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New Delay-dependent Stability Criteria for Uncertain Stochastic Systems with Time-varying Delays

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Abstract - In this paper, the problem of delay-dependent stability of uncertain stochastic systems with time-varying delay is considered. The uncertainties are assumed to be norm-bounded. Based on the Lyapunov stability theory, new delay-dependent stability criteria for the system are derived in terms of LMI(linear matrix inequality). Two numerical examples are given to show the effectiveness of proposed method.

Key Words : Time-varying delays, Linear matrix inequalities, Lyapunov method, Stochastic systems.

1. Introduction

Time-delays frequently occurred in many industrial systems such as chemical processes, network controled systems, large-scale systems, cellular neural networks, synchronization between two chaotic systems, and so on. It is well known delay-dependent stability criteria, which includes the information on the size of delays, are generally less conservative than delay-independent ones. Therefore, many attention has been paid to the stability analysis of systems with time-delays. For example, see [1-8] and references therein.

On the other hand, the stability analysis of stochastic systems with delays have been investigated by many researches since stochastic modeling came to play an important role in many fields of science and engineering applications. In this field, an important index for chencking the conservatism of stability criteria is the maximum delay bound for guaranteeing the asymptotic stability for the concerned system. Li et al [1] studied a delay-dependent and parameter-dependent robust stability criterion for stochastic time-delay systems with polytopic uncertainties. Yang et al [2] improved the stability criteria by fractioning delay intervals. Yue and Won [3] proposed new stability criterion for time-delay stochastic system with nonlinear uncertainties by using the neutral model transformation. Chen et al [4] proposed new delay-dependent stability criteria for stochastic systems with multiple delays by using a descriptor model transformation of the system and by applying Moon's inequality for bounding cross terms. In Yan et al [7] and Zhang et al [8], free weighting matrices are employed to reduce the conservatism of stability criteria for stochastic system with time-varying delays. However, there are still room for further improvement to the stability criteria for stochastic systems with time-varying delays.

In this paper, we propose new improved delay-dependent stability criteria for uncertain stochastic systems with time-varying delays. Additional stochastic perturbations are considered as two cases: 1) trace bounded and 2) linear function and norm-bounded. By constructing a suitable Lyapunov-Krasovskii functional and employing appropriate free weighting matrices, new delay-dependent stability criteria are derived in terms of LMIs which can be solved efficiently by using the interior-point algorithm [9]. To reduce the conservatism of stability criteria, the integral terms obtained by calculating the stochastic differential of Lyaponov-Krasovskii's functionals are divided into two terms with different free weighting matrices. Two numerical examples are given and compared with the very recent ones to show the effectiveness of the proposed method.

Notation : $\mathbf{R}^n$ is the n-dimensional Euclidean space, $\mathbf{R}^{m\times n}$ denotes the set of $m\times n$ real matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices X and Y, the notation $X>Y$ is positive definite, (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents

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the elements below the main diagonal of a symmetric matrix. I is the identity matrix with appropriate dimension. A^T means the transpose of the matrix A. For $h>0$, $C([-h,\ 0], R^n)$ means the family of continuous functions ϕ from $[-h,\ 0]$ to R^n with the norm $\|\phi\| = \sup_{-h \le s \le 0}|\phi(s)|$. Let $(\Omega, F, \{F_t\}_{t \ge 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t \ge 0}$ satisfying the usual conditions (i.e. it is right continuous and F_0 and contains all P-pull sets). $L^p_{F_0}([-h,\ 0], R^n)$ the family of all F_0-measurable $C([-h,\ 0], R^n)$-valued random variables$\xi = \{\xi(\theta) : -h \le \theta \le 0\}$ such that $\sup_{-h \le s \le 0}\mathbf{E}|\xi(\theta)|^p < \infty$ where $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. Denote by $C^{2,1}(R^n \times R^+, R^+)$ the family of all nonnegative functions $V(x,t)$ on $R^n \times R^+$ which are continuously twice differentiable in x and differentiable in t.

2. Problem Statements

Consider the following uncertain stochastic systems with time-varying delays:

$$\begin{aligned} dx(t) &= [(A+\Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-h(t))]dt \\ &\quad + \sigma(t, x(t), x(t-h(t))d\omega(t) \\ x(s) &= \phi(s),\ s \in [-h_U,\ 0]. \end{aligned} \tag{1}$$

Here, $x(t) \in R^n$ is the state vector, $\phi(s) \in C([-h,\ 0], R^n)$ is the initial function, A and A_d are known real constant matrices with appropriate dimensions, $w(t)$ is a m-dimensional Wiener Process (Brownian Motion) on $(\Omega, F, \{F_t\}_{t \ge 0}, P)$ which satisfies $\mathbf{E}\{dw(t)\} = 0$ and $\mathbf{E}\{dw^2(t)\} = dt$. The delays, $h(t)$, are time-varying continuous function that satisfies

$$0 \le h(t) \le h_U,\ \dot{h}(t) \le h_D, \tag{2}$$

where h_U is a positive constant and h_D is any constant one. $\Delta A(t)$, and $\Delta A_d(t)$ are the uncertainties of system matrices of the form:

$$[\Delta A(t) \quad \Delta A_d(t)] = DF(t)[E_1 \quad E_2] \tag{3}$$

in which the time-varying nonlinear function $F(t)$ satisfies

$$F^T(t)F(t) \le I, \quad \forall t \ge 0, \tag{4}$$

and D, E_1, and E_2 are known constant matrices. $\sigma(t, x(t), x(t-h(t))) \in R^{n \times m}$ is the nonlinear uncertainties satisfying

$$\begin{aligned} &trace(\sigma^T(t, x(t), x(t-h(t))\sigma(t, x(t), x(t-h(t))) \\ &\le \|G_1 x(t)\|^2 + \|G_2 x(t-h(t))\|^2 \end{aligned} \tag{5}$$

where G_1, and G_2 are constant matrices with appropriate dimensions.

Now, system (1) can be written as:

$$\begin{aligned} dx(t) &= [-Ax(t) + A_d x(t-h(t)) + Dp_1(t)]dt \\ &\quad + \sigma(t, x(t), x(t-h(t)))dw(t), \\ p_1(t) &= F(t)q_1(t), \\ q_1(t) &= E_1 x(t) + E_2 x(t-h(t)). \end{aligned} \tag{6}$$

Before deriving our results, we state the following facts, lemma and definition.

Fact 1. (Schur complement) Given constant matrices Σ_1, Σ_2, Σ_3 where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Fact 2. For any real vectors a, b and any matrix $Q>0$ with appropriate dimensions, it follow that:

$$2a^T b \le a^T Q a + b^T + b^T Q^{-1} b.$$

Definition 1. For the uncertain stochastic systems (1) and every $\phi \in L^2_{F_0}([-h,\ 0], R^n)$, the trivial solution is asymptotically stable in the mean square if, for all admissible uncertainties,

$$\lim_{t \to \infty} \mathbf{E}|x(t, \phi)|^2 = 0$$

In deriving our main results, Itô's formula plays a key role in stability analysis of stochastic systems (See [10-12] for details).

3. Main results

In this section, we propose new delay-dependent stability criteria for uncertain stochastic systems with time-varying delays described by (6).

Before introducing Theorem 1, we define the following notations for simplicity:

$$\begin{aligned}
&\zeta_1^T(t) = [x^T(t)\ \ x^T(t-h(t))\ \ x^T(t-h_U)\ \ y^T(t)\ \ \ p_1^T(t)], \\
&\Sigma_1 = [\Sigma_{1(i,j)}],\ i,j = 1,...,5, \\
&\Sigma_{1(1,1)} = \rho G_1^T G_1 + N + R_2 + \varepsilon E_1^T E_1 + P_1 A + A^T P_1^T, \\
&\Sigma_{1(1,2)} = \varepsilon E_1^T E_2 + P_1 A_d,\ \Sigma_{1(1,3)} = 0,\ \Sigma_{1(1,4)} = R_1 - P_1 + P_2 A, \\
&\Sigma_{1(1,5)} = P_1 D,\ \Sigma_{1(2,2)} = \rho G_2^T G_2 - (1-h_D)R_2 + \varepsilon E_2^T E_2, \\
&\Sigma_{1(2,3)} = 0,\ \Sigma_{1(2,4)} = A_d^T P_2^T,\ \Sigma_{1(2,5)} = 0,\ \Sigma_{1(3,3)} = -N, \\
&\Sigma_{1(3,4)} = 0,\ \Sigma_{1(3,5)} = 0,\ \Sigma_{1(4,4)} = h_u Q - P_2 - P_2^T,\ \Sigma_{1(4,5)} = P_2 D, \\
&\Sigma_{1(5,5)} = -\varepsilon I, \\
&X_1 = [X_1^T \quad X_2^T \quad X_3^T \quad 0 \quad 0]^T,\ Y_1 = [Y_1^T \quad Y_2^T \quad Y_3^T \quad 0 \quad 0]^T, \\
&\Gamma_1 = [X_1 \quad -X_1 + Y_1 \quad -Y_1 \quad 0 \quad 0],\ \Xi_1 = [E_1 \quad E_2 \quad 0 \quad 0 \quad 0].
\end{aligned} \tag{7}$$

Now, we have the following Theorem 1.

Theorem 1 For given $h_U > 0$, h_D, and the stochastic system (6) is asymptotically stable in the mean square if there exist positive definite matrices $R_i (i=1,2)$, N, Q, a positive scalar ε, ρ, any matrices X_i, $Y_i (i=1,2,3)$, and $P_i (i=1,2)$ satisfying the following LMIs:

$$R_1 \le \rho I \tag{8}$$

$$\begin{bmatrix} \Sigma_1+\Gamma_1+\Gamma_1^T & h_U \boldsymbol{Y_1} \\ \star & -h_U Q \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} \Sigma_1+\Gamma_1+\Gamma_1^T & h_U \boldsymbol{X_1} \\ \star & -h_U Q \end{bmatrix} < 0. \tag{10}$$

where Σ_1, Γ_1, $\boldsymbol{X}_1$ and $\boldsymbol{Y}_1$ are defined in Eq.(7).

Proof. First of all, let us define

$$y(t) = Ax(t) + A_d x(t-h(t))) + Dp_1(t),$$
$$g(t) = \sigma(t, x(t), x(t-h(t))). \tag{11}$$

Then, system (6) can be described as

$$dx(t) = y(t)dt + g(t)d\omega(t). \tag{12}$$

From [11], the following two equations

$$z_1(t): x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} y(s)ds - \int_{t-h(t)}^{t} g(s)dw(s) = 0,$$
$$z_2(t): x(t-h(t)) - x(t-h_U) - \int_{t-h_U}^{t-h(t)} y(s)ds - \int_{t-h_U}^{t-h(t)} g(s)dw(s) = 0. \tag{13}$$

hold.

For positive definite matrices R_1, R_2, N, Q, let us consider the Lyapunov-Krasovskii functional candidate:

$$V(x(t),t) = \sum_{i=1}^{3} V_i(x(t),t) \tag{14}$$

where

$$V_1(x(t),t) = x^T(t)R_1x(t) + \int_{t-h_U}^{t} x^T(s)Nx(s)ds,,$$
$$V_2(x(t),t) = \int_{t-h(t)}^{t} x^T(t)R_2(s)ds$$
$$V_3(x(t),t) = \int_{t-h_U}^{t}\int_{s}^{t} y^T(u)Qy(u)duds.$$

Then, by the weak infinitesimal operator $\mathcal{L}$, $\mathcal{L}V_i(x(t),t)(i=1,...,4)$ can obtained as

$$\mathcal{L}V_1(x(t),t) = 2x^T(t)R_1y(t) + trace(g^T(t)R_1g(t)) + x^T(t)Nx(t) - x^T(t-h_U)Nx(t-h_U),$$
$$\mathcal{L}V_2(x(t),t) = x^T(t)R_2x(t) - (1-\dot{h}(t))x^T(t-h(t))R_2x(t-h(t)),$$
$$\mathcal{L}V_3(x(t),t) = h_Uy^T(t)Qy(t) - \int_{t-h_U}^{t} y^T(s)Qy(s)ds. \tag{15}$$

If the inequality (5) holds, then

$$trace(g^T(t)R_1g(t)) \le \rho trace(g^T(t)g(t)) \le \rho(x^T(t)G_1^TG_1x(t) + x^T(t-h(t))G_2^TG_2x(t-h(t))). \tag{16}$$

Note that

$$-\int_{t-h_U}^{t} y^T(s)Qy(s)ds = -\int_{t-h(t)}^{t} y^T(s)Qy(s)ds - \int_{t-h_U}^{t-h(t)} y^T(s)Qy(s)ds. \tag{17}$$

By utilizing Fact 2 and Eq.(12), the integral term $-\int_{t-h(t)}^{t} y^T(s)Qy(s)ds$ can be estimated as

$$-\int_{t-h(t)}^{t} y^T(s)Qy(s)ds$$
$$\le h(t)\zeta_1^T(t)\boldsymbol{X_1}Q^{-1}\boldsymbol{X}_1^T\zeta_1(t) + 2\zeta_1^T(t)\boldsymbol{X_1}\int_{t-h(t)}^{t} y(s)ds$$
$$= h(t)\zeta_1^T(t)\boldsymbol{X_1}Q^{-1}\boldsymbol{X}_1^T\zeta_1(t) + 2\zeta_1^T(t)\boldsymbol{X_1}[x(t) - x(t-h(t))]$$
$$-2\zeta_1^T(t)\boldsymbol{X_1}\int_{t-h(t)}^{t} g(s)d\omega(s) \tag{18}$$

Similarly, an upper bound of the term $-\int_{t-h_U}^{t-h(t)} y^T(s)Qy(s)ds$ can be

$$-\int_{t-h_U}^{t-h(t)} y^T(s)Qy(s)ds$$
$$\le (h_U - h(t))\zeta_1^T(t)\boldsymbol{Y_1}Q^{-1}\boldsymbol{Y}_1^T\zeta_1(t)$$
$$+2\zeta_1^T(t)\boldsymbol{Y_1}[x(t-h(t)) - x(t-h_U)] - 2\zeta_1^T(t)\boldsymbol{Y_1}\int_{t-h_U}^{t-h(t)} g(s)d\omega(s). \tag{19}$$

Since the following inequality holds from (3), (4) and (6),

$$p_1^T(t)p_1(t) \le q_1^T(t)q_1(t) = \zeta_1^T(t)\Xi_1^T\Xi_1\zeta_1(t), \tag{20}$$

there exist a positive scalar ε satisfying the following inequality

$$\varepsilon[\zeta_1^T(t)\Xi_1^T\Xi_1\zeta_1(t) - p_1^T(t)p_1(t)] \ge 0, \tag{21}$$

where Ξ_1 is defined in (7).

As a tool of reducing the conservatism of stability criterion, we add the following zero equation with any matrices P_1 and P_2:

$$0 = 2[x^T(t)P_1 + y^T(t)P_2][-y(t) + Ax(t) + A_dx(t-h(t) + Dp_1(t)]. \tag{22}$$

From (10)-(22) and by applying S-procedure [9], the $\mathcal{L}V(x(t),t) = \sum_{i=1}^{3} V_i(x(t),t)$ has a new upper bound as

$$\mathcal{L}V(x(t),t) \le \zeta_1^T(t)(\Sigma_1 + \Gamma_1 + \Gamma_1^T + h(t)\boldsymbol{X_1}Q^{-1}\boldsymbol{X}_1^T + (h_U - h(t))\boldsymbol{Y_1}Q^{-1}\boldsymbol{Y}_1^T)\zeta_1(t) + \xi(d\omega(t) \tag{23}$$

where

$$\xi(d\omega(t) - 2\zeta_1^T(t)\boldsymbol{X_1}\int_{t-h(t)}^{t} g(s)d\omega(s) - 2\zeta_1^T(t)\boldsymbol{Y_1}\int_{t-h_U}^{t-h(t)} g(s)d\omega(s).$$

Let us define

$$\Omega(h(t)) = \Sigma_1 + \Gamma_1 + \Gamma_1^T + h(t)\boldsymbol{X_1}Q^{-1}\boldsymbol{X_1^T} + (h_U - h(t))\boldsymbol{Y_1}Q^{-1}\boldsymbol{Y_1^T}. \tag{24}$$

Since $\Omega(h(t))$ is a convex combination of the matrices $\boldsymbol{X}_1Q^{-1}\boldsymbol{X}_1^T$ and $\boldsymbol{Y}_1Q^{-1}\boldsymbol{Y}_1^T$, $\Omega(h(t)) < 0$ can be handled by the two LMIs $\Omega(0) < 0$ and $\Omega(h_U) < 0$. Using Fact 1, these two LMIs are equivalent to the LMIs (9) and (10) in Theorem 1. Note that $\boldsymbol{E}\{\xi(\mathrm{d}\omega(\mathrm{t})\} = 0$ [5].

Therefore, if LMIs (8)-(10) hold, by taking the mathematical expectation on both side of (23), there exists a positive scalar γ satisfying

$$\mathbf{E}\{\mathcal{L}V(x(t),t)\} \le \mathbf{E}\{\zeta_1^T(t)\Omega(h(t))\zeta_1(t)\} \le -\gamma\mathbf{E}|x(t)|^2. \tag{25}$$

The obtained inequality (25) indicates that the system (6) is asymptotically stable in the mean square. This complete our proof. ■

Remark 1. The solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem. In this paper, we utilize Matlab's LMI Control Toolbox [13] which implements the interior-point algorithm. This algorithm is faster than classical convex optimization algorithms [9].

Remark 2. By iteratively solving the LMI given in Theorem 1 with respect to h_U for h_D, one can find the maximum upper bound of time delay h_U for guaranteeing asymptotic stability of system (6).

Remark 3. If V_2 is not considered in Theorem 1, the obtained stability criterion do not need the information of the upper bound of time-derivative of delays. Thus, Theorem 1 without V_2 can be applicable to the uncertain stochastic system with fast time-varying delays.

For a special case, if $\sigma(t,x(t),x(t-h(t))$ is a linear function, that is, system (1) has the following form:

$$\begin{aligned} dx(t) &= [Ax(t)+A_d x(t-h(t))+Dp_1(t)]dt \\ &\quad +[Hx(t)+H_d x(t-h(t))+Dp_2(t)]dw(t), \\ p_1(t) &= F(t)q_1(t), \\ p_2(t) &= F(t)q_2(t), \\ q_1(t) &= E_1 x(t)+E_2 x(t-h(t)). \\ q_2(t) &= E_3 x(t)+E_4 x(t-h(t)). \end{aligned} \tag{26}$$

For the system (26), a delay-dependent stability criterion can be obtained by considering the same Lyapunov-Krasovskii's functional and using the similar method presented in the proof of Theorem 1, which will be shown as Corollary 1. We also introduce the following notations for simplicity:

$$\begin{aligned} &\zeta_2^T(t) = \big[x^T(t)\ \ x^T(t-h(t))\ \ x^T(t-h_U) \\ &\qquad y^T(t)\quad g^T(t)\quad p_1^T(t)\quad p_2^T(t)\big], \\ &\Sigma_2 = [\Sigma_{2(i,j)}],\ i,j=1,\dots,7, \\ &\Sigma_{2(1,1)} = N+R_2+\varepsilon_1 E_1^T E_1+\varepsilon_2 E_3^T E_3+P_1 A+A^T P_1+P_3 H+H^T P_3^T, \\ &\Sigma_{2(1,2)} = \varepsilon_1 E_1^T E_2+\varepsilon_2 E_3^T E_4+P_1 A_d+P_3 H_d,\ \Sigma_{2(1,3)}=0, \\ &\Sigma_{2(1,4)} = R_1-P_1+P_2 A,\ \Sigma_{2(1,5)} =-P_3+H^T P_4^T,\ \Sigma_{2(1,6)}=P_1 D, \\ &\Sigma_{2(1,7)} = P_3 D,\ \Sigma_{2(2,2)} =-(1-h_D)R_2+\varepsilon_1 E_2^T E_2+\varepsilon_2 E_4^T E_4, \\ &\Sigma_{2(2,3)}=0,\ \Sigma_{2(2,4)}=A_d^T P_2^T,\ \Sigma_{2(2,5)}=H_d^T P_4^T,\ \Sigma_{2(2,6)}=0, \\ &\Sigma_{2(2,7)}=0,\ \Sigma_{2(3,3)}=-N,\ \Sigma_{2(3,4)}=0,\ \Sigma_{2(3,5)}=0,\ \Sigma_{2(3,6)}=0, \\ &\Sigma_{2(3,7)}=0,\ \Sigma_{2(4,4)}=h_u Q-P_2-P_2^T,\ \Sigma_{2(4,5)}=0,\ \Sigma_{1(4,6)}=P_2 D, \\ &\Sigma_{2(4,7)}=0,\ \Sigma_{2(5,5)}=R_1-P_4-P_4^T,\ \Sigma_{2(5,6)}=0, \\ &\Sigma_{2(5,7)}=P_4 D,\ \Sigma_{2(6,6)}=-\varepsilon_1 I,\ \Sigma_{2(6,7)}=0,\ \Sigma_{1(7,7)}=-\varepsilon_2 I \\ &\boldsymbol{X}_2 = \begin{bmatrix} X_1^T & X_2^T & X_3^T & 0 & 0 & 0 & 0\end{bmatrix}^T, \\ &\boldsymbol{Y}_2 = \begin{bmatrix} Y_1^T & Y_2^T & Y_3^T & 0 & 0 & 0 & 0\end{bmatrix}^T, \\ &\Gamma_2 = \begin{bmatrix} \boldsymbol{X}_2 & -\boldsymbol{X}_2+\boldsymbol{Y}_2 & -\boldsymbol{Y}_2 & 0 & 0 & 0 & 0\end{bmatrix}, \end{aligned} \tag{27}$$

Then, we have the following Theorem 2.

Theorem 2 For given $h_U>0$, h_D, and the stochastic system (28) is asymptotically stable in the mean square if there exist positive definite matrices $R_i(i=1,2)$, N, Q, positive scalars $\varepsilon_i(i=1,2)$, any matrices X_i, $Y_i(i=1,2,3)$, and $P_i(i=1,\dots,4)$ satisfying the following LMIs:

$$\begin{bmatrix} \Sigma_2+\Gamma_2+\Gamma_2^T & h_U \boldsymbol{Y}_2 \\ \star & -h_U Q \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} \Sigma_2+\Gamma_2+\Gamma_2^T & h_U \boldsymbol{X}_2 \\ \star & -h_U Q \end{bmatrix} < 0. \tag{29}$$

Proof. Let us define $g(t)=Hx(t)+H_d x(t-h(t))+Dp_2(t)$ and consider the same Lyapunov-Krasovskii functional (13). With the following zero equation

$$0=2[x^T(t)P_3+g^T(t)P_4][-g(t)+Hx(t)+H_d x(t-h(t)+Dp_2(t)], \tag{30}$$

augmentation vector $\zeta_2(t)$, and similar method in the proof of Theorem 1, we can easily show that if the two LMIs (28) and (29) hold, then the stochastic system (26) is asymptotically stable in the mean square. This complete our proof. ■

Remark 4. In Theorem 2, we used $g(t)$ as augmented variables and added Eq.(30) to $\mathcal{L}V(x(t),t)$ to reduce the conservatism of stability criteria, which is not considered in [7-8]. This additional equation may lead to give less conservatism of stability criteria. Through Example 2, we will show Theorem 2 gives improved results than the ones [7-8].

4. Numerical Example

Example 1. Consider the stochastic system (6) with

$$\begin{aligned} &A=\begin{bmatrix}-2 & 0\\ 1 & -1\end{bmatrix},\quad A_d=\begin{bmatrix}-1 & 0\\ -0.5 & -1\end{bmatrix},\ H=0,\ H_d=0, \\ &D=I,\ E_1=E_2=0.1I,\ G_1=G_2=\sqrt{0.1}\,I,\ h(t)=h_U. \end{aligned} \tag{31}$$

By applying Theorem 1 to the system (31), one can obtain the maximum delay bound h_U as 1.9576. Table 1 lists the maximum delay bounds obtained by the previous methods. From Table 1, one can see that Theorem 1 provides larger delay bounds than the ones in [2-6].

표 1 예제 1에서 안정성을 보장하는 h_U의 상한 값

Table 1 Delay bounds h_U in Example 1.

Methods	h_U
Yue et al [3].	0.8635
Yang et al [2].	1.1
Chen et al [4].	1.3640
Xu et al [5].	1.5270
Gershon et al [6]	1.56
Theorem 1	2.1491

Example 2. Consider the uncertain stochastic neural networks (26) with time-invariant delay

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\ H=0,\ H_d=0,$$

$$D=0.2I,\ E_1=E_2=E_3=E_4=I,\ h(t)=h_U, h_D=0. \qquad (32)$$

By applying Theorem 2 to the above system, the maximum delay bounds of $h(t)$ with different values of h_D are shown in Table 2. "-" means that the result is not applicable to the condition $h_D=1$. From Table 2, Theorem 2 give larger delay bounds than the ones in [7-8] under different values of h_D .

표 2 예제 1에서 다양한 h_D값에 따른 안정성을 보장하는 h_U의 상한 값

Table 2 Delay bounds h_U with different values of h_D in Example 2.

h_D	0.3	0.5	0.9	1
Yan et al [7].	0.7288	0.5252	0.1489	-
Zhang et al [8]	1.2950	1.1006	0.9434	0.9424
Theorem 2	1.5476	1.3565	1.1529	1.1440

5. Conclusion

In this paper, the problem of the delay-dependent stability for uncertain stochastic systems with time-varying delays is considered. A linear matrix inequality approach has been proposed to derive the new improved delay-dependent stability criteria. Through two numerical examples, the superiority of our results are shown.

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