0$, $f^{\Delta} < 0$, $f^{\Delta} \geq 0$, $f^{\Delta} \leq 0$ for all $t \in [a, b)$, then f is increasing, decreasing, nondecreasing, nonincreasing on $[a, b]$, respectively.

For $a, b \in \mathbb{T}$ and a differentiable function f, the Cauchy integral of f^{Δ} is defined by

$$\int_a^b f^{\Delta}(t)\Delta t = f(b) - f(a)$$

and define improper integral by

$$\int_a^{\infty} f(s)\Delta s = \lim_{t \to \infty} \int_a^t f(s)\Delta s$$

provided this limit exists.

We say that a function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$ holds. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted in this paper by

$$\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R}).$$

We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \text{ for all } t \in \mathbb{T}\}$.

If $p \in \mathcal{R}$, then we define the exponential function by

$$e_p(t, s) = exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\right) \ for \ s, t \in \mathbb{T},$$

where $\xi_h(z) = \begin{cases} \frac{1}{h} Log(1 + zh), & h \neq 0, \\ z, & h = 0. \end{cases}$

For properties of this exponential function see [2]. For instance, if $p \in \mathcal{R}$, then $e_p(t, s)$ is real valued and nonzero on $\mathbb{T}$, and if $p \in \mathcal{R}^+$, then $e_p(t, s)$ is positive.

By a solution of (1.1), we mean a nontrivial real-valued function x satisfying (1.1) for $t \geq t_0$. A solution x of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Equation (1.1) is called oscillatory if all solutions are oscillatory. Our attention is restricted to those solutions x of (1.1) which exist on half line $[t_x, \infty)$ with $\sup\{|x(t)| : t \geq t_0\} \neq 0$ for any $t_0 \geq t_x$.

2. Main results

Note that for Equation (1.1),

$$e_r(t, t_0)x^{\triangle\triangle} + e_r(t, t_0)r(t)x^{\triangle^\sigma} + e_r(t, t_0)q(t)f(x \circ g) = 0,$$

i.e.,

$$(e_r(t, t_0)x^{\triangle})^{\triangle} + p(t)f(x \circ g) = 0,$$

where $p(t) = e_r(t, t_0)q(t)$. Let $a(t) = e_r(t, t_0) > 0$. Then the above equation can be written as

$$(a(t)x^{\triangle})^{\triangle} + p(t)f(x \circ g) = 0. \tag{2.1}$$

Before we state and prove our main oscillation results we first introduce the following lemmas.

Lemma 1 [11]. *Assume that* $g(t) \leq t$,

$$\int_{t_0}^{\infty} \frac{\Delta t}{a(t)} = \infty,$$

and

$$\int_{t_0}^{\infty} g(t)p(t)\Delta t = \infty,$$

and assume that (2.1) has a positive solution x *on* $[t_0, \infty)_{\mathbb{T}}$. *Then there exists a* $T \in [t_0, \infty)_{\mathbb{T}}$ *sufficiently large, so that*

(1) $x^{\triangle}(t) > 0, \ x(t) > tx^{\triangle}(t)$ *for* $t \in [T, \infty)_{\mathbb{T}}$;

(2) x *is strictly increasing and* $x(t)/t$ *is strictly decreasing on* $[T, \infty)_{\mathbb{T}}$.

Lemma 2[2]. *Let $a, b \in \mathbb{T}$ and assume $f, g \in C_{rd}$. Then*

$$\int_a^b f(\sigma(t))g^{\triangle}(t)\triangle t = [f(t)g(t)]_a^b - \int_a^b f^{\triangle}(t)g(t)\triangle t.$$

Lemma 3[2]. *Assume $g : \mathbb{T} \to \mathbb{R}$ is delta differentiable on $\mathbb{T}$. Assume further $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable. Then $f \circ g : \mathbb{T} \to \mathbb{R}$ is delta differentiable and satisfies*

$$(f \circ g)^{\triangle}(t) = \left\{\int_0^1 f'(g(t) + h\mu(t)g^{\triangle}(t))dh\right\} g^{\triangle}(t).$$

Theorem 1. *Assume that $\int_{t_0}^{\infty} \frac{\triangle t}{a(t)} = \infty,\ g(t) \le t,\ \int_{t_0}^{\infty} g(t)p(t)\triangle t = \infty$. If there exists a positive $\triangle-$differentiable function δ such that*

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left[\frac{M\delta^{\sigma}(s)p(s)g(s)}{\sigma(s)} - \frac{(\delta^{\triangle}(s))^2 a(s)\sigma(s)}{4s\delta^{\sigma}}\right] \triangle s = \infty, \tag{2.2}$$

then every solution of (1.1) is oscillatory.

Proof. Suppose to the contrary that Eq.(1.1) has a nonoscillatory solution $x(t)$. Without loss of generality, we may assume that x is an eventually positive solution of Eq.(1.1). Then from Lemma 1, there exists T such that $x^{\triangle}(t) > 0$ holds for $t \ge T$. Define the function

$$\omega(t) = \delta(t)\frac{a(t)x^{\triangle}(t)}{x(t)} > 0. \tag{2.3}$$

$\triangle-$differentiation of Eq.(2.3) gives

$$\begin{aligned}
\omega^{\triangle}(t) &= \delta^{\triangle}(t)[\frac{a(t)x^{\triangle}(t)}{x(t)}] + \delta^{\sigma}(t)[\frac{a(t)x^{\triangle}(t)}{x(t)}]^{\triangle} \\
&= \frac{\delta^{\triangle}(t)\omega(t)}{\delta(t)} - \frac{\delta^{\sigma}(t)p(t)f(x(g(t)))}{x(\sigma(t))} - \frac{\delta^{\sigma}(t)a(t)(x^{\triangle}(t))^2}{x(t)x(\sigma(t))} \\
&\le -\delta^{\sigma}(t)p(t)M\frac{x(g(t))}{x(\sigma(t))} + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta^{\sigma}(t)a(t)(x^{\triangle}(t))^2}{x(t)x(\sigma(t))} \\
&< -\delta^{\sigma}(t)p(t)M\frac{g(t)}{\sigma(t)} + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta^{\sigma}(t)\omega^2(t)}{\delta^2(t)a(t)} \cdot \frac{1}{1+\mu(t)\frac{x^{\triangle}(t)}{x(t)}} \\
&< -\delta^{\sigma}(t)p(t)M\frac{g(t)}{\sigma(t)} + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta^{\sigma}(t)\omega^2(t)}{\delta^2(t)a(t)} \cdot \frac{t}{t+\mu(t)} \\
&= -M\delta^{\sigma}(t)p(t)\frac{g(t)}{\sigma(t)} + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{Q(t)\omega^2(t)}{\delta^2(t)},
\end{aligned}$$

where $Q(t) = \frac{\delta^\sigma(t)t}{a(t)\sigma(t)} > 0$. Further we get

$$\begin{aligned}\omega^\triangle(t) &\le -M\delta^\sigma(t)p(t)\frac{g(t)}{\sigma(t)} + \frac{(\delta^\triangle(t))^2}{4Q(t)} - [\frac{\sqrt{Q(t)}}{\delta(t)}\omega(t) - \frac{\delta^\triangle(t)}{2\sqrt{Q(t)}}]^2 \\ &< -[M\delta^\sigma(t)p(t)\frac{g(t)}{\sigma(t)} - \frac{(\delta^\triangle(t))^2}{4Q(t)}].\end{aligned}$$

Integrating the above equation from T to t, we obtain

$$-\omega(T) < \omega(t) - \omega(T) < -\int_T^t [\frac{M\delta^\sigma(s)p(s)g(s)}{\sigma(s)} - \frac{(\delta^\triangle(s))^2}{4Q(s)}]\triangle s = -\infty,$$

which is a contradiction. The proof is complete. □

Corollary 1. *If we choose $\delta(t) = t$, and (2.2) can be written as*

$$\limsup_{t\to\infty} \int_{t_0}^t [Mp(s)g(s) - \frac{a(s)}{4s}]\triangle s = \infty, \tag{2.4}$$

then every solution of (1.1) is oscillatory.

Example 1. Consider the equation

$$x^{\triangle\triangle} + \frac{1-t}{t^2}x^{\triangle^\sigma} + \sqrt{t}e^{\frac{ln^2 t}{2ln2}}x(\frac{t}{2}) = 0,\ t > t_0 = 1, t \in \mathbb{T}$$

where $\mathbb{T} = 2^{N_0}$, $r(t) = \frac{1-t}{t^2}$, $q(t) = \sqrt{t}e^{\frac{ln^2 t}{2ln2}}$, $g(t) = \frac{t}{2}$, $M = 1$, then

$$a(t) = e_r(t, t_0) = \prod_{s\in(0,t)\bigcap\mathbb{T}} (1 + \tfrac{1-s}{s}) = \prod_{n=0}^{k-1} \tfrac{1}{2^n} = 2^{\frac{k}{2}} \cdot 2^{-\frac{k^2}{2}},$$

where we put $t = 2^k$, substituting $t = 2^k$, we finally get that

$$a(t) = \sqrt{t}e^{-\frac{ln^2 t}{2ln2}}, \qquad p(t) = e_r(t,1)q(t) = t,$$

$$\limsup_{t\to\infty} \int_1^t [s \cdot \frac{s}{2} - \frac{1}{4\sqrt{s}e^{\frac{ln^2 s}{2ln2}}}]\triangle s = \infty.$$

By corollary 1, this equation is oscillatory. □

Theorem 2. *Assume* $\int_{t_0}^\infty \frac{\triangle t}{a(t)} = \infty$, $a^\triangle(t) \ge 0$, $g(t) \ge \sigma(t)$ *if there exists a positive $\triangle-$differentiable function δ such that for any constant $M_1 > 1$*

$$\limsup_{t\to\infty} \int_{t_0}^t [M\delta^\sigma(s)p(s) - \frac{(\delta^\triangle(s))^2 a(s)(s + M_1\mu(s))}{4s\delta^\sigma}]\triangle s = \infty, \tag{2.5}$$

then every solution of (1.1) is oscillatory.

Proof. Without loss of generality, we may assume that x is an eventually positive solution of Eq.(1.1). We next show that $x^\triangle(t) > 0$. From (2.1) we know that

$$(a(t)x^\triangle(t))^\triangle = -p(t)f(x \circ g) \le 0.$$

Thus $a(t)x^{\triangle}(t)$ is nonincreasing. If there exists t_1 such that $x^{\triangle}(t_1) \le 0$, since $p(t) \not\equiv 0$, there exists t_2 such that $a(t_2)x^{\triangle}(t_2) < a(t_1)x^{\triangle}(t_1) \le 0$. For $t > t_2$, $a(t)x^{\triangle}(t) \le a(t_2)x^{\triangle}(t_2)$, i.e., $x^{\triangle}(t) \le \dfrac{a(t_2)x^{\triangle}(t_2)}{a(t)}$. Integrating the above equation we get

$$x(t) \le x(t_2) + \int_{t_2}^{t} \frac{a(t_2)x^{\triangle}(t_2)}{a(s)} \triangle s.$$

Letting $t \to \infty$, then $x(t) \to -\infty$. So $x^{\triangle}(t) > 0$ holds for $t \ge T$. From $(a(t)x^{\triangle}(t))^{\triangle} = a^{\triangle}(t)x^{\triangle}(t) + a(\sigma(t))x^{\triangle\triangle}(t) \le 0,\ a^{\triangle}(t) > 0$, we have $x^{\triangle\triangle}(t) < 0$ and

$$x(t) - x(T) = \int_T^t x^{\triangle}(s)\triangle s > x^{\triangle}(t)(t-T).$$

So there exist T_1 and $M_1 > 1$ such that $\frac{x^{\triangle}(t)}{x(t)} < \frac{M_1}{t}$ holds for $t > T_1$. Let $\omega(t) = \delta(t)\dfrac{a(t)x^{\triangle}(t)}{x(t)} > 0$, similar to the proof of Theorem 1. We get

$$\begin{aligned}
\omega^{\triangle}(t) &\le -\delta(\sigma(t))\frac{p(t)Mx(g(t))}{x(\sigma(t))} + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta(\sigma(t))a(t)(x^{\triangle}(t))^2}{x(t)x(\sigma(t))} \\
&< -\delta(\sigma(t))p(t)M + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta(\sigma(t))\omega^2(t)}{\delta^2(t)a(t)} \cdot \frac{1}{1+\mu(t)\frac{x^{\triangle}(t)}{x(t)}} \\
&< -\delta(\sigma(t))p(t)M + \frac{\delta^{\triangle}(t)}{\delta(t)}\omega(t) - \frac{\delta(\sigma(t))\omega^2(t)}{\delta^2(t)a(t)} \cdot \frac{1}{1+\mu(t)\frac{M_1}{t}}.
\end{aligned}$$

Let $Q(t) = \frac{\delta(\sigma(t))}{a(t)} \cdot \frac{t}{t+M_1\mu(t)}$. Then

$$\omega^{\triangle}(t) < \frac{(\delta^{\triangle}(t))^2}{4Q(t)} - \delta(\sigma(t))p(t)M.$$

Integrating the above equation from T_1 to t, we get

$$-\omega(T_1) < \omega(t) - \omega(T_1) < -\int_{T_1}^{t} [M\delta^{\sigma}(s)p(s) - \frac{(\delta^{\triangle}(s))^2}{4Q(s)}]\triangle s.$$

This is a contradiction. The proof is complete. □

Corollary 2. *If we choose $\delta(t) = t$, and (2.5) can be written as*

$$\limsup_{t\to\infty} \int_{t_0}^{t} [M\sigma(s)p(s) - \frac{a(s)(s+M_1\mu(s))}{4s\sigma(s)}]\triangle s = \infty, \tag{2.6}$$

then every solution of (1.1) is oscillatory.

Theorem 3. *Assume that $g(t) \ge \sigma(t)$, $\int_{t_0}^{\infty} \frac{\triangle t}{a(t)} = \int_{t_0}^{\infty} p(t)\triangle t = \infty$, and f is increasing. Then Eq.(1.1) is oscillatory.*

Proof. From the proof of Theorem 2, we know $x(t) > 0, x^{\triangle}(t) > 0$ hold for $t \geq T$. Now dividing by $f(x \circ \sigma)$ and integrating (2.1) yields

$$\frac{a(t)x^{\triangle}(t)}{f(x(t))} - \int_T^t a(s)x^{\triangle}(s)(\frac{1}{f \circ x})^{\triangle}(s)\triangle s + \int_T^t p(s)\frac{f(x(g(s)))}{f(x(\sigma(s)))}\triangle s = \frac{a(T)x^{\triangle}(T)}{f(x(T))},$$

where $t \geq T$. Since $x^{\triangle} > 0$, f is increasing, $\frac{f(x(g(s)))}{f(x(\sigma(s)))} \geq 1$.

$$\frac{a(t)x^{\triangle}(t)}{f(x(t))} + \int_T^t p(s)\triangle s \leq \frac{a(T)x^{\triangle}(T)}{f(x(T))} + \int_T^t a(s)x^{\triangle}(s)(\frac{1}{f \circ x})^{\triangle}(s)\triangle s.$$

But

$$\int_T^t a(s)x^{\triangle}(s)(\frac{1}{f \circ x})^{\triangle}(s)\triangle s = -\int_T^t a(s)x^{\triangle}(s)\frac{(f \circ x)^{\triangle}(s)}{f(x(s))f(x(\sigma(s)))}\triangle s$$

$$= -\int_T^t a(s)x^{\triangle}(s)\{\int_0^1 f^{'}(x(s) + h\mu(s)x^{\triangle}(s))dh\}\frac{x^{\triangle}(s)}{f(x(s))f(x(\sigma(s)))}\triangle s \leq 0.$$

So

$$\frac{a(t)x^{\triangle}(t)}{f(x(t))} + \int_T^t p(s)\triangle s \leq \frac{a(T)x^{\triangle}(T)}{f(x(T))},$$

we get a contradiction. □

Remark. If $g(t) = \sigma(t)$, $\mathbb{T} \neq \mathbb{R}$, Theorem 3 is same with M.Bohner's result for nonlinear second order dynamic equations [3].

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