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Delay-dependent Robust Stability of Uncertain Dynamic Systems with Time-varying Delays

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Abstract – In this paper, the stability analysis for uncertain dynamic systems with time-varying delays is considered. By constructing a new Lyapunov functional, a novel stability criterion is established in terms of linear matrix inequalities (LMIs). Two numerical examples are carried out to support the effectiveness of the proposed method.

Key Words : Robust Stability, Time-varying Delays, Uncertain Dynamic Systems, Lyapunov Method, LMI

1. Introduction

Time delays often occur in many industrial systems such as large-scale systems, chemical processes, tandem rolling mills, cellular neural networks, network control systems, and so on. It is well known that the existence of time delays may cause poor performance and even instability [1]-[3]. Therefore, many attentions have been paid to stability analysis of dynamic systems with either constant time-delays or time-varying delays [1]-[19]. Especially, delay-dependent stability criteria, which are less conservative than delay-independent ones when the size of time-delays are small, have been investigated by many researchers during the last decade [3]-[19].

In this field, an important issue is to enlarge the feasible region for guaranteeing the stability in a given interval of time-delays. Therefore, how to choose Lyapunov-Krasovskii functionals and derive the stability condition by calculating the upper bounds of time derivative of Lyapunov-Krasovskii functionals play key roles to increase the maximum allowable bound of time delays. In this regard, Park [10] proposed a new bounding technique and showed the proposed stability criteria of time-delay systems have larger time-delay interval. Kim [11] investigated a new stability criterion for systems with time-varying delays and Yue and Won [12] improved the results of Kim [11]. Fridman and Shaked [13] proposed a descriptor system approach which considers zero equations including the state equation. Jing et al. [14] reduces the conservatism of delay-dependent stability criterion for uncertain linear systems with time-varying delays by constructing a new Lyapunov-Krasovskii functionals with a proper distribution of the time delays. Kwon and Park [15] used a new parameter model transformation to reduce the conservatism of stability and stabilization criteria of dynamic systems with constant time-delays. He et al. [16] introduced free-weighting matrices in zero equations in driving a less conservative results. Kim and Yi [17] investigated a new delay-dependent stability criterion of time-varying delays. In [18], an improved stability criterion for systems with time-varying delays was proposed by taking integral terms as augmentation variables. Recently, Cheng and Peng [19] studied delay-dependent stability criterion with interval time-varying delays, which the lower bound of delay bounds is not restricted to be zero, was proposed.

In this paper, we propose a new delay-dependent stability criterion of uncertain dynamic systems with time-varying delays. The considered system is assumed to have norm bounded parameter uncertainties. In order to derive a less conservative results, a new Lyapunov functional which fractions delay interval is proposed. Then, a novel delay-dependent robust stability criterion is derived in terms of LMIs (Linear Matrix Inequalities) which can be solved efficiently by various convex optimization algorithms [20]. Two numerical examples are given to show the effectiveness of the proposed method. Throughout this paper, ★ represents the elements below the main diagonal of a symmetric matrix. The notation

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$X > Y$, where X and Y are matrices of same dimensions, means that the matrix $X-Y$ is positive definite, I denotes the identity matrix whose dimensions can be determined from the context. R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix. $C_{n,h}([-h,\ 0),R^n)$ denotes the Banach space of continuous vector functions which maps the interval $[-h,0)$ into R^n.

2. Problem Statements

Consider the following uncertain dynamic systems with time-varying delays:

$$\dot{x}(t) = (A+\Delta A)x(t) + (A_d+\Delta A_d)x(t-h(t)) \qquad (1)$$
$$x(s) = \phi(s),\ s\in[-h_U,0],$$

where $x(t)\in R^n$ is the state vector, $A,\ A_d\in R^{n\times n}$ are known system matrices, $\phi(s)\in C_{n,h}$ is a given vector valued initial function, $h(t)$ represents time-varying delays which satisfies $0\le h(t)\le h_U$, $\dot{h}(t)\le h_D$. The parameter uncertainties ΔA, ΔA_d are assumed to be of the form

$$[\Delta A\ \ \Delta A_d] = DF(t)[E E_d] \qquad (2)$$

where D, E, E_d are known real constant matrices of appropriate dimensions, and $F(t)$ are unknown matrices, which satisfy

$$F^T(t)F(t) \le I. \qquad (3)$$

Let us define

$$p(t) = F(t)q(t), \qquad (4)$$
$$q(t) = Ex(t) + E_d x(t-h(t)).$$

Then, system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Dp(t), \qquad (5)$$
$$p(t) = F(t)q(t),$$
$$q(t) = Ex(t) + E_d x(t-h(t)).$$

The objective of this paper is to develope a delay-dependent stability criterion for system (5).

Before deriving the main result, we need the following fact and lemma.

Fact 1. (Schur Complement) Given constant symmetric matrices $\Sigma_1,\Sigma_2,\Sigma_3$ where $\Sigma_1=\Sigma_1^T$ and $0<\Sigma_2=\Sigma_2^T$, then $\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$ if and only if

$$\begin{bmatrix}\Sigma_1 & \Sigma_3^T\\ \Sigma_3 & -\Sigma_2\end{bmatrix}<0, \text{or } \begin{bmatrix}-\Sigma_2 & \Sigma_3\\ \Sigma_3^T & -\Sigma_1\end{bmatrix}<0. \qquad (6)$$

Fact 2. For any real vectors a, b and any matrix $Q>0$ with appropriate dimension, the following inequality

$$\pm 2a^T b \le a^T Qa + b^T Q^{-1} b$$

is always satisfied.

To derive a less conservative stability criterion, let us introduce a integral inequality bounding lemma which will be used to take an upper bound of time derivative of Lyapunov function.

Lemma 1. For any scalar $h(t)>0$, and positive matrix Q, the following inequality holds:

$$-\int_{t-h(t)}^{t} \dot{x}^T Q\dot{x}(s)ds \le \qquad (7)$$
$$+h(t)\zeta^T(t)XQ^{-1}X^T\zeta(t) + 2\zeta^T(t)X[x(t)-x(t-h(t))]$$

where

$$\zeta^T(t) = \left[x^T(t)\ x^T(t-h(t))\ x^T(t-\frac{h_U}{2})\ x^T(t-h_U)\ \dot{x}^T(t)\ p^T(t)\right]^T, \qquad (8)$$

and X is free variable matrix with appropriate dimension.

Proof. From Fact 2, the following inequality with Q>0 holds:

$$-\int_{t-h(t)}^{t} 2(X^T\zeta(t))^T\dot{x}(s)ds \le \qquad (9)$$
$$\int_{t-h(t)}^{t}[\dot{x}^T(s)Q\dot{x}(s) + (X^T\zeta(t))^T Q^{-1}(X\zeta(t))]ds$$

The inequality (9) can be written as

$$\int_{t-h(t)}^{t}\zeta^T(t)XQ^{-1}X^T\zeta(t)ds$$
$$+2\zeta^T X\int_{t-h(t)}^{t}\dot{x}(s)ds + \int_{t-h(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds$$
$$= h(t)\zeta^T(t)XQ^{-1}X^T\zeta(t)$$
$$+2\zeta^T X[x(t)-x(t-h(t)] + \int_{t-h(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds$$
$$\ge 0. \qquad (10)$$

Theferore, from (10), the inequality (6) can be obtained. This completes the proof of Lemma 1. ■

3. Main Results

For simplicity of matrix dimension, let us define the following notations.

$$\Sigma = [\Sigma_{(i,j)}],(i,j=1,\dots,6)$$
$$\Sigma_{(1,1)} = R_2+R_4+P_1+P_1^T,\ \Sigma_{(1,2)} = P_1A_d,$$
$$\Sigma_{(1,3)} = 0,\ \Sigma_{(1,4)} = R_1,\ \Sigma_{(1,5)} = -P_1+A^TP_2^T,$$
$$\Sigma_{(1,6)} = P_1D,\ \Sigma_{(2,2)} = -(1-h_D)R_4,\ \Sigma_{(2,3)} = 0,$$
$$\Sigma_{(2,4)} = 0,\ \Sigma_{(2,5)} = A_d^TP_2^T,\ \Sigma_{(2,6)} = 0,$$
$$\Sigma_{(3,3)} = -R_2+R_3,\ \Sigma_{(3,4)} = 0,\ \Sigma_{(3,5)} = 0,$$
$$\Sigma_{(3,6)} = 0,\ \Sigma_{(4,4)} = -R_3,\ \Sigma_{(4,5)} = 0,$$
$$\Sigma_{(5,5)} = \frac{h_U}{2}(Q_1+Q_2)-P_2-P_2^T,\ \Sigma_{(5,6)} = P_2D,\ \Sigma_{(6,6)} = -H,$$
$$X = [X_1^T X_2^T X_3^T X_4^T 0\ 0],\ Y = [Y_1^T Y_2^T Y_3^T Y_4^T 0\ 0],$$
$$Z = [Z_1^T Z_2^T Z_3^T Z_4^T 0\ 0],\ \Xi = [X\ \ -X+Y\ \ -Y+Z\ \ -Z\ 0\ 0],$$
$$\tilde{X} = [\tilde{X}_1^T \tilde{X}_2^T \tilde{X}_3^T \tilde{X}_4^T 0\ 0],\ \tilde{Y} = [\tilde{Y}_1^T \tilde{Y}_2^T \tilde{Y}_3^T \tilde{Y}_4^T 0\ 0],$$
$$\tilde{Z} = [\tilde{Z}_1^T \tilde{Z}_2^T \tilde{Z}_3^T \tilde{Z}_4^T 0\ 0],\ \tilde{\Xi} = [\tilde{X}\ \ -\tilde{Y}+\tilde{Z}\ \ -\tilde{X}+\tilde{Y}-\tilde{Z}\ 0\ 0],$$
$$\Psi = [E\ E_d\ 0\ 0\ 0\ 0]. \qquad (11)$$

Then, we have the following theorem.

Theorem 1. For given $h_U>0$, and h_D, the system (5) is asymptotically stable for $0\le h(t)\le h_U$ and $\dot{h}(t)\le h_D$ if there exist positive matrices $R_i(i=1,\dots,4)$, $Q_i(i=1,2)$, H and any matrices X_i, Y_i, Z_i, $\overline{X}_i$, $\overline{Y}_i$, $\overline{Z}_i$, $(i=1,\dots,4)$ such that such that the following LMIs hold

$$\begin{bmatrix} \Sigma+\Xi+\Xi^T\Psi^T H & \frac{h_U}{2}X & \frac{h_U}{2}Y & \frac{h_U}{2}Z \\ \star & -H & 0 & 0 & 0 \\ \star & \star & -\frac{h_U}{2}Q_1 & 0 & 0 \\ \star & \star & \star & -\frac{h_U}{2}Q_1 & 0 \\ \star & \star & \star & \star & -\frac{h_U}{2}Q_2 \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \Sigma+\tilde{\Xi}+\tilde{\Xi}^T\Psi^T H & \frac{h_U}{2}\tilde{X} & \frac{h_U}{2}\tilde{Y} & \frac{h_U}{2}\tilde{Z} \\ \star & -H & 0 & 0 & 0 \\ \star & \star & -\frac{h_U}{2}Q_1 & 0 & 0 \\ \star & \star & \star & -\frac{h_U}{2}Q_2 & 0 \\ \star & \star & \star & \star & -\frac{h_U}{2}Q_2 \end{bmatrix} < 0. \quad (13)$$

Proof. For positive matrices $R_i(i=1,\ldots,4)$, and $Q_i(i=1,2)$ let us consider the following Lyapunov function defined by

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t) \quad (14)$$

where

$$\begin{aligned} V_1(t) &= x^T(t)R_1x(t), \\ V_2(t) &= \int_{t-\frac{h_U}{2}}^{t} x^T(s)R_2x(s)ds+\int_{t-h_U}^{t-\frac{h_U}{2}} x^T(s)R_3x(s)ds, \\ V_3(t) &= \int_{t-h(t)}^{t} x^T(s)R_4x(s)ds, \\ V_4(t) &= \int_{t-\frac{h_U}{2}}^{t}\int_s^t \dot{x}^T(u)Q_1\dot{x}(u)duds \\ &\quad +\int_{t-h_U}^{t-\frac{h_U}{2}}\int_s^t \dot{x}^T(u)Q_2\dot{x}(u)duds. \end{aligned} \quad (15)$$

First, the time derivative of $V_1(t)$ along the solution of Eq. (5) is obtained by

$$\dot{V}_1(t)=2x^T(t)R_1\dot{x}(t). \quad (16)$$

Second, differentiating $V_2(t)$ leads to

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)R_2x(t)-x^T(t-\frac{h_U}{2})R_2x(t-\frac{h_U}{2}) \\ &\quad + x^T(t-\frac{h_U}{2})R_3x(t-\frac{h_U}{2})-x^T(t-h_U)R_3x(t-h_U) \end{aligned} \quad (17)$$

Third, the upper bound of $\dot{V}_3(t)$ can be obtained as

$$\begin{aligned} \dot{V}_3(t) &= x^T(t)R_4x(t)-(1-\dot{h}(t))x^T(t-h(t))R_4x(t-h(t)) \\ &\le x^T(t)R_4x(t)-(1-h_D)x^T(t-h(t))R_4x(t-h(t)). \end{aligned} \quad (18)$$

Lastly, the upper bound of $\dot{V}_4(t)$ can be calculated

$$\begin{aligned} \dot{V}_4(t) &= \frac{h_U}{2}\dot{x}^T(t)Q_1\dot{x}(t)-\int_{t-\frac{h_U}{2}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds \\ &\quad +\frac{h_U}{2}\dot{x}^T(t)Q_2\dot{x}(t)-\int_{t-h_U}^{t-\frac{h_U}{2}}\dot{x}^T(s)Q_2\dot{x}(s)ds. \end{aligned} \quad (19)$$

Using Lemma 1, the upper bounds of $-\int_{t-\frac{h_U}{2}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds$ and $-\int_{t-h_U}^{t-\frac{h_U}{2}}\dot{x}^T(s)Q_2\dot{x}(s)ds$ can be estimated as follows respectively.

(i) Case 1: $0 \le h(t) \le \frac{h_U}{2}$

$$\begin{aligned} &-\int_{t-\frac{h_U}{2}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds \\ &= -\int_{t-h(t)}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds-\int_{t-\frac{h_U}{2}}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}(s)ds \\ &\le h(t)\zeta^T(t)XQ_1^{-1}X^T\zeta(t)+2\zeta^T(t)X[x(t)-x(t-h(t))] \\ &\quad +(\frac{h_U}{2}-h(t))\zeta^T(t)YQ_1^{-1}Y^T\zeta(t) \\ &\quad +2\zeta^T(t)Y[x(t-h(t))-x(t-\frac{h_U}{2})] \\ &\le \frac{h_U}{2}\zeta^T(t)XQ_1^{-1}X^T\zeta(t)+2\zeta^T(t)X[x(t)-x(t-h(t))] \\ &\quad +\frac{h_U}{2}\zeta^T(t)YQ_1^{-1}Y^T\zeta(t) \\ &\quad +2\zeta^T(t)Y[x(t-h(t))-x(t-\frac{h_U}{2})] \end{aligned} \quad (20)$$

$$\begin{aligned} &-\int_{t-h_U}^{t-\frac{h_U}{2}}\dot{x}^T(s)Q_2\dot{x}(s)ds \\ &\le \frac{h_U}{2}\zeta^T(t)ZQ_2^{-1}Z^T\zeta(t)+2\zeta^T(t)Z[x(t-\frac{h_U}{2})-x(t-h_U)] \end{aligned} \quad (21)$$

Note that $\zeta(t)$, X, Y, Z are defined in (11).
To obtain a less conservative results, we add the following zero equations with any matrices P_1 and P_2 to be chosen as

$$\begin{aligned} 0 &= [x^T(t)P_1+\dot{x}^T(t)P_2] \\ &\quad \times[-\dot{x}(t)+Ax(t)+A_dx(t-h(t)+Dp(t)] \end{aligned} \quad (22)$$

Since the following inequality holds from (3) and (5),

$$p^T(t)p(t) \le q^T(t)q(t), \quad (23)$$

there exist a positive matrix H satisfying the following inequality

$$\zeta^T(t)\Psi^T H\Psi\zeta(t)-p^T(t)Hp(t) \ge 0, \quad (24)$$

where Ψ is defined in (11).
From (15) -(25) and applying S-procedure[15], the upper bound of $\dot{V}(t)=\sum_{i=1}^{4}\dot{V}_i(t)$ for the case $0 \le h(t) \le \frac{h_U}{2}$ can be obtained as

$$\begin{aligned} \dot{V}(t) &\le \zeta^T(t)[\Sigma+\Xi+\Xi^T+\Psi^T H\Psi+\frac{h_U}{2}XQ_1^{-1}X^T \\ &\quad +\frac{h_U}{2}YQ_1^{-1}Y^T+\frac{h_U}{2}ZQ_2^{-1}Z^T]\zeta(t) \\ &= \zeta^T(t)\Omega_1\zeta(t) \end{aligned} \quad (25)$$

Using Fact 1, $\Omega_1<0$ is equivalent to (12).

(ii) Case 2: $\frac{h_U}{2} \le h(t) \le h_U$.

$$-\int_{t-\frac{h_U}{2}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds \tag{26}$$
$$\le \frac{h_U}{2}\zeta^T(t)\tilde{X}Q_1^{-1}\tilde{X}^T\zeta(t)+2\zeta^T(t)\tilde{X}[x(t)-x(t-\frac{h_U}{2})]$$
$$+(h_U-h(t))\zeta^T(t)\tilde{Z}Q_2^{-1}\tilde{Z}^T\zeta(t)$$
$$+2\zeta^T(t)\tilde{Z}[x(t-h(t))-x(t-h_U)]$$
$$\le \frac{h_U}{2}\zeta^T(t)\tilde{Y}Q_2^{-1}\tilde{Y}^T\zeta(t)$$
$$+2\zeta^T(t)\tilde{Y}[x(t-\frac{h_U}{2})-x(t-h(t))]$$
$$+\frac{h_U}{2}\zeta^T(t)\tilde{Z}Q_2^{-1}\tilde{Z}^T\zeta(t)$$
$$+2\zeta^T(t)\tilde{Z}[x(t-h(t))-x(t-h_U)].$$
$$-\int_{t-h_U}^{t-\frac{h_U}{2}}\dot{x}^T(s)Q_2\dot{x}(s)ds$$
$$=-\int_{t-h(t)}^{t-\frac{h_U}{2}}\dot{x}^T(s)Q_2\dot{x}(s)ds-\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_2\dot{x}(s)ds$$
$$\le (h(t)-\frac{h_U}{2})\zeta^T(t)\tilde{Y}Q_2^{-1}\tilde{Y}^T\zeta(t)$$
$$+2\zeta^T(t)\tilde{Y}[x(t-\frac{h_U}{2})-x(t-h(t))] \tag{27}$$

Note that $\tilde{X}, \tilde{Y}, \tilde{Z}$ are defined in (11).
From (15)-(19), (22)-(24), (26)-(27) and applying S-procedure [15], the upper bound of $\dot{V}(t)=\sum_{i=1}^{4}\dot{V}_i(t)$ for the case $\frac{h_U}{2}\le h(t)\le h_U$ can be obtained as

$$\dot{V}(t)\le \zeta^T(t)[\Sigma+\Xi+\Xi^T+\Psi^T H\Psi+\frac{h_U}{2}\tilde{X}Q_1^{-1}\tilde{X}^T$$
$$+\frac{h_U}{2}\tilde{Y}Q_2^{-1}\tilde{Y}^T+\frac{h_U}{2}\tilde{Z}Q_2^{-1}\tilde{Z}^T]\zeta(t)$$
$$\equiv \zeta^T(t)\Omega_2\zeta(t) \tag{28}$$

Using Fact 1, $\Omega_2<0$ is equivalent to (13) in Theorem 1. Therefore, if LMIs (12) and (13) hold, then the system (5) with $0\le h(t)\le h_U$ and $\dot{h}(t)\le h_D$ is asymptotically stable. This completes the proof of Theorem 1. ■

Remark 1. The problem for solving LMIs (12) and (13) in Theorem 1 is to determine whether the problem is feasible or not. It is called the feasibility problem. The solutions of the LMIs (12) and (13) can be found by using various efficient convex optimization algorithms which can be used to check the feasibility of the LMIs in Theorem 1. In this paper, in order to solve the LMIs of Theorem 1, we utilize Matlab's LMI Control Toolbox [16], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithm [15].

Remark 2. In Eq. (15), the new Lyapunov functional which divided into two intervals $[0,\frac{h_U}{2}]$ and $[\frac{h_U}{2},h_U]$ are proposed. Therefore, by taking the states $x(\frac{t-h_U}{2})$ and $x(t-h_U)$ as augmented variables simultaneously, the stability criterion in Theorem 1 utilizes more information on state variables. And in deriving the upper bound of $\dot{V}_4(t)$, the two cases $0\le h(t)\le\frac{h_U}{2}$ and $\frac{h_U}{2}\le h(t)\le h_U$ are considered with different free variables, which are not considered in other literature. To consider the above two cases in upper bound of time-derivative values of $\dot{V}_4(t)$, we utilize Lemma 1 proposed in this paper. These consideration mentioned above may lead to provide larger delay bounds than the previous ones.

Remark 3. If we do not consider $V_3(t)$ in Theorem 1, then we can easily obtained a delay-dependent stability criterion of systems (5) with no delay-derivative information. In other words, the stability criterion without $V_3(t)$ in Theorem 1 do not need the condition $\dot{h}(t)\le h_D$, which means *fast time-varying delays.*

4. Numerical Examples

Example 1. Consider the following systems (5) with

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}, D=\lambda I,\ E=E_d=I. \tag{29}$$

For the condition $\lambda=0$, by applying Theorem 1 to the above system, delay bounds with different h_D are listed in Table 1. For the conditions $\lambda=0.2$ and $h_D=0$, the results are shown in Table 2. From these two tables, one can see the proposed Theorem 1 provides larger delay bounds than the ones in other literature.

Example 2. Consider the following systems (5) with

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_d+\Delta A_d)x(t-h) \tag{30}$$

where

$$A=\begin{bmatrix}-0.6 & -2.3\\ 0.8 & -1.2\end{bmatrix},\ A_d=\begin{bmatrix}-0.9 & 0.6\\ 0.2 & 0.1\end{bmatrix}, D=\lambda I,\ E=E_d=I. \tag{31}$$

When $\lambda=0.6$, the maximum delay bounds for guaranteeing stability was 0.2139 in [19]. However, by applying Theorem 1 to the system (30) with the same $\lambda=0.6$, one can obtain delay bound as 0.3298. In Table 3, the delay bound of h_U for different λ is given and the results are compared with the ones in [4], [8] and [17]. From Table 3, one can see our result gives a larger upper bound of h_U for guaranteeing the asymptotic stability of above system than those in other literature.

Example 3. Consider the following systems (5) with

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_d+\Delta A_d)x(t-h(t)) \tag{32}$$

where

$$A=\begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}, \quad (33)$$
$$D=I,\ E=diag\{1.6,\ 0.05\},\ E_d=\mathrm{diag}\{0.1,\ 0.3\}.$$

For the above system, the delay bound of h_U for various h_D is given in Table 4, which shows Theorem 1 provides larger delay bounds than the ones in previous literatures [18]-[19].

Example 4. Consider the following systems (5) with

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_d+\Delta A_d)x(t-h(t)) \quad (34)$$

where

$$A=\begin{bmatrix}-0.5 & -2\\ 1 & -1\end{bmatrix},\ A_d=\begin{bmatrix}-0.5 & -1\\ 0 & 0.6\end{bmatrix}, D=0.2I,\ E=E_d=I. \quad (35)$$

When $h_D=0.5$ and 0.9, the maximum delay bounds were 0.4243 and 0.4095, respectively. However, the delay bounds obtained by applying Theorem to the system (34) with the same condition are 0.4284 and 0.4268, respectively. Therefore, when $h_D \geq 0.5$, Theorem 1 for this example give larger delay bounds than the ones in [19].

Example 5. Consider the following state-space model

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}x(t)+\begin{bmatrix}0\\ 0.1\end{bmatrix}u(t) \quad (36)$$

It is assumed the system (36) is controlled through a network by state-feedback controller $u(t)=-[3.75\ 11.5]x(t)$. In [23], the maximum allowable transfer interval (MATI) which guarantees the asymptotic stability of system (39) with the controller $u(t)=-[3.75\ 11.5]x(t)$ was 1.0081. However, by applying Theorem 1 to the above system, one can obtain MATI as 1.0428, which is larger than the one in [23].

5. Conclusions

In this paper, a new delay-dependent stability criterion for uncertain dynamic systems with time-varying delays was proposed. To obtain a less conservative result, a new Lyapunov functional was proposed and an integral inequality lemma, which includes free variables, was utilized in obtaining an upper bound of the integral term. Through two numerical examples, the effectiveness of the proposed stability criterion was shown.

표 1 예제 1에서 $\lambda=0$일때 h_D값에 따른 안정성을 보장하는 h_U의 상한 값

Table 1 Upper bounds of h_U for guaranteeing stability in Example 1 with $\lambda=0$ and different h_D.

h_D	0	0.1	0.5	0.9	≥1
Ref. [4]	4.472	3.604	2.008	1.180	0.999
Ref. [9]	4.472	3.652	2.008	1.183	0.999
Ref. [23]	-	-	-	-	1.3453
Theorem 1	4.865	3.914	2.191	1.568	1.568

표 2 예제 1에서 $\lambda=0.2$ 및 $h_D=0$ 일때 안정성을 보장하는 h_U의 상한 값

Table 2 Upper bounds of h_U for guaranteeing stability in Example 1 with $\lambda=0.2$ and $h_D=0$.

	Upper bounds of h_U
Ref. [5]	1.77
Ref. [16]	2.39
Ref. [6]	2.3970
Ref. [7]	2.4317
Ref. [18]	2.4390
Ref. [4]	2.7651
Theorem 1	2.9658

표 3 예제 2에서 λ 변화에 따른 안정성을 보장하는 h_U의 상한 값

Table 3 Upper bounds of h_U for guaranteeing stability in Example 1 with different λ.

λ	0.3	0.4	0.5	0.6
Ref. [8]	0.9514	0.7950	0.6426	0.2087
Ref. [17]	1.0243	0.8102	0.6459	0.2097
Ref. [4]	1.1286	0.8809	0.6883	0.3298
Theorem 1	1.2120	0.9222	0.7070	0.3298

표 4 예제 3에서 h_D 변화에 따른 안정성을 보장하는 h_U의 상한 값

Table 4 Upper bounds of h_U for guaranteeing stability in Example 1 with various h_D.

h_D	0	0.5	0.9	≥1
Ref. [18]	1.1623	0.9264	0.6954	-
Ref. [19]	1.1623	0.9322	0.7590	-
Ref. [23]	-	-	-	0.9442
Theorem 1	1.2875	1.1052	1.0459	1.0459

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